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Loss Aversion and Wealth Allocation

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Abstract

We study how the wealth-allocation decisions and the loss aversion of non-professional investors change subject to behavioral factors. The optimal wealth assignment between risky and risk-free assets results within a VaR portfolio model, where risk is individually assessed according to an extended prospect-theory framework. We show how the past performance and the portfolio evaluation frequency impact investor behavior. Myopic loss aversion holds at different evaluation frequencies. One year is the optimal frequency at which, under practical constraints, risky holdings are maximized. Previous research using standard VaR-significance levels may underestimate the loss aversion of individual investors.

Keywords: prospect theory, myopic loss aversion, Value-at-Risk, portfolio evaluation, capital allocation

JEL Classification: G10, G11, D81, E27

1 Introduction

The main concern of investors in financial markets is how to optimally allocate money among different (types of) assets. Portfolio theory teaches us that the optimal allocation results from the maximization of expected portfolio returns subject to given levels of market risk. In spite of the appealing intuition and mathematical tractability, such an optimization is not an easy task, especially for laymen. The reason is that it involves the selection from a huge variety and quantity of financial instruments existent in practice, and it often requires advanced mathematical skills. The natural response of real financial environments to this difficulty has been the specialization of the investment activity between professionals and non-professionals. Non-professional investors – in other words people whose main occupation does not concern financial investments and/or who lack the necessary knowledge, expertise, time or any combination of them for making more sophisticated investment decisions – rely often on the help of professional portfolio managers in devising an optimal mix of risky assets. In other words, they often delegate the security and asset allocation to professional managers.[1]

In particular, one can think of the decision process of non-professional investors as unfolding in two main steps: First, they determine the total sum of money to be invested in financial markets (in technical terms the budget constraint). Second, in order to optimally split this money among different financial instruments, they ask for professional advice. In so doing, they commit the technical details of the optimization of their asset portfolio to professional managers, who dispose of sufficient resources to this end. Moreover, non-professional investors provide managers with information about the level of risk they are ready to bear (the risk constraint). Acting on this information, managers finally derive the optimal capital allocation for their particular clients. What is important for non-professionals in this context is simply how their wealth can be (optimally) split between

[1]In essence, this practical tendency of work division between non-professionals and professionals conforms with portfolio theory. According to the top-down strategy, portfolio optimization can be described by means of a threefold decision procedure: A first step, referred to as the *capital allocation decision*, deals with the choice between risky and risk-free assets. A second so-called *asset allocation decision* focuses on the further selection of different classes of risky assets. The third *security allocation decision* is concerned with the specific securities to be held within each particular risky asset class chosen before. In practice, the last two decisions are usually made by professional portfolio managers with no intervention of their (non-professional) clients. By contrast, as far as the first decision is concerned the participation of these non-professionals becomes necessary, since it allows managers to determine the capital allocation that best fits the individual risk-profiles of their clients.

risky and risk-free assets.

It is the *decision process of the non-professional investors* that our paper focuses on. This process – although of indisputable practical importance – has been somewhat neglected in the literature sofar. The extensive research on portfolio optimization deals with more sophisticated details, such as of choosing among different categories of risky assets, that we consider to usually be the responsibility of the portfolio managers.

In particular, we are interested in how non-professionals split their money between risky and risk-free assets. Since this decision depends on the individual risk profile, we also study the investors' attitude towards financial losses. Note that our work does not contribute to the understanding of professional investors' decisions, but gives insight into how non-professional investors "operate" on financial markets. In our setting, non-professional investors start from questioning what is their acceptable monetary loss from risky investments. This information depends on individual risk profiles that affect the quantity of money that they are going to invest in risky assets. Also, the frequency of evaluating risky portfolio changes the risk profile and hence their overall performance.

Our paper extends the portfolio optimization setting in Campbell, Huisman, and Koedijk (2001), where risk is quantified in form of Value-at-Risk, by explicitly accounting for the formation of what we denote as the *individual VaR* (VaR*). Our VaR* relies on subjective perceptions of the non-professional investors and is formulated in line with the extended prospect theory in Barberis, Huang, and Santos (2001).

We first analyze how non-professional investors set their subjective VaR*, specifically contingent upon their loss aversion, the past performance of the risky portfolio, the current value of the risky investment, and the expected risk premium. We show how VaR* flows into the portfolio optimization undertaken by the professional manager in form of a risk constraint. We derive the optimal wealth percentages to be invested in the risky portfolio and in risk-free assets and study how they vary in time and subject to different portfolio evaluation frequencies. Furthermore, we introduce an extended measure, termed as the global first-order risk aversion (gRA), that attempts to provide additional information concerning the loss attitude of non-professional investors. We comment on how the frequency of evaluating risky performance can directly and indirectly impact on the investors' attitude towards risky investments and on how this twofold influence can be estimated.

Our theoretical results are supported and amended by findings relying on the S&P 500 index and the US three-month treasury bills between 1982-2006. The past performance appears to drive the current perception of the risky portfolio. Investors allocate on average between 0-35% of their wealth to risky assets, where the main source of this substantial fluctuation is the portfolio evaluation frequency. The proportion of risky investments decreases fast when portfolio performance is checked more often than once a year, which complies with the notion of myopic loss aversion introduced in Benartzi and Thaler (1995).

Furthermore, we conduct an extended analysis of the perceived utility of the risky portfolio and of the loss attitude in what we denote as the evaluation-frequency domain. Specifically, we propose a twofold segmentation in dependence on the portfolio evaluation frequency of both the prospective value and the global first-order risk aversion. Merely evaluation frequencies higher than one year are of practical relevance. In this context, both variables suggest the frequency of one year as being optimal for generating positive attitudes towards risky investments.

Finally, variables aimed at providing an equivalence between the traditional VaR-approach and the estimates in our VaR*-framework – such as equivalent significance levels, loss aversion coefficients, and investments in risky assets – point out that the actual reluctance towards financial losses of non-professional investors might be underestimated.

The remainder of the paper is organized as follows. Section 2 presents the main theoretical considerations. We start with a brief review of the optimal portfolio selection model with exogenous VaR* by Campbell, Huisman, and Koedijk (2001), then take on the value function formulation in Barberis, Huang, and Santos (2001). The notion of VaR* is subsequently introduced. Finally, concentrating on how individual investors perceive the value of the risky portfolio, we derive the prospective value and introduce our extended measure of loss aversion. Section 3 illustrates the implementation of our theoretical model. We discuss the impact of the evaluation frequency and of the past performance on the wealth percentages invested in the risky portfolio. Also, we extensively investigate the evolution of the prospective value and of the extended loss-attitude measure subject to various evaluation frequencies. Our model is further restated in terms of previous research with VaR risk constraints. Section 4 summarizes the results and concludes. Graphs and further findings are included in the Appendix.

2 Theoretical model

This section contains the main theoretical considerations of our work. We start by reviewing the portfolio selection model in Campbell, Huisman, and Koedijk (2001). This model uses VaR as its measure of risk. Our own setting, subsequently formulated, incorporates the individual perception of risky projects as captured in the extended prospect theory framework of Barberis, Huang, and Santos (2001). We detail the construction of our measure of individual loss aversion VaR* and its implications for the wealth allocation decisions of non-professional investors. We also add to the formal representation of the investors' attitude towards financial losses by introducing the notion of global first-order risk aversion (gRA). Moreover, we briefly address how the prospective value and this extended loss-attitude measure may vary subject to different portfolio evaluation frequencies.

2.1 Optimal portfolio selection with "exogenous" VaR

The model in Campbell, Huisman, and Koedijk (2001) follows the common procedure of portfolio optimization, where market risk is assessed by means of Value-at-Risk (abbr. VaR). In particular, financial assets are chosen in order to maximize expected returns, subject to a twofold restriction: the budget and risk constraints. Investors can borrow or lend extra money at the fixed market interest rate – which is equivalent with an investment in risk-free assets. The maximum expected loss from holding the risky portfolio should not exceed what we call an *exogenous* VaR (abbr. VaR^{ex}). This VaR^{ex} stands for the risk level that the non-professional client is disposed to bear. It is indicated to the portfolio manager in form of a single, fixed number.[2] In this model, managers do *not* account for how VaR^{ex} forms in the client perception. They consider it as constraint, exogenous to the optimization problem.[3]

The objective of the optimization problem in Campbell, Huisman, and Koedijk (2001)

[2]The VaR^{ex} further enters the portfolio optimization problem in form of a threshold level, being thus *exogenous* to it.

[3]Specifically, managers interpret the client indication (a single number) in terms of the theoretical concept of VaR, i.e. of two elements: a confidence level and an investment horizon.

is maximizing the next-period wealth W_{t+1}. This wealth results from what the components of the risky portfolio and the risk-free assets are expected to return. The risky portfolio consists of $i = 1, \ldots, n$ financial assets with single time-t prices $p_{i,t}$ and portfolio weights $w_{i,t}$, such that $\sum_{i=1}^{n} w_{i,t} = 1$. Moreover, $a_{i,t}$ is the number of shares of the asset i contained in the portfolio at time t.[4] Formally, we can state the portfolio optimization problem as follows:

$$W_{t+1}(w_t) = (W_t + B_t)E_t[R_{t+1}(w_t)] - B_t R_f \underset{w_t}{\longrightarrow} \max. \tag{1}$$

$$\text{s.t.} \quad \begin{aligned} & W_t + B_t = \sum_{i=1}^{n} a_{i,t} p_{i,t} = a_t' p_t \quad \text{(budget constraint)} \\ & P_t[W_{t+1}(w_t) \leq W_t - \mathrm{VaR}^{\mathrm{ex}}] \leq 1 - \alpha \quad \text{(risk constraint)}, \end{aligned} \tag{2}$$

where $R_{t+1}(w_t)$ stands for the portfolio gross returns at the next trade and $E_t[R_{t+1}(w_t)]$ for the corresponding expected returns. Henceforth, we refer to the gross returns of the risky portfolio by "returns" or "portfolio returns" .

In the above Equations (1) and (2), B_t denotes the risk-free investment, i.e. the sum of money that can be borrowed ($B_t > 0$) or lent ($B_t < 0$) at the fixed risk-free gross return rate R_f. Note that the maximization in Equation (1) is carried over the weights of the risky portfolio w_t but *not* over B_t. The risk-free investment results as a by-product of the optimization procedure.[5] Finally, P_t denotes the conditional probability given the information at time t, and $1 - \alpha$ the chosen confidence level.

After some manipulations, Campbell, Huisman, and Koedijk (2001) obtain the optimal weights of the risky portfolio as:

$$w_t^{\mathrm{opt}} \equiv \arg\max_{w_t} \frac{E_t[R_{t+1}(w_t)] - R_f}{W_t R_f - W_t q_t(w_t, \alpha)}, \tag{3}$$

where $q_t(w_t, \alpha)$ represents the quantile of the distribution of portfolio gross returns $R_{t+1}(w_t)$ for the confidence level $1 - \alpha$ (or significance α), i.e. $P_t[R_{t+1}(w_t) \leq q_t(w_t, \alpha)] \leq 1 - \alpha$. Thus, the optimal mix of risky assets depends merely on the distribution of the portfolio

[4] Clearly, $a_{i,t} = w_{i,t}(W_t + B_t)/p_{i,t}$.

[5] See the comments concerning the two-fund separation below.

gross returns and on the significance level α.

Equation (3) shows that, similarly to the traditional mean-variance framework, the *two-fund separation theorem* applies: Neither the (non-professional) investors' initial wealth nor the desired risk level VaR^{ex} affects the maximization procedure. In other words, investors first determine the optimal risky portfolio (i.e. the optimal allocation among different risky assets) and second, they decide upon the extra amount of money to be borrowed or lent (i.e. invested in risk-free assets). The latter reflects by how much the portfolio VaR, that is defined as:

$$\text{VaR}_t = W_t\Big(q_t(w_t^{\text{opt}}, \alpha) - 1\Big), \tag{4}$$

varies according to the investor degree of loss aversion measured by the selected (desired) VaR^{ex}-level.[6]

The optimal investment in risk-free assets can be then written as:

$$B_t = \frac{\text{VaR}^{\text{ex}} + \text{VaR}_t}{R_f - q_t(w_t^{\text{opt}}, \alpha)}, \tag{5}$$

and hence the value of the risky investment at time $t+1$ yields:

$$S_{t+1} = (W_t + B_t)R_{t+1}. \tag{6}$$

Since we consider that non-professionals are mainly concerned with how to split their money between risky and risk-free assets, the optimal investments in risk-free and risky assets in Equations (5) and (6) represent fundamental variables in our model. Note that we do not further elaborate on the optimal weights of the risky assets in Equation (3), as the details of wealth allocation among the different risky portfolio components are assumed to be left in charge of portfolio managers.

[6]Note that VaR^{ex} is imposed by the client *prior* to the portfolio formation and enters the portfolio optimization problem in form of a constraint. By contrast, the portfolio VaR is an *output* of this optimization and measures the actual maximum loss that can be incurred at time t at the confidence level $1-\alpha$ for the obtained optimal portfolio w_t^{opt}.

2.2 The individual loss level VaR*

Coming from the main ideas of the setting in Campbell, Huisman, and Koedijk (2001), our model goes a step further by asking how non-professional investors actually arrive at their desired level of loss aversion. We elaborate on the construction of an *individual loss level*, that we denote as VaR*, and on its implications for the wealth allocation between risky and risk-free assets. As far as the optimization procedure presented in the above Section 2.1 is concerned, we can think of VaR* formally replacing VaR^{ex} in the above equations, but remaining an exogenous input (constraint). However, the value of this risk constraint forms now, according to our approach, on the basis of individual behavioral parameters and affects the final wealth allocation between risky and risk-free assets, as apparent from Equation (5). This extension of the allocation problem[7] motivates us to denote VaR* as the *endogenous* individual loss level.

2.2.1 The value function

Investors' desires and attitudes – hence their subjective level VaR* – depend on their perception of the value of financial investments. The prospect theory (abbr. PT) in Kahneman and Tversky (1979) and Tversky and Kahneman (1992) suggests how individual perceptions of financial performance can be formalized by means of the so-called *value function* v.[8] Accordingly, human minds take for actual carriers of value not the absolute outcomes of a project, but their changes defined as departures from an individual reference point. The deviations above (below) this reference are labeled as gains (losses). Thus, the value function is kinked at the reference point and exhibits distinct profiles in the domains of gains and losses, being steeper for losses (a property known as *loss aversion*). It also shows diminishing sensitivity in both domains, i.e. it is concave for gains but convex for losses.

As noted in Barberis, Huang, and Santos (2001), individual perceptions can be addi-

[7] Now, the allocation problem incorporates not only the portfolio optimization in the strict sense, as performed by managers, but also the earlier decision of non-professional investors with respect to the desired risk level.

[8] Note that the concepts on which we base our setting are fully elaborated only in the cumulative prospect theory (CPT) of Tversky and Kahneman (1992). Since we are not particularly interested in the formal details and most of these concepts are already present in the original PT in Kahneman and Tversky (1979), we refer to both theories as PT.

tionally influenced by the past performance of risky investments. This past performance is captured by the *cushion* concept. Formally, the cushion corresponds to the difference between the current value of the risky investment S_t and a historical benchmark level of the risky value Z_t (that can e.g. be the price at which the assets were purchased, a more recent value of the risky holdings, or a combination of them).[9] When this difference is positive, investors made money from investing in risky assets in the past, otherwise they made losses.

Our approach relies on the extended formulation of the value function proposed in Equations (15) and (16) by Barberis, Huang, and Santos (2001). In the following, we refer to $x_t = R_{t+1} - R_{ft}$ as the *risk premium*, to $S_t - Z_t$ as the (absolute) *cushion*, and to $z_t = Z_t/S_t$ as the *relative cushion*. The positive (negative) past performance corresponds to a positive (negative) cushion that can be termed as $Z_t \leq S_t$ ($Z_t > S_t$) or equivalently as $z_t \leq 1$ ($z_t > 1$). The value function takes different courses in dependence on the past performance and can be expressed as follows:[10]

$$v_{t+1} = \begin{cases} v_{t+1}^{\text{prior gains}} & \text{, for } z_t \leq 1 \\ v_{t+1}^{\text{prior losses}} & \text{, for } z_t > 1, \end{cases} \tag{7}$$

where:

$$v_{t+1}^{\text{prior gains}} = \begin{cases} S_t x_{t+1} & \text{, for } x_{t+1} + (1 - z_t)R_{ft} \geq 0 \\ \lambda S_t x_{t+1} + (\lambda - 1)(S_t - Z_t)R_{ft} & \text{, for } x_{t+1} + (1 - z_t)R_{ft} < 0, \end{cases} \tag{8}$$

and

$$v_{t+1}^{\text{prior losses}} = \begin{cases} S_t x_{t+1} & \text{, for } x_{t+1} \geq 0 \\ \lambda S_t x_{t+1} + k(Z_t - S_t)x_{t+1} & \text{, for } x_{t+1} < 0. \end{cases} \tag{9}$$

The parameter λ in Equations (8) and (9) is termed the *coefficient of loss aversion*. According to PT, investors are loss averse when $\lambda > 1$, while $\lambda = 1$ points to loss neutrality. The parameter $k \geq 0$ captures the influence of previous losses on the perception of current

[9]For more details with respect to the interpretation of Z_t see Barberis, Huang, and Santos (2001), p. 9.

[10]Where we restate the term in the condition of Equation (15) by Barberis, Huang, and Santos (2001) as: $R_{t+1} - z_t R_{ft} = x_{t+1} + (1 - z_t)R_{ft}$.

ones: The larger the previous losses are, the more painful the next losses become. We denote it as the *sensitivity to past losses*.

Note that the gain branches of both value functions in Equations (8) and (9) are invariable to the past performance z_t. The loss branches are yet distinct. However, they both contain a first term $\lambda S_t(R_{t+1} - R_{ft})$ that resembles the original PT, but also a second one revealing the impact of the cushion $S_t - Z_t$. Also, the reference point[11] shifts in dependence on the past performance.

Henceforth, we use the following probability notations:

$$\begin{aligned} \pi_t &= P_t(z_t \leq 1) \\ \omega_t &= P_t(x_{t+1} \geq 0 | z_t > 1) \\ \psi_t &= P_t(x_{t+1} + (1 - z_t)R_{ft} \geq 0 | z_t \leq 1), \end{aligned} \tag{10}$$

where π_t stands for the probability of past gains, and ω_t for the probability of a positive premium given past losses. Finally, we can term ψ_t as the probability of obtaining a return premium $x_{t+1} + (1 - z_t)R_{ft}$, higher than the risk premium x_{t+1}, that expresses raised expectations resulting from recurrent gains.

2.2.2 The derivation of VaR*

In Equation (5), the risk-free investment depends, among others, on the risk level VaR^{ex} indicated by the non-professional client to the portfolio manager. The traditional approach does not account for the way in which non-professionals ascertain this level. This ascertainment should take place according to individual perceptions of financial losses which can, in line with PT, substantially differ from the actual losses. In this section, we define a new measure of the *individually accepted* (or *desired*) *loss level* that we denote as VaR*.

In so doing, we start from the literal definition of VaR*: the maximum loss that can be a-priori expected by someone investing in risky assets. We concentrate on the terms "loss", "individual", and "maximum" encompassed by this definition. First, VaR* quantifies *losses*. According to PT, what actually counts for individual (non-professional)

[11] That can be observed in the conditions of the two value functions in Equations (8) and (9).

investors is not the absolute magnitude of a loss, but rather the subjectively perceived one, as captured by the value function described above. Hence, VaR* should rely on the *subjective value* of losses expressed in the loss branches of the value functions in Equations (8) and (9). It thus depends on individual features, originating in the subjective view over gains and losses, and can vary over time, for instance with the past performance of risky investments. Moreover, we are looking for a *maximal* value such that, in calculating VaR*, investors must ascribe a maximal occurrence probability to the losses in the value function, which can be formally rendered as: $\pi_t(1-\psi_t)+(1-\pi_t)(1-\omega_t)\overset{!}{=}1$.[12] Finally, VaR* should correspond to the concept of Value-at-Risk and hence represent a *quantile*, namely, according to the above considerations, a quantile of the subjective loss distribution.

Therefore, we suggest the following formal definition for the individual loss level:[13]

$$
\begin{aligned}
\mathrm{VaR}^*_{t+1} &= E_t[\text{loss-value}_{t+1}] - \varphi\sqrt{Var_t[\text{loss-value}_{t+1}]} \\
&= \lambda S_t E_t[x_{t+1}] - kE_t[x_{t+1}](S_t - Z_t) \\
&+ \sqrt{\pi_t(1-\psi_t)}\Big(\sqrt{\pi_t(1-\psi_t)} - \varphi\sqrt{1-\pi_t(1-\psi_t)}\Big)\Big((\lambda-1)R_{ft} + kE_t[x_{t+1}]\Big)(S_t - Z_t) \\
&= \lambda S_t E_t[x_{t+1}] + \Big(\zeta_t(\lambda-1)R_{ft} + (\zeta_t - 1)kE_t[x_{t+1}]\Big)(S_t - Z_t)
\end{aligned}
\tag{11}
$$

where "loss-value" stands for the subjective value ascribed to financial losses according to the loss branch of the value functions in Equations (8) and (9), and the subjectively perceived losses are assumed to follow a distribution (e.g. normal or Student-t)[14] with the lower quantile φ. Moreover, $E_t[x_{t+1}] = E_t[R_{t+1}] - R_{ft}$ denotes the expected risk premium. The last expression in Equation (11) is obtained using the simplifying notation $\zeta_t = \sqrt{\pi_t(1-\psi_t)}\Big(\sqrt{\pi_t(1-\psi_t)} - \varphi\sqrt{1-\pi_t(1-\psi_t)}\Big)$.

We distinguish two terms of the VaR*-expression in Equation (11): The first one accounts for the expected risky return (relative to the risk-free rate) $S_t E_t[x_{t+1}]$, weighted by the loss aversion coefficient λ. As it consequently resembles the prospective value

[12] This condition requires that the *absolute* probability of making a loss, i.e. *independently of the prior performance*, is one.

[13] The derivation of the expectation and the variance of the loss utility is deferred to Appendix 5.1.

[14] Although VaR is a very popular measure of risk, it has been criticized because it does not satisfy one of the four properties for coherent risk measures, namely the subadditivity (see Artzner, Delbaen, Eber, and Heath (1999), Rockafellar and Uryasev (2000) and Szegö (2002)). However, according to Embrechts, McNeil, and Straumann (1999), VaR becomes subadditive, and hence coherent, for elliptic joint distributions, such as normal and Student-t with finite variance.

according to the original PT, we denote this term as the *PT-term*. The last term is responsible for the influence of previous performance represented by the cushion $S_t - Z_t$ in Barberis, Huang, and Santos (2001). For this reason, we denote it as the *cushion term*. The corresponding weight is a linear combination of the expected risky and the risk-free returns.

Once the non-professional investors set their minds about the desired VaR*, they communicate it to the portfolio manager. In the view of the latter, this client indication represents an exogenous risk level that corresponds to VaR^{ex} in Equation (5) and is applied in order to determine the optimal level of borrowing or lending B_t. When VaR* is lower in absolute value than the portfolio VaR, B_t is negative, which formalizes the profile of more risk-averse investors who prefer to increase the proportion of wealth invested in risk-free assets. By contrast, for a VaR* higher than VaR in absolute value, investors augment their risky investments by borrowing extra money, i.e. they are less risk averse. Thus, analyzing the evolution of B_t (or equivalently of S_t/W_t, as conducted in the subsequent Section 3) can shed some light on the behavior of non-professional investors confronted with financial losses.

A further interesting topic to investigate lies in estimating the equivalent loss aversion parameter λ_t^* that can be obtained for a fixed $\overline{\text{VaR}^*}$ under the traditional approach.[15] Common assumptions of this approach are significance levels of 1%, 5%, or 10% and no dependency on past performance $k = 0$. The formula of λ^* is then immediate from the definition in Equation (11), where k is taken to be zero[16] and yields:

$$\lambda_{t+1}^* = \frac{\overline{\text{VaR}^*} + \zeta_t R_{ft}(S_t - Z_t)}{S_t E_t[x_{t+1}] + \zeta_t R_{ft}(S_t - Z_t)}. \tag{12}$$

2.3 The prospective value of the risky investment

The estimation of the individually maximum acceptable loss level VaR* represents only the first step in our analysis. As discussed above, it directly enters the optimal risk-free investment derived (by the professional manager) as a byproduct of the portfolio optimization procedure. Thus, VaR* dictates the optimal choice of the non-professional

[15] In other words, the loss aversion that equivalently results under the manager assumption of a fixed, exogenous risk level.

[16] This holds since λ_{t+1}^* depends on the *fixed* $\overline{\text{VaR}^*}$.

investors in terms of wealth percentages allocated between risky and risk-free assets.

However, we are also interested in the attitude of non-professional investors towards financial losses in general, as this attitude influences the level of the individual VaR*. The loss attitude results from the perception of the utility generated by financial investments.[17] The corresponding PT-concept of (subjectively) expected utility is the so-called *prospective value* V. In our framework, the prospective value of the risky portfolio can be formulated as:[18]

$$\begin{aligned}
V_{t+1} &= \pi_t E_t[v_{t+1}^{\text{prior gains}}] + (1-\pi_t)E_t[v_{t+1}^{\text{prior losses}}] \\
&= \pi_t[\psi_t S_t E_t[x_{t+1}] + (1-\psi_t)(\lambda S_t E_t[x_{t+1}] + (\lambda-1)(S_t - Z_t)R_{ft})] \\
&+ (1-\pi_t)[\omega_t S_t E_t[x_{t+1}] + (1-\omega_t)(\lambda S_t E_t[x_{t+1}] + k(Z_t - S_t)E_t[x_{t+1}])] \\
&= \Big(\pi_t\psi_t + (1-\pi_t)\omega_t + (\pi_t(1-\psi_t) + (1-\pi_t)(1-\omega_t))\lambda\Big) S_t E_t[x_{t+1}] \\
&+ \Big(\pi_t(1-\psi_t)(\lambda-1)R_{ft} - (1-\pi_t)(1-\omega_t)kE_t[x_{t+1}]\Big)(S_t - Z_t).
\end{aligned} \tag{13}$$

Note the existence of a twofold effect in the prospective-value formula, that is similar to the one discussed for VaR*: The first term of the last expression in Equation (13), that we subsequently denote as the *PT-effect*, captures the expected risky-investment value relative to the safe bank investment $S_t E_t[x_{t+1}]$. The corresponding probability weight can be rendered as the sum of the perceived gain and loss probabilities, laxly put as: $P_t(\text{gain}) + \lambda P_t(\text{loss})$. It points out that, as in PT, losses loom larger than gains, being additionally penalized by the loss aversion coefficient λ.

The last term of the prospective value in Equation (13) covers the cushion influence and we refer to it as the *cushion effect*. The weight of the cushion in this term is a combination of expected losses under the consideration of the performance history. Specifically, when current losses follow past gains – which occurs with the joint probability $\pi_t(1-\psi_t)$ – the past performance (given by the cushion) is valued at the risk-free rate R_{ft} and is amended by how much the loss aversion coefficient λ exceeds the loss-neutral value of 1. Indeed,

[17] In contrast to Barberis, Huang, and Santos (2001), our investors derive utility merely from financial wealth fluctuations, being *not* concerned with consumption.

[18] We also applied a slightly different definition of the prospective value. Accordingly, gains continue to be considered as possible events and are hence weighted by the occurrence probability. Losses are instead assessed in what we can call a "worst case scenario", i.e. with maximum probability. This is equivalent to saying that losses are accounted for in the form of VaR*. The obtained results, available upon request, were qualitatively similar to applying Equation (13).

if risky investments were successful in the past, a current loss has value only compared to the alternative of having put the entire money in risk-free assets. When losses extend from past to present – where $(1-\pi_t)(1-\omega_t)$ is the joint probability of current and past losses – the valuation implies a comparison of the risk-free rate to the risky performance $-E_t[x_{t+1}]$ in view of the sensitivity to past losses k.

We are interested in the evolution of the prospective value not only in time but also for different portfolio evaluation frequencies. The rationale for this is that revising portfolio performance at different time intervals implies first drawing back on distinct return values, hence on different return premia. Second, these return changes implicitly impact the later values of further model parameters, such as the cushion and the probabilities of past and current gains and losses. Therefore, the prospective value in Equation (13) is affected in multiple ways. We analyze this topic theoretically in Section 2.4 and in an applied context in Sections 3.2 and 3.3.

In so doing, we apply a further notion referring to the investor attitudes towards financial risks that attempts to capture more complex dependencies than the simple coefficient of loss aversion λ. According to PT, loss aversion corresponds to risk aversion of first order in the loss domain. In the same spirit, we term the first derivative of the prospective value with respect to the expected risk premium as the *global first-order risk aversion* (abbr. gRA). Formally, gRA yields:

$$\begin{aligned}\mathrm{gRA}_t &= \frac{\partial V_{t+1}}{\partial E_t[x_{t+1}]} \\ &= \Big(\pi_t\psi_t + (1-\pi_t)\omega_t + (\pi_t(1-\psi_t) + (1-\pi_t)(1-\omega_t))\lambda\Big)S_t - (1-\pi_t)(1-\omega_t)k(S_t - Z_t).\end{aligned} \tag{14}$$

Thus, gRA reflects the sensitivity – in terms of first-order changes – of the prospective value to the variation of expected returns (or equivalently to the expected risk premium).[19] Due to the linearity of our prospective value in the expected risk premium $E_t[x_{t+1}]$, gRA is independent of this premium.

Moreover, since gRA directly reflects changes in the prospective value – which is proportional to the attractiveness of financial investments – higher gRA-values point to a more

[19] As the prospective value is the PT-counterpart of the classic concept of investment utility, gRA is the pendant of a marginal utility with respect to the expected premium.

relaxed loss attitude. This can be formally recognized in Equation (14): The first term increases with the sum invested in risky assets S_t and the second is inversely proportional to the cushion $S_t - Z_t$. Note yet that this second term accounts for the situation when past losses are followed by current losses – which occurs with the probability $(1-\pi_t)(1-\omega_t)$ – and when, most probably, cushions are negative $S_t - Z_t \leq 0$. Thus, smaller negative cushions render this second term higher. In sum, gRA grows both when investors put more money in risky assets and when they manage to reduce recurrent losses.

2.4 The impact of the portfolio evaluation frequency

We assume that the frequency at which the risky-portfolio performance is evaluated affects the investors' loss attitude and leads to different investment decisions. Intuitively, the higher the frequency of performance checks, the higher the volatility of the risky portfolio will be. This makes risky returns less likely to be significantly different from the risk-free rate. In consequence, the investors' disappointment concerning the risky portfolio performance becomes more pronounced. Since according to PT, registered losses are perceived as more painful than gains of similar size, risky investments become even less attractive.

The tendency of performing such frequent checks is termed as *myopia* or *narrow framing*.[20] The idea that the joint effect of the myopia over financial decisions and the reluctance to make losses can dramatically affect the risk perception and hence the subjective desirability of risky investments comes in line with the concept of *myopic loss aversion* (abbr. mLA) introduced in Benartzi and Thaler (1995).[21]

We are interested in testing for the existence of myopic loss aversion in our framework and, more generally, in observing how wealth allocation decisions and loss attitudes vary at different portfolio evaluation frequencies. To this end, in the applied context of Section

[20] According to Barberis and Huang (2006), myopia refers strictly to annual evaluations of gains and losses, hence the term of *narrow framing* would be better suited to describing the underlying phenomenon. In a financial context, narrow framing illustrates the isolated evaluation of stock market risk (i.e. unrelated to the overall wealth risk). As underlined in Barberis and Huang (2004), this isolated evaluation entails an underestimation of the stock desirability, even though, viewed in a wide utility-risk frame, stocks represent a good diversification modality.

[21] The occurrence of mLA has received support from numerous direct experimental tests, such as Thaler, Tversky, Kahneman, and Schwartz (1997a), Gneezy and Potters (1997), Gneezy, Kapteyn, and Potters (2003), or Haigh and List (2005).

3 we examine how the wealth allocation to risky and risk-free assets given by S_t and B_t, the prospective value V, and the extended measure of the attitude towards financial losses gRA change at various evaluation horizons (ranging from one day to eight years). For reasons that we will more extensively comment on Section 3.2, our focus will lie on high evaluation frequencies, which we consider to be more plausible in practice, such as one day, one week, one month, two months and more, up to one year.

We suggest a modality of quantifying the impact of the evaluation frequency on the loss attitude, the understanding of which necessitates some further explanations. As already mentioned in Section 2.3, the evaluation frequency affects our model's variables (and hence investors' decisions and attitudes) mainly due to their dependence on expected returns. These returns directly depend on the *time horizon* τ over which they are computed, or equivalently on the portfolio *evaluation frequency* $1/\tau$. Thus, τ influences our model variables, in particular V and gRA, directly through expected returns. We denote this as the *direct transmission mechanism* of the evaluation frequency. However, other model variables, such as the cushions, the past and current gain probabilities, etc., are affected by past values of returns and hence indirectly depend on the evaluation frequency. We refer to this as the *indirect transmission mechanism*.

Theoretically, the direct dependence (i.e., on returns) could be studied by holding all model parameters, besides current return expectations, invariable to the evaluation frequency. Note however that this is technically impossible, as multiple other parameters are indirectly affected by the evaluation frequency. Yet, the direct effect can be discarded by eliminating the current returns. This is rendered possible by gRA, that represents by definition a derivative with respect to expected returns, where the direct impact is no longer contained. Consequently, studying how the prospective value and gRA vary with respect to the evaluation frequency amounts to examining the *total* and the *indirect* mechanism, respectively.

A further related issue is the following: Given that the portfolio evaluation frequency appears to affect investor perceptions over financial losses (and thus the level of risky investments), could the reverse causality hold as well? In other words, for a certain loss aversion value (at time t), is there an evaluation frequency that is *optimal* in the sense that it leads to the most relaxed attitude towards risky investments? If this is the case, financial advisors – whose interest is to attract clients, thus to raise capital – could for instance recommend to their clients to undertake performance checks with this "optimal" frequency that maximizes their risky investments and hence the budget at the manager's disposal.[22]

We will address this question in parallel with the examination of the changes in the loss attitude at different evaluation frequencies. Specifically, we will proceed in two steps: First, we study the total transmission mechanism by considering the prospective value V. As this variable formalizes the perceived utility of risky assets, we search for a generally valid specification $V(\tau)$ that maximizes it. Second, we concentrate on the indirect transmission mechanism by analyzing gRA. An "optimal" frequency in terms of the minimization of the reluctance towards financial losses is to be found by maximizing gRA(τ).

3 Application

This section presents findings complying with the theoretical results derived in Section 2 and based on market data. In particular, we consider daily values of the S&P 500 index, corrected for dividends and stock splits, and of the US three-month treasury-bill nominal returns. These two financial instruments – the stock index and the T-bill – serve as proxies for the risky and the risk-free investment, respectively. Both data series range from 01/02/1962 to 03/09/2006 (11,005 observations).[23]

As a consequence of the financial reform in 1979, which significantly changed the trading conditions, the early 80s mark the beginning of a new era of financial markets. We therefore reckon that only the second part of the data is relevant for inferring current market evolutions and divide our sample into two parts: The "active" data set (from 03/01/1982[24] to 03/09/2006, 6,010 observations) and the "inactive" data (consisting of the first part of the sample from 01/02/1962 to 03/01/1982). The subsequent investigations are based on the active set, while the previous observations provide a basis for estimating the empirical mean and the standard deviation of the portfolio returns at the "date zero" of trade (03/01/1982). The data contains an outlier, corresponding to the October 1987

[22]In the same context, Gneezy and Potters (1997) suggest that managers could manipulate the evaluation period of prospective clients.

[23]Descriptive statistics can be found in Tables 5 and 6 of the Appendix.

[24]As it took several years until the financial reform became operative.

market crash, which may distort the results. Since this market data serves in our work merely as support for simulating trading behaviors – that we view as more general – this outlier is smoothened out by replacing it with the mean of the ten before and after data points.[25]

We consider that non-professional investors perceive risky investments according to the value functions in Equations (8) and (9) and calculate their maximum expected loss level according to Equation (11). The active data set allows us to run the model on the basis of Sections 2.1 and 2.2.1 and to derive the desired VaR*, as well as the wealth proportion invested in the risky portfolio (i.e., in the S&P 500 index). The remaining money is assumed to be automatically put in the risk-free 3-month T-bill. Moreover, we assume that investors start trading with an even initial wealth allocation between the risky portfolio and the risk-free asset.[26] We also take the number of investors to be constant, i.e., no investors can enter or exit the market during the trading interval.[27]

We construct daily, weekly, monthly, and up to eleven months (increasing one month at a time), then yearly and further lower frequency returns ranging from one to eight years (with a one-year increment). The case commented throughout the application section of this paper relies on values considered in Barberis, Huang, and Santos (2001) for the loss-aversion coefficient and the sensitivity to past losses, namely $\lambda = 2.25$ and $k = 3$.[28] The expected portfolio gross returns are taken to be the unconditional mean returns until the last date before the decision time.[29] Further details with respect to the parameter choice for the presented results will be given in the text.

[25]We consider that this method is appropriate for preserving some of the particularities of less probable market events such as crashes, while at the same time allowing for circumvention of excessive impacts due to extreme outliers.

[26]A similar assumption is made in Thaler, Tversky, Kahneman, and Schwartz (1997b).

[27]This assumption implies that the evaluation period is shorter than the lifetime of our loss averse agents or, equivalently, that investors are long-lived beyond the VaR horizon. Identical assumptions are made in Basak and Shapiro (2001), Berkelaar, Kouwenberg, and Post (2004), and Berkelaar and Kouwenberg (2006).

[28]We performed parallel simulations for all values $\lambda \in \{0.5; 1; 2.25; 3\}$ and $k \in \{0; 3; 10; 20\}$. The results are qualitatively similar and available upon request.

[29]We also performed simulations for the cases where expected portfolio gross returns were computed as a zero mean process, or as an AR(1) process. The results, available upon request, are qualitatively similar. As unsophisticated investors (such as our non-professional traders) are more likely to rely on simple descriptive statistics from past data, we concentrate here on the case when expected returns are derived from average past returns.

3.1 The evolution of the risky investment

In this section we address how the risky investment develops subject to different portfolio evaluation frequencies and to distinct ways of assessing the cushion.

3.1.1 The combined impact of the portfolio evaluation frequency and of the cushion

According to Benartzi and Thaler (1995), loss-averse investors – who evaluate the performance of their portfolios once a year and employ linear value functions with conventional PT parameter values – give rise to a market evolution that can explain the equity return premium observed in practice. In the same spirit, we analyze how wealth allocation decisions of our non-professional investors change due to variations in the *portfolio evaluation frequency*. As in our framework these decisions are intrinsically linked to the past performance of the risky portfolio, we study at the same time the *cushion impact*.

In particular, we are interested in how different ways of assessing the cushion contribute to determining the amount of wealth to be invested in risky vs. risk-free assets at different evaluation frequencies. To this end, we apply two cushion definitions: *myopic* and *dynamic cushions*.[30] In calculating myopic cushions, we fix the benchmark level of past performance to be identical to the last-period risky holdings $Z_t = S_{t-1}$, so that the myopic cushion expression yields $S_t - S_{t-1}$. The *dynamic cushions* are based on Equation (18) in Barberis, Huang, and Santos (2001), which assumes a more complicated benchmark formula, in particular $Z_t = \eta Z_{t-1}\bar{R} + (1-\eta)S_t$. Hence, the dynamic cushion results in $\eta(S_t - Z_{t-1}\bar{R})$, where the parameter η measures how far in the past the investors' memory stretches.[31] In line with the same authors, we subsequently concentrate on the case where $\eta = 0.9$.[32] We moreover take the variable $\bar{R}$ in the definition of the dynamic cushion as the mean

[30] We also consider other cushion definitions. For instance, *cumulative cushions* amass from the date zero of the trade, so that $Z_t = Z_1 = S_0$ (e.g. the purchase price). Moreover, we also define *new myopic cushions* assuming $Z_t = Z_{t-1}R_t$. The corresponding results are available upon request.

[31] See Barberis, Huang, and Santos (2001), p. 15. This parameter allows for adjustments of the benchmark, wherefrom the denomination of "dynamic" cushions. Specifically, lower η-values put increased weight on the current risky value S_t relative to past evolutions captured by $Z_{t-1}\bar{R}$, which corresponds to a more myopic view. By contrast, higher η-values denote a more pronounced conservativeness in assessing the past performance benchmark, as the current term S_t losses in importance relative to the past-oriented $Z_{t-1}\bar{R}$.

[32] In fact, we considered three values of η, namely 0.1, 0.5, and 0.9. The results are qualitatively similar and are available upon request.

gross return.[33]

Following Campbell, Huisman, and Koedijk (2001), we start by computing the portfolio VaR in Equation (4) for gross returns of the risky portfolio that are either (standard) normally or Student-t (with five degrees of freedom) distributed, and for a significance level of 5%. We take π_t, ψ_t, and ω_t to be the empirical frequencies of the cases where $z_t \leq 1$ (i.e. past gains), $x_{t+1} + (1 - z_t)R_{ft} \geq 0 | z_t \leq 1$ (a premium that is acceptable under a history of gains), and $x_{t+1} \geq 0 | z_t > 1$ (a positive premium, conditional on the cases with past losses), respectively. We derive VaR* according to Equation (11) using either myopic or dynamic cushions. This value is then plugged into Equation (5) in order to determine the optimal level B_t of borrowing ($B_t > 0$) or lending ($B_t < 0$), that depends on the degree of loss aversion of non-professional investors.

Table 1 presents averages of the wealth percentages S_t/W_t invested in the risky portfolio, for both myopic and dynamic cushions, normally distributed and Student-t distributed portfolio gross returns R_t, and at different portfolio evaluation horizons τ up to one year. The current value of the risky investment S_t is derived according to Equation (6).

Table 1: Average wealth percentages invested in S&P 500.

	Myopic cushions		**Dynamic cushions**	
Evaluation	**Portfolio returns**		**Portfolio returns**	
frequency	Normal	Student-t	Normal	Student-t
1 year	34.51	25.79	30.50	24.48
6 months	20.23	15.67	19.92	16.08
4 months	16.96	13.23	16.30	13.16
3 months	13.42	10.55	13.00	10.52
1 month	7.70	6.21	7.69	6.29
1 week	3.85	3.13	3.85	3.15
1 day	1.90	1.55	1.90	1.56

This table presents the average wealth percentages S_t/W_t invested in the risky portfolio at different evaluation horizons τ up to one year for both myopic cushions $S_t - S_{t-1}$ and dynamic cushions $\eta(S_t - Z_{t-1}\bar{R})$, and standard normal and Student-t with 5 degrees of freedom distributed portfolio gross returns R_t. Other parameter values used are $\lambda = 2.25$, $k = 3$, $\eta = 0.9$ and $\bar{R} = \text{mean}[R_t]$.

[33] As no dividend data is available to our analysis, we could not apply the simultaneous estimation procedure of Barberis, Huang, and Santos (2001). Note also that due to the fact that the mean and median of our return sample lie very close to each other, the results with $\bar{R} = \text{mean}[R_t]$ and $\bar{R} = \text{median}[R_t]$ are almost identical.

Accordingly, our non-professional investors allocate almost no money to over 30% of their wealth in risky assets, where the substantial fluctuation of these sums is mainly caused by the frequency at which risky performance is evaluated. Specifically, more frequent checks entail lower investments in the risky portfolio, independent of the way in which our investors account for past performance (i.e., of the cushion type). This result is consistent with previous findings, such as Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001), summarized under the notion of mLA. It suggests that loss-averse investors who perform high frequency evaluations and narrow-frame financial projects – by overly focusing on long series of past performances – become extremely loss averse.

At the evaluation frequency of one year, non-professional investors who dynamically assess cushions appear to be more loss averse than their myopic peers, as they allocate less money to the risky portfolio. This difference becomes however negligible at higher evaluation frequencies. Moreover, independently of the cushion type, the investors' reluctance towards risky investments is higher for normally distributed than for Student-t distributed portfolio gross returns.

Since VaR has been proven to be an adequate market risk measure in the case where returns follow a normal distribution and since our VaR* follows the VaR concept, we henceforth focus on the case with normally distributed returns.

3.1.2 The importance of the cushion

In this section we study the importance of the past performance of the risky portfolio – a concept added to the initial PT representation by Barberis, Huang, and Santos (2001) and formalized by the notion of cushion – for wealth allocation.[34] In contrast to the above section, we are now interested in the *magnitude* of the cushion effect and its evolution over time.

In order to analyze this issue, we fix the evaluation frequency at one year and plot in Figure 1 (Figure 7 in Appendix 5.3) the annual returns of the index S&P 500, the evolution of the myopic (dynamic) cushion generated by a series of past gains or losses, and the

[34]Gneezy and Potters (1997) test for the influence of experienced gains and losses on risk behavior, but find no significant effect. However, as they note on p. 641, their experimental framework deviates from real market settings.

resulting yearly wealth percentages invested in the risky portfolio. These figures point to a positive correlation of the three variables (returns, cushions, and risky investments).

In line with the idea that loss aversion is sensitive to past performance, we observe in panels c of Figures 1 and 7 that the lower the cushions are, the more loss-averse investors become, since they dispose of less back-up for later contingent losses.

At this point, a further interesting empirical question arises: How long does it take for an investor performing frequent evaluations to quit the risky market? Figure 2 (Figure 8 in Appendix 5.3) emphasizes the dramatic effect of high evaluation frequencies for investors who act upon myopic (dynamic) cushions (see panels c). Specifically, investors who check their portfolio performance every single day put less than 5% of their wealth in risky assets, beginning soon after they start investing. The reason is that each day can bring substantial changes in the perceived past performance. Therefore, although non-professionals do not completely quit the risky market, their risky holdings are kept at very low levels during the entire trading interval.

3.2 The evolution of the prospective value

In this section we analyze the evolution of the prospective value focusing on the influence of the evaluation frequency. As observed in Section 2.4, the evaluation horizon τ affects the prospective value of the risky investment in Equation (13) through two transmission mechanisms: a direct one that refers to the expected returns and thus to the expected risk premium, and an indirect one that relies on other model parameters influenced by past return values, such as the cushion or the probabilities of past and current gains and losses. The prospective value sheds light on the *total* impact of the evaluation frequency on investors' behavior.

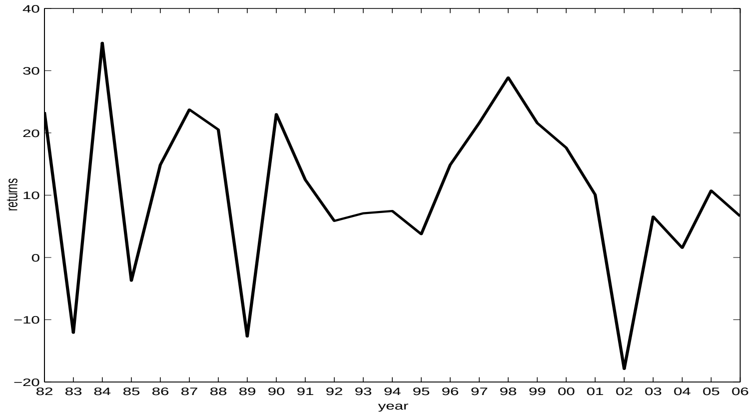
Henceforth we refer to the descriptions of variables in dependence on the frequency at which the risky performance is checked as representations in the *evaluation frequency domain*.

3.2.1 The impact of the portfolio evaluation frequency

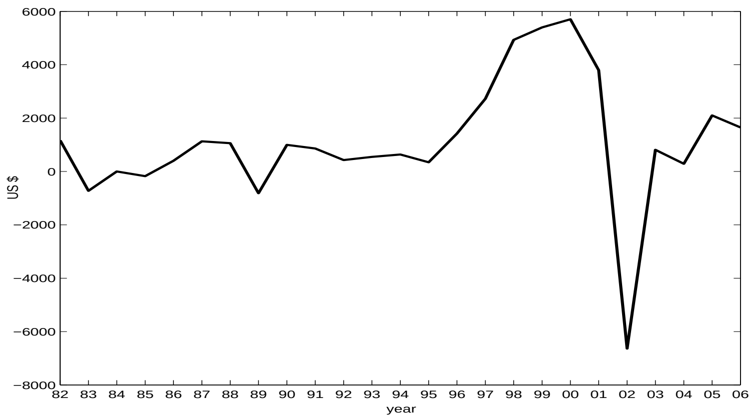
We commence our analysis by shortly considering the time evolution of the prospective value in Equation (13) and its two components, in order to ascribe the importance of the

Figure 1: Evolution of risky returns, myopic cushions, and wealth percentages invested in the risky portfolio for yearly portfolio evaluations.

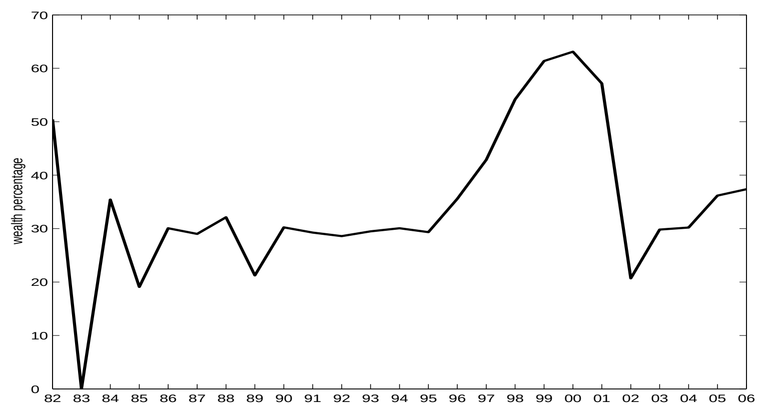
(a) Yearly S&P 500 log-returns.



(b) Yearly myopic cushions.



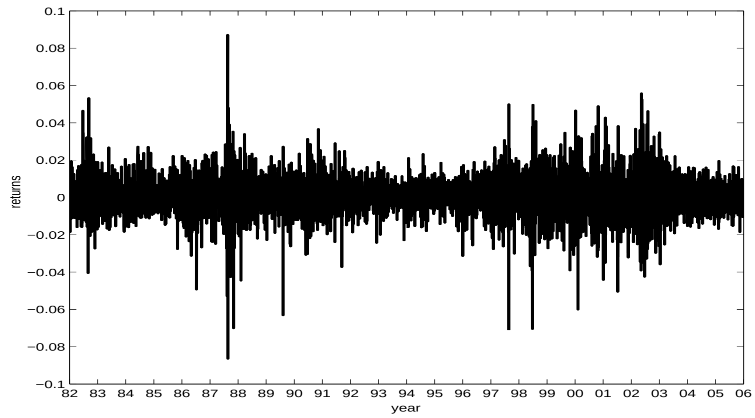
(c) Yearly wealth percentages invested in S&P 500.



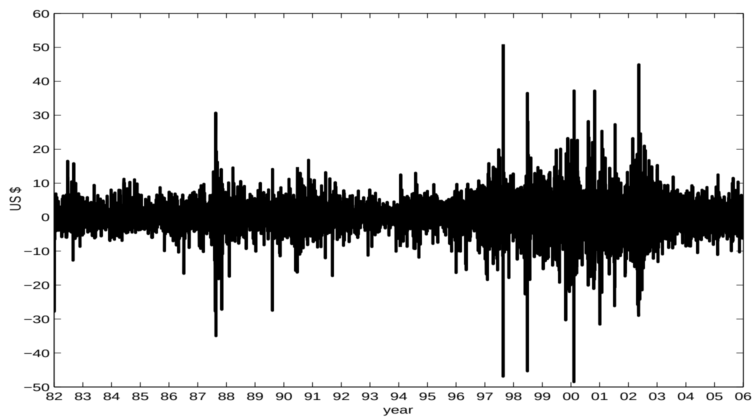
This figure illustrates the annual log-returns R_t of the index S&P 500, the corresponding yearly evolution (in US \$) of the past performance encompassed by the myopic cushion $S_t - S_{t-1}$, and the resulting yearly wealth percentages S_t/W_t invested in the risky portfolio. The wealth percentages are obtained from Equation (5), where VaR^{ex} is replaced by the VaR*-values from Equation (11) and the risky investment S_t results from Equation (6). We assume $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\ldots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, and $k = 3$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to 1st March of each year.

Figure 2: Evolution of risky returns, myopic cushions, and percentages invested in the risky portfolio for daily portfolio evaluations.

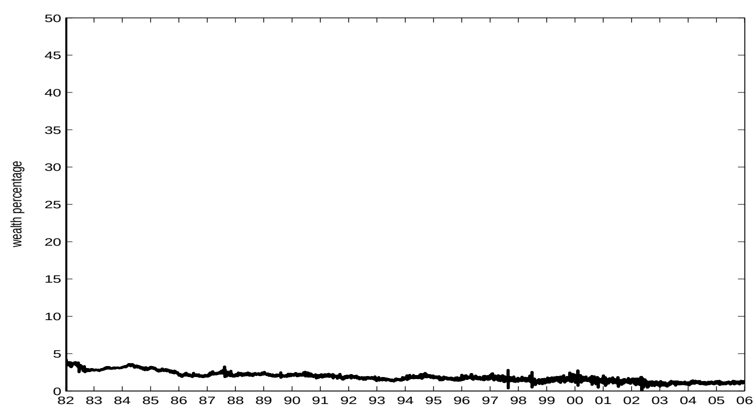
(a) Daily S&P 500 returns.



(b) Daily myopic cushions.



(c) Daily percentage investments in S&P 500.

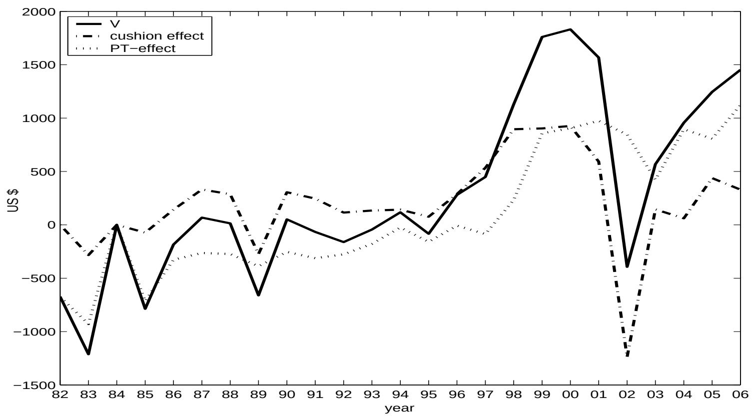


This figure illustrates the daily log-returns R_t of the index S&P 500, the corresponding daily past performance (in US \$) encompassed by the myopic cushion $S_t - S_{t-1}$, and the resulting daily wealth percentages S_t/W_t invested in the risky portfolio. The wealth percentages are obtained from Equation (5), where $\mathrm{VaR}^{\mathrm{ex}}$ is replaced by the VaR*-values from Equation (11) and the risky investment S_t results from Equation (6). We assume $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\ldots,t}{\mathrm{mean}}[R_s]$, $\lambda = 2.25$, and $k = 3$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to 1st March of each year.

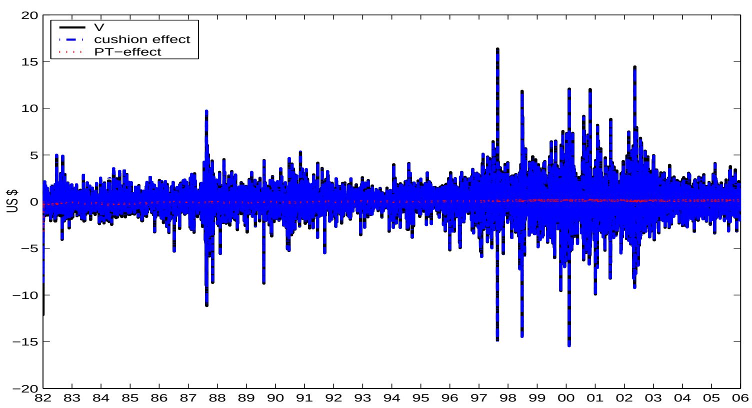
cushion and PT-effects. Figure 3 (Figure 9 in Appendix 5.3) illustrates these variables for myopic (dynamic) cushions and evaluation frequencies of one year and one day. Note that at both frequencies, as long as cushions are sufficiently high in absolute value, it is the cushion effect that dictates the shape of the prospective value. This lead role is even more pronounced for daily evaluations where the expected return premium is very small and hence the PT-effect weak.[35]

Figure 3: Prospective value evolution for yearly and daily evaluations.

(a) Yearly evaluations.



(b) Daily evaluations.

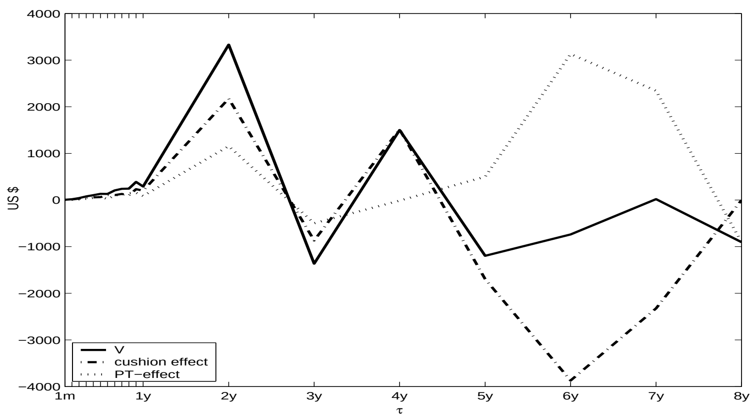


This figure illustrates the yearly and daily evolution of the prospective value V (in US \$) from Equation (13) and of its two components, the PT-effect and the cushion effect, that correspond to the two terms in this equation. The PT-effect corresponds to the representation of V in PT, without accounting for the influence of past performance, which is encompassed by the cushion-effect. We assume myopic cushions $S_t - S_{t-1}$, $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\dots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, and $k = 3$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to 1st March of each year.

[35]Specifically, in this case the prospective value (black) cannot be practically disentangled from the cushion effect (blue).

In Figure 4 (Figure 10, panel a, in Appendix 5.3), we plot the prospective value and its two components again, but now as functions of the evaluation horizon τ. This horizon ranges from one month to eight years, where we consider monthly increments of up to one year and yearly increments thereafter.[36]

Figure 4: Prospective value evolution for different evaluation frequencies.



This figure illustrates the prospective value V (in US \$) from Equation (13) and its two components, the PT-effect and the cushion effect, as functions of the portfolio evaluation horizon τ. V reflects the perceived utility of risky investments and captures the total impact of τ on investor behavior (through expected returns and other model variables). The PT-effect corresponds to the representation of V in PT, without accounting for the influence of past performance, which is encompassed by the cushion-effect. Higher V-values point to an increased utility of risky investments as perceived by non-professional investors. We assume myopic cushions $S_t - S_{t-1}$, $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\dots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, and $k = 3$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006).

It is apparent in Figures 4 and 10 that up to two years the perceived attractiveness of financial investments increases with the evaluation horizons. This tendency is consistent with mLA and it characterizes the evolution of both the PT-effect and the cushion effect at higher evaluation frequencies. In fact, the PT-effect is upward sloping across all considered evaluation frequencies, which supports the coherency of mLA within the framework initially suggested by PT.

However, the prospective value yields even negative values for higher evaluation horizons (such as three, five, or six years). This is motivated by the leading role of the cushion effect discussed above, and by the fact that for lower evaluation frequencies cushion values are negative and sufficiently high in order to counterbalance the PT-effect and to

[36] In order to obtain a suggestive graphic representation, we consider all frequencies from one to twelve months and discard the observations for one day and one week. An evaluation frequency of eight years implies that investors can only make three portfolio checks during our estimating sample. Therefore, a further increase of the evaluation time becomes senseless.

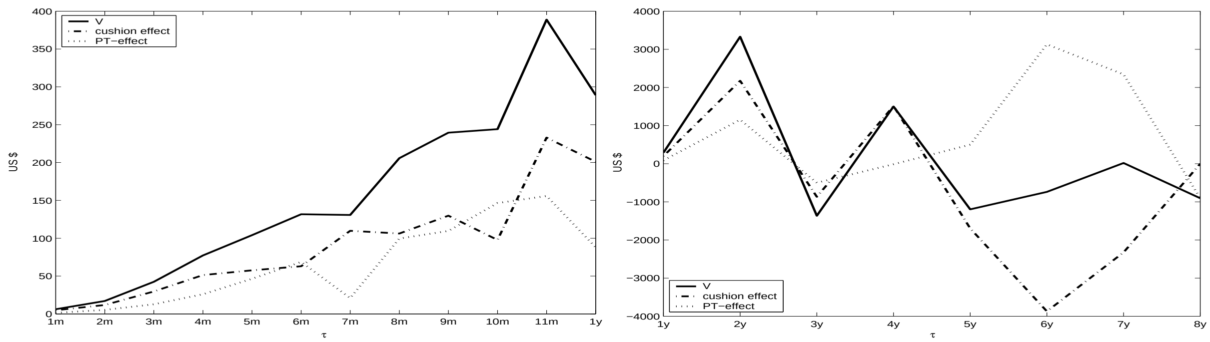
dramatically reduce the perceived value of risky investments. Intuitively, when risky performance is checked at longer time intervals, the decision flexibility is lower, since current decisions fix the portfolio composition over the entire coming interval of several years. Thus, investors would be more wary of the possibility of registering current losses for lower evaluation frequencies. As the cushion effect accounts for the perception of possible current losses – a perception which varies depending on the past performance, see the cushion weight in Equation (13) – it increases in absolute value for more seldom portfolio checks, but its sign is given by the sign of the cushion. Here, investors create negative cushions, which gives rise to the observed drop in the cushion effect and consequently in the prospective value. In sum, checking risky performance less often than once every one or two years appears to deteriorate the perception of the utility of risky investments.

Indeed as documented in Benartzi and Thaler (1995), a decade ago investors used to perform in practice yearly portfolios checks. Nowadays, due to the high amount of information available at almost no cost and to the enhanced dynamics of market events, financial decisions may be reconsidered more often. One year remains however an important anchor in the investors' minds given that, on one hand, various events (such as the release of annual activity reports, taxes, etc.) take place with this frequency and, on the other hand, non-professional investors may not be sufficiently impatient (perhaps because they do not dispose of sufficient time, financial resources, knowledge, experience or the combination of any of them) to perform much more frequent portfolio checks. In our opinion, non-professional investor perceptions reasonably rely on evaluation horizons of one year and less.

Based on these ideas, we delimitate *two distinct segments* of the prospective value in the evaluation horizon domain depicted in Figures 4 and 10. These segments meet at the "critical" frequency of *one year* and are characterized by different evolutions. We denote the segment lying to the left of the frequency of one year as the *left segment* and, as we view it as the (only) one relevant in practice, our subsequent analysis will concentrate on it. The part of the prospective value in the frequency domain encompassing evaluation horizons higher than one year is referred to as the *right segment*. Figure 5 (Figure 10, panels b and c, in Appendix 5.3) illustrate these two segments separately, for myopic (dynamic) cushions.

Figure 5: Prospective value evolution on the two evaluation-frequency segments.

(a) $\tau \leq 1$ year. (b) $\tau \geq 1$ year.



This figure illustrates the prospective value V (in US \$) from Equation (13) and its two components, the PT-effect and the cushion effect, as functions of the portfolio evaluation horizon τ. V reflects the perceived utility of risky investments and captures the total impact of τ on investor behavior (through expected returns and other model variables). The PT-effect corresponds to the representation of V in PT, without accounting for the influence of past performance, which is encompassed by the cushion effect. Higher V-values point to an increased utility of risky investments as perceived by non-professional investors. Panel a depicts the evolution of V for evaluation horizons up to one year (in monthly increments) and panel b for evaluation horizons from one to eight years (in yearly increments). We assume myopic cushions $S_t - S_{t-1}$, $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\ldots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, and $k = 3$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006).

In the left segment, the perceived risky value appears to increase on average with the evaluation horizon. In effect, the curve $V(\tau)$ in panel a of Figure 5 (panel b of Figure 10) is acceptably well described by a polynomial of first order.[37] Accordingly, the subjectively perceived utility of the non-professional investors – captured by the prospective value – should be maximized at the highest frequency of this domain, which is one year.[38] One year can be hence designated as the *optimal evaluation frequency* with respect to minimizing loss aversion and hence maximizing risky investments.

In the same spirit, the highest evaluation frequency of one day entails a minimal expected value of the risky portfolio, pushing investors to step out of the risky market and to allocate (almost) all their money to risk-free assets. In other words, loss-averse investors should check the performance of their risky investments *as seldom as possible*

[37]Specifically, the adjusted R-squared yields 91.69% (77.44%) for myopic (dynamic) cushions. The estimations are based on polynomial regression fitting performed with the Matlab Curve Fitting Toolbox. All findings in this section are robust across different parameter specifications, such as of the loss aversion coefficient, the sensitivity to past losses, the cushion, returns distribution, expected returns, etc. Further results are available upon request.

[38]In fact, the prospective value in the left segment in Figures 5 and 10 attains its maximum at eleven months. As this value lies closely to the predicted maximum point of one year and as one year is a much more noticeable value in investor perception, we consider one year as a sufficiently good approximation for the optimum.

in order to maximize the corresponding prospective value of their investments. Under the practical informational constraints that govern financial markets nowadays, *one year* appears to be the most reasonable evaluation time that would increase the perceived returns of risky investments.

3.3 The evolution of the actual attitude towards financial losses

In this section, we extend the analysis in the frequency domain to our new measure of loss attitude gRA. In so doing, we study the *indirect* transmission mechanism mentioned in Section 2.4. Being a derivative of a linear variable gRA does not contain any direct influence (i.e., through the expected risk premium) of the evaluation frequency. The variation of gRA captures thus the collateral impact of τ on other model parameters, such as the cushion $S - Z$, the probability of past gains π, the probability of a positive risk premium given past losses ω, and that of an acceptable premium given past losses ψ.

Panel a of Figure 6 (Figure 11 in Appendix 5.3) illustrates the gRA course for evaluation frequencies ranging from one month to eight years and myopic (dynamic) cushions.[39] On average, gRA appears to increase with the evaluation horizon, pointing to a more relaxed attitude towards financial losses as portfolio performance is checked less often. Note that this occurs at all frequencies and not only in the left segment, as was the case for the prospective value in Section 3.2.1. Thus, while the impact of the evaluation frequency on the loss perception can be ambiguous in a context where both direct and indirect transmission mechanisms are considered, the indirect mechanism consistently supports the concept of mLA.

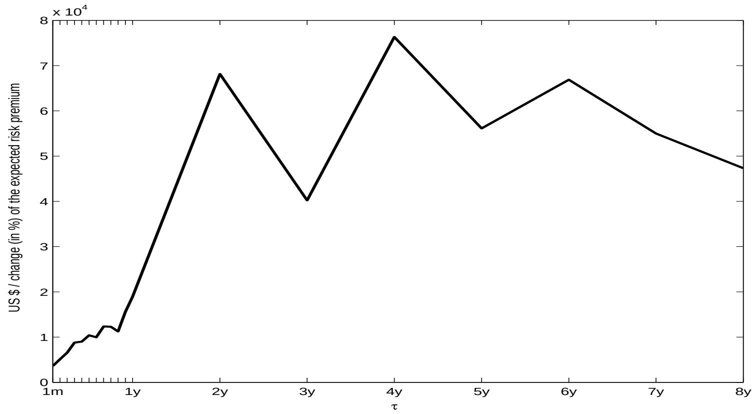
The ambiguity of the total transmission mechanism reported for the prospective value appears to be therefore given by its direct component, i.e., through expected returns. The cushion effect, that is highly dependent on returns, distorts the evolution of the prospective value for very seldom portfolio checks, making it extremely sensitive to past performance.

Similarly to the prospective value, we consider a segmentation of gRA at the evaluation horizon of one year (see panels b and c in Figures 6 and 11). In the left segment (panels

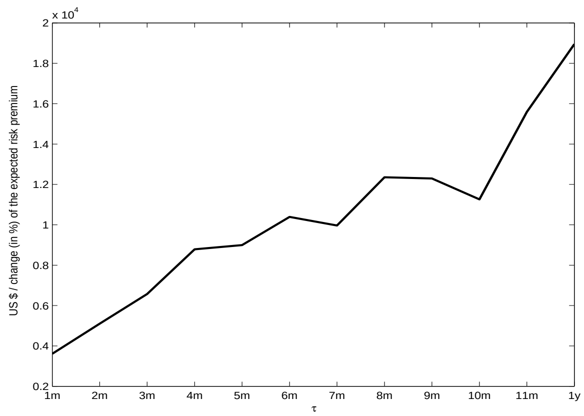
[39] All findings in this section are robust across different parameter specifications. Further results are available upon request.

Figure 6: Evolution of the global first-order risk aversion for different evaluation frequencies.

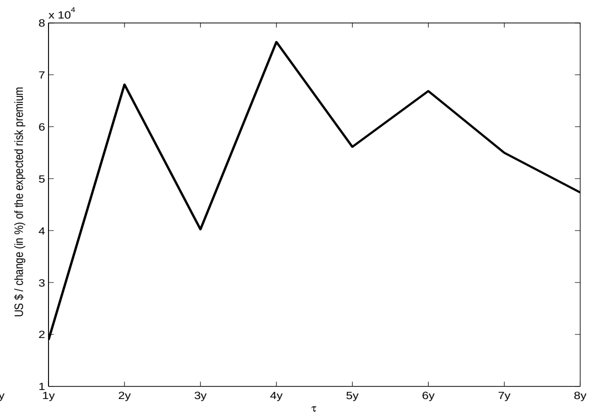
(a) All evaluation frequencies.



(b) $\tau \leq 1$ year.



(c) $\tau \geq 1$ year.



This figure illustrates the evolution of our measure of the loss attitude gRA (in US \$ per percentage change of the expected risk premium) from Equation (14) as function of the portfolio evaluation horizon τ. gRA reflects the sensitivity of the prospective value to the variation of expected returns. It captures merely the indirect impact of τ on investor behavior, i.e. through channels other than expected returns, such as the cushion, the probabilities of past gains and losses, etc. Higher gRA-values point to a more relaxed loss attitude. Panel a depicts gRA for all evaluation horizons, panel b focuses on horizons up to one year (in monthly increments), and panel c on horizons from one to eight years (in yearly increments). We assume myopic cushions $S_t - S_{t-1}$, $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\ldots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, and $k = 3$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006).

b), simple lines appear to fit the data acceptably well.[40] Our measure gRA attains its maximum for the lowest frequency of this segment, i.e. of one year.[41]

As mentioned in Section 2.3, higher gRA-values represent the result of a more relaxed attitude towards financial losses. Thus, minimizing the loss aversion – as measured by gRA – requires again that portfolio performance should be checked *as seldom as possible*. For the left segment, this is consistent with the recommendation derived from the perception of risky investments – as captured by the prospective value – in Section 3.2.1.

In the right evaluation-frequency segment, the course of gRA is more complex, so that second-order polynomials are necessary in order to acceptably describe the data.[42] The maximum of these parabolas is achieved at an evaluation frequency of around five years, which might recommend this frequency as an optimal one in this segment.[43] Nevertheless, as stressed above, we consider the right segment to be of less practical importance.

In sum, both the total and the indirect mechanisms by which the evaluation frequency impacts perceptions and decisions suggest that, under practical information constraints, an improvement in the investors' attitude towards risky holdings can be achieved for yearly performance checks.

3.4 A comparison with the portfolio optimization framework

This section proposes a way to "translate" the results obtained in our framework in terms of the portfolio optimization "language" spoken by professional managers. Specifically, our investors *individually ascertain* the maximum sustainable level of losses VaR* on the basis of subjective behavioral parameters. By contrast, in practice, managers *standardize* the risk definition that could not (sufficiently) account for individual characteristics of their clients. For instance, when risk is measured by means of the VaR concept, it can be reduced to specific confidence levels and time horizons. In order to provide a comparison of these two frameworks – termed in Section 2 as "endogenous" and "exogenous", respectively – we confront the VaR* in our model with the standard VaR used by portfolio managers.

[40] Specifically, the adjusted R-squared yields 90.5% (91.57%) for myopic (dynamic) cushions.
[41] A statement which is now consistent both with the data and the fitted curve.
[42] Specifically, the adjusted R-squared yields 49.61% (60.74%) for myopic (dynamic) cushions.
[43] Specifically, this frequency yields 4.9859 (5.3178) for myopic (dynamic) cushions.

In particular, we perform twofold equivalence computations: First, we start from our VaR*-estimates and derive equivalent significance levels α from the VaR-formula. Second, we apply confidence levels commonly used (such as 1% and 10%) to the same VaR-formula and obtain equivalent average coefficients of loss aversion and equivalent wealth percentages invested in the risky portfolio, on the basis of the corresponding formulas and estimates in our model.

3.4.1 VaR*-equivalent significance levels

Portfolio managers consider the risk level indicated by their clients VaR^* in terms of the standard concept of VaR, specifically as VaR^{ex}. In other words, VaR* is equated with the lower quantile of the portfolio gross returns at a given (i.e. fixed) significance level that we denote by α^* (or confidence $1-\alpha^*$). According to Equation (5), if the portfolio VaR at time t corresponds to an $\alpha_t > \alpha^*$ (or equivalently, to a confidence level $1-\alpha_t < 1-\alpha^*$), then too much risk would arise by putting the entire wealth in the risky portfolio. The portfolio manager will conclude that a percentage of the investors' wealth should be lent (i.e. invested in the risk-free asset) $B_t < 0$. On the contrary, if $\alpha_t < \alpha^*$, then the portfolio risk meets the individual risk requirements – being lower than the subjective risk threshold – and investors borrow extra money $B_t > 0$ in order to increase their S&P 500-holdings.

In this section, we apply the formulas suggested by portfolio theory in order to determine the significance levels that would correspond to the estimates of VaR^*_{t+1} derived from Equation (11) on the basis of real market data. The corresponding averages of α^* over time are listed in Table 2 for different portfolio evaluation frequencies, normally distributed and Student-t distributed gross returns, myopic cushions and dynamic cushions.

The results are striking: The equivalent significance level α^* lies below the commonly acceptable interval (being practically zero). Thus, the assumption of classical portfolio selection models based on the VaR-concept that investors choose significance levels α in the interval $[1, 10]\%$ appears to be at odds with the findings in our VaR*-framework, for any evaluation frequency higher than one year. Even the lowest significance level of 1% used in standard portfolio models is not able to capture the loss aversion of non-professional investors acting according to our setting. In other words, investors may be substantially more risk averse in practice than actually considered in theory.

Table 2: Portfolio-equivalent significance levels of the estimated VaR^*_{t+1} (α^*).

	Myopic cushions		**Dynamic cushions**	
Evaluation	**Portfolio returns**		**Portfolio returns**	
frequency	Normal	Student-t	Normal	Student-t
1 year	0.00	0.00	0.00	0.00
6 months	0.00	0.00	0.00	0.00
4 months	0.00	0.00	0.00	0.00
3 months	0.00	0.00	0.00	0.00
1 month	0.00	0.00	0.00	0.00
1 week	0.00	0.00	0.00	0.00
1 day	0.00	0.00	0.00	0.00

This table presents the portfolio-equivalent significance levels α^* of the estimated VaR* from Equation (11) at different evaluation horizons τ up to one year for both myopic cushions $S_t - S_{t-1}$ and dynamic cushions $\eta(S_t - Z_{t-1}\bar{R})$, and standard normal and Student-t with 5 degrees of freedom distributed portfolio gross returns R_t. Other parameter values used are $\lambda = 2.25$, $k = 3$, $\eta = 0.9$ and $\bar{R} = \text{mean}[R_t]$.

3.4.2 Portfolio-equivalent indices of loss aversion

The same equivalence issue can also be addressed from the opposite viewpoint: we determine the values of λ^*_{t+1} in Equation (12) and the average investment in risky assets that result from our VaR*-formula in Equation (11). These values correspond to the risk levels used (by managers) according to the conventional VaR-procedure using common significance levels α of 1% and 10%.

Tables 3 and 4 (Tables 7 and 8 in Appendix 5.3) present the results of this analysis for normally distributed and Student-t distributed portfolio returns and myopic (dynamic) cushions. Recall that the portfolio VaR in Equation (4) is estimated using a 5% significance that is considered the benchmark for the values in these tables (i.e. it corresponds to 100% risky investments).

Accordingly, the equivalent recommendations concerning the money to be invested in risky assets that result from the optimal portfolio allocation under VaR at 1% (10%) significance lie well below (above) the benchmark VaR at 5%. This points out a higher (lower) loss aversion in our endogenous VaR*-framework – after restating it in terms of the exogenous-VaR model – relative to the portfolio risk measured by VaR. Comparing Tables 3 and 4 (Tables 7 and 8 in Appendix 5.3), we can observe that the lower the significance

Table 3: Wealth percentages invested in S&P 500 and the average λ^*, for $\alpha = 1\%$.

	Wealth %		λ^*	
Evaluation frequency	**Portfolio returns** Normal	Student-t	**Portfolio returns** Normal	Student-t
1 year	60.99	36.48	1.02	1.01
6 months	59.72	34.63	0.91	0.90
4 months	59.40	34.17	1.00	0.82
3 months	59.30	34.01	1.43	1.67
1 month	59.04	33.65	0.90	1.62
1 week	58.82	33.34	0.80	0.58
1 day	58.70	33.20	1.00	1.02

This table presents the average wealth percentages invested in the risky portfolio (first two columns) and the average loss-aversion coefficients λ^* from Equation (12) (last two columns) equivalent to a portfolio significance level $\alpha = 1\%$ at different evaluation horizons τ up to one year. The 100% level for the wealth percentages invested in risky assets corresponds to $\alpha = 5\%$ and $\lambda = 1$ points to a loss-neutral attitude. We assume myopic cushions $S_t - S_{t-1}$, standard normal and Student-t with 5 degrees of freedom distributed portfolio gross returns R_t, $\lambda = 2.25$, and $k = 3$.

Table 4: Wealth percentages invested in S&P 500 and the average λ^*, for $\alpha = 10\%$.

	Wealth %		λ^*	
Evaluation frequency	**Portfolio returns** Normal	Student-t	**Portfolio returns** Normal	Student-t
1 year	120.80	125.37	1.20	1.02
6 months	121.47	126.11	0.88	1.06
4 months	121.64	126.29	1.00	1.00
3 months	121.70	126.36	1.00	1.00
1 month	121.84	126.50	1.00	1.00
1 week	121.96	126.63	1.00	1.00
1 day	122.00	126.67	1.00	1.00

This table presents the average wealth percentages invested in the risky portfolio (first two columns) and the average loss-aversion coefficients λ^* from Equation (12) (last two columns) equivalent to a portfolio significance level $\alpha = 10\%$ at different evaluation horizons τ up to one year. The 100% level for the wealth percentages invested in risky assets corresponds to $\alpha = 5\%$ and $\lambda = 1$ points to a loss-neutral attitude. We assume myopic cushions $S_t - S_{t-1}$, standard normal and Student-t with 5 degrees of freedom distributed portfolio gross returns R_t, $\lambda = 2.25$, and $k = 3$.

(or the higher the confidence level) is, the more risk averse the non-professional investors become, as the proportion of wealth invested in the risky portfolio is smaller than 100%.

However, even the lowest percentages in Tables 3 and 7 are still much higher than those in Table 1, where VaR* is treated as endogenous, mainly for high frequency revisions.

Interestingly, the results for $\alpha = 1\%$ are qualitatively consistent with our previous findings supporting mLA, since the wealth percentages invested in risky assets decrease for higher evaluation frequencies, although their variation is much weaker than for the case with $\alpha = 5\%$ considered in Section 3.1.1. By contrast, when the confidence level increases to $\alpha = 10\%$, this phenomenon is reversed and investors appear to allocate slightly more money to the risky portfolio for more frequent evaluations. As mLA is a widely documented phenomenon, we can conclude that the traditional portfolio optimization framework fails once more to capture the real investor behavior in a consistent way. This problem appears to become more acute for more relaxed assumptions regarding the risk attitude.

Similar conclusions can be drawn with respect to the equivalent loss-aversion coefficient λ^* derived for conventional significance levels assumed in previous research. Its values in Tables 3 and 4 (Tables 7 and 8 in Appendix 5.3) for myopic (dynamic) cushions are much lower than the empirical level estimated in the original PT and largely used in previous empirical research[44] of 2.25. For the majority of the considered combinations of α-values and evaluation frequencies, we obtain $\lambda^* \simeq 1$, a level that indicates identical perception over gains and losses according to the value function from Equation (13) (and recalling that $k = 0$, i.e. no influence of past losses). Actually, this "neutral" level of 1 is exceeded rarely for some evaluation frequencies for $\alpha = 1\%$ and 10%. This reinforces our earlier claim that even assuming low significance levels (e.g. $\alpha = 1\%$ as is the common case in previous portfolio optimization research) entails an *underestimation* of the loss attitude of real investors captured by the specific coefficient λ.

4 Summary and conclusions

This paper investigates the behavior of non-professional investors facing the problems of fixing a maximal acceptable level for financial losses and of optimally allocating wealth between a risk-free asset and a risky portfolio. We assume that these investors are loss

[44] Such as Barberis, Huang, and Santos (2001) and Benartzi and Thaler (1995).

averse, narrowly frame financial investments, and perceive future portfolio returns as being influenced by past portfolio performance.

We extend the portfolio allocation model developed in Campbell, Huisman, and Koedijk (2001) in order to incorporate the effect of a desired risk level that is now subjectively assessed (VaR*). The first task of our non-professional investors consists of fixing their individual VaR*-level. This is subsequently communicated to professional portfolio managers in charge of finding the optimal composition of the risky portfolio. Based on VaR*, portfolio managers derive the optimal sum of money to be invested in risk-free assets. The non-professional clients are thus provided with a recommendation of how to allocate money between risky and risk-free assets that corresponds to their personal perceptions and preferences.

In modeling the investors' perception over the risky investment that yields the subjective VaR*, we employ the extended subjective valuation of prospective risky investments proposed in Barberis, Huang, and Santos (2001) and rely on the notion of myopic loss aversion introduced in Benartzi and Thaler (1995). The contribution of our paper is to integrate these behavioral explanations in the portfolio decision framework mentioned above. Moreover, we enrich the two models with original findings that stem both from theoretical considerations and results obtained on the basis of real market data (specifically, the S&P 500 and the US 3-month T-bill returns).

Considering that investors are merely concerned with financial investments as a source of utility, we build on the theoretical modelling of their perceptions of risky assets (the value function) and define the maximum individually sustainable level of financial losses (VaR*). This VaR*-level serves in deciding upon the optimal amount of money to be invested in the risky portfolio. Also, we propose a way in which non-professional investors can assess the utility of risky prospects (the prospective value). Moreover, we introduce an extended measure, the global first-order risk aversion, that attempts to better capture the actual attitude towards financial losses of real investors. We finally investigate how the portfolio evaluation frequency impacts, through different mechanisms, the prospective value and this further measure of the loss attitude. In this context, we also suggest a way to derive the horizon of performance revisions that maximizes risky investments.

The theoretical results are supported and extended by our application. We show that, in sum, our non-professional investors demonstrate myopic loss aversion. They allocate the main part of their wealth to risk-free assets, while a smaller sum (always lower than 35% of wealth) is put into the risky portfolio. This latter sum substantially decreases at higher frequencies at which the portfolio performance is evaluated.

Furthermore, financial wealth fluctuations determined by the success or failure of previous decisions (the cushion) exert a significant impact on the current portfolio allocation. They make investors without substantial cushion gains firmly refuse holding a large fraction of risky assets.

One year appears to be a critical evaluation frequency under practical market constraints, commonly used in practice, and optimal from the viewpoint of maximizing risky holdings in consequence of a more relaxed attitude. The individual perception of risky investments, captured by the prospective value, and the loss attitude measured by the global first-order risk aversion can be split into two segments with qualitatively distinct evolutions around this frequency of one year. Myopic loss aversion holds for the segment of evaluation frequencies of at least one year that can be considered as being the only one of practical relevance. However, the prospective value, which reflects the total impact of the evaluation frequency, reveals a somewhat ambiguous behavior when portfolio performance is checked at time intervals longer than one year. This is apparently due to the direct component of this total impact (i.e. the manifestation through expected returns), since the indirect component, that can be measured by means of the global first-order risk aversion, shows a more consistent evolution across all analyzed evaluation frequencies.

Finally, the estimation of variables aimed at establishing an equivalence between the theoretical portfolio optimization under exogenous VaR-constraints and our extended framework with individual VaR* (such as significance levels, loss aversion coefficients, and investments in risky assets) suggests an underestimation of the attitude of non-professional investors towards financial losses.

5 Appendix

5.1 Expectation and variance of the loss utility

According to Equations (8) and (9), we have:

$$\begin{aligned} E_t[\text{loss-value}_{t+1}] &= \pi_t(1-\psi_t)(\lambda S_t E_t[x_{t+1}] + (\lambda-1)(S_t - Z_t)R_{ft}) \\ &+ (1-\pi_t)(1-\omega_t)(\lambda S_t E_t[x_{t+1}] - k(S_t - Z_t)E_t[x_{t+1}]) \\ &\underset{\text{cond.}}{=} \lambda S_t E_t[x_{t+1}] + \Big(\pi_t(1-\psi_t)((\lambda-1)R_{ft} + kE_t[x_{t+1}]) - kE_t[x_{t+1}]\Big)(S_t - Z_t). \end{aligned} \tag{15}$$

Also, the expectation of the squared loss value and consequently the loss variance result in:

$$\begin{aligned} E_t[\text{loss-value}_{t+1}^2] &= \pi_t(1-\psi_t)(\lambda S_t E_t[x_{t+1}] + (\lambda-1)(S_t - Z_t)R_{ft})^2 \\ &+ (1-\pi_t)(1-\omega_t)(\lambda S_t E_t[x_{t+1}] - k(S_t - Z_t)E_t[x_{t+1}])^2 \\ &= (\lambda S_t E_t[x_{t+1}])^2 + \Big(\pi_t(1-\psi_t)((\lambda-1)^2 R_{ft}^2 - k^2 x_{t+1}^2) + k^2 x_{t+1}^2\Big)(S_t - Z_t)^2 \\ &+ 2\Big(\pi_t(1-\psi_t)((\lambda-1)R_{ft} + kE_t[x_{t+1}]) - kE_t[x_{t+1}]\Big)\lambda S_t E_t[x_{t+1}](S_t - Z_t), \end{aligned}$$

and

$$\begin{aligned} Var_t[\text{loss-value}_{t+1}] &= E_t[\text{loss-value}_{t+1}^2] - E_t^2[\text{loss-value}_{t+1}] \\ &= \pi_t(1-\psi_t)\Big(1 - \pi_t(1-\psi_t)\Big)\Big((\lambda-1)R_{ft} + kE_t[x_{t+1}]\Big)^2 (S_t - Z_t)^2. \end{aligned} \tag{16}$$

The variance of the loss value is exclusively based on past performance, being generated only by the cushion $S_t - Z_t$.

5.2 Descriptive statistics

Table 5: Log-difference of the S&P 500 index.

S&P 500	**Evaluation frequency**	
	Quarterly	Yearly
Mean	0.017	0.066
Median	0.018	0.071
Std.Dev.	0.079	0.136
Kurtosis	2.661	-0.9659
Skewness	-0.671	-0.205
Max.	0.290	0.345
Min.	-0.302	-0.207
Obs.	175	43

This table presents descriptive statistics of the log-difference R_t of the S&P 500 index for quarterly and yearly portfolio evaluations, corrected for dividends and stock splits. This index is used as proxy for the risky investment. The data series range from 01/02/1962 to 03/09/2006.

Table 6: 3-month T-bill log-returns.

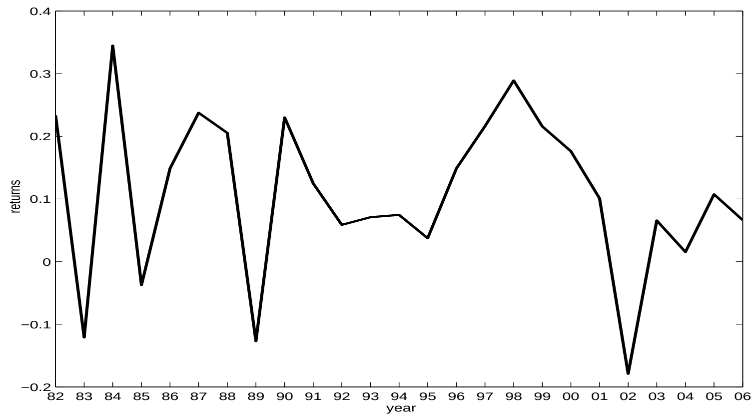
3-month T-bill	**Evaluation frequency**	
	Quarterly	Yearly
Mean	0.017	0.073
Median	0.017	0.070
Std.Dev.	0.006	0.026
Kurtosis	0.623	0.974
Skewness	0.951	1.042
Max.	0.036	0.142
Min.	0.009	0.037
Obs.	175	43

This table presents descriptive statistics of the 3-month T-bill log-returns R_f for quarterly and yearly portfolio evaluations, corrected for dividends and stock splits. This index is used as proxy for the risk-free investment. The data series range from 01/02/1962 to 03/09/2006.

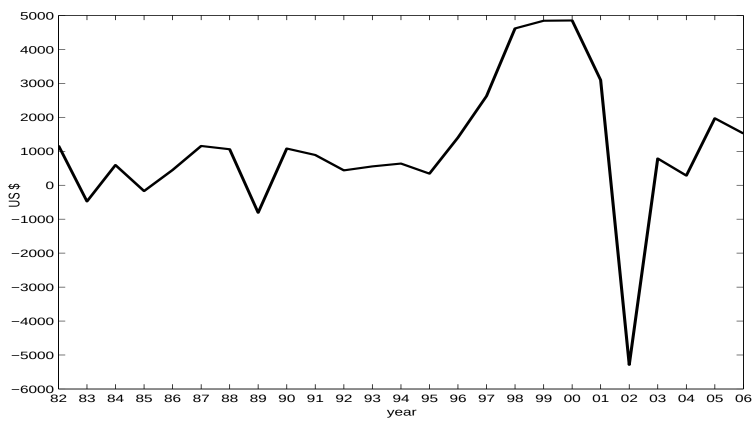
5.3 Results for the dynamic cushion

Figure 7: Evolution of risky returns, dynamic cushions, and wealth percentages invested in the risky portfolio for yearly portfolio evaluations.

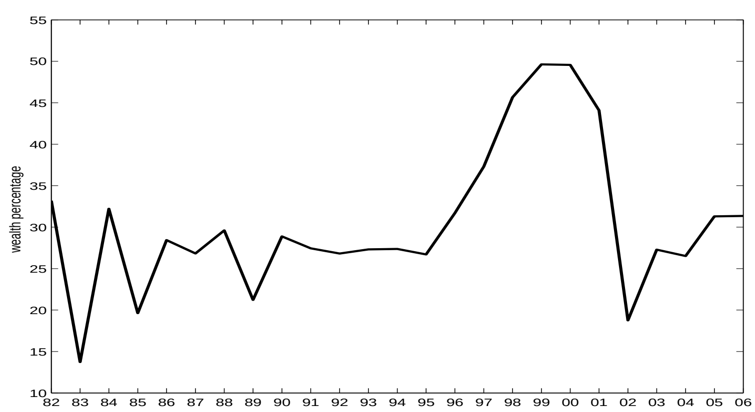
(a) Yearly S&P 500 log-returns.



(b) Yearly dynamic cushions.



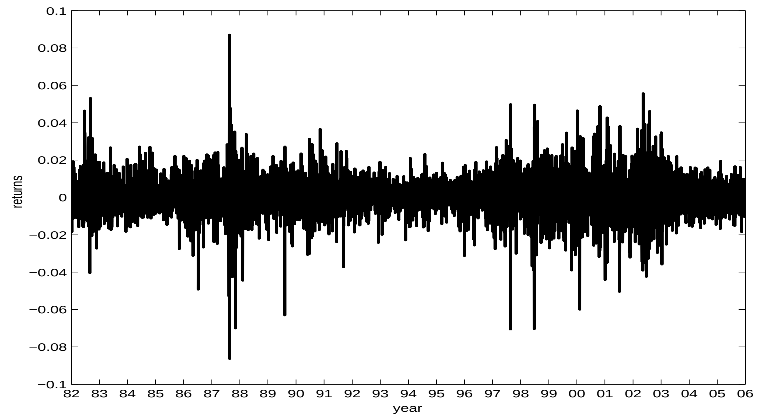
(c) Yearly wealth percentage invested in S&P 500.



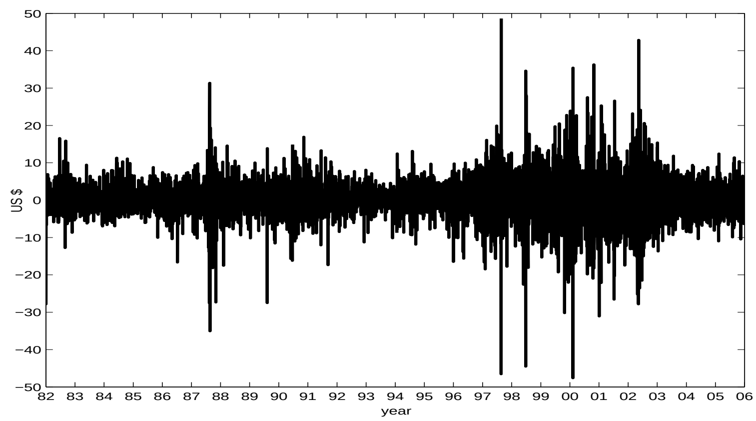
This figure illustrates the annual log-returns R_t of the index S&P 500, the corresponding yearly evolution (in US \$) of the past performance encompassed by the dynamic cushions $\eta(S_t - Z_{t-1}\bar{R})$, and the resulting yearly wealth percentages S_t/W_t invested in the risky portfolio. The wealth percentages are obtained from Equation (5), where VaR^{ex} is replaced by the VaR*-values from Equation (11) and the risky investment S_t results from Equation (6). We assume $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\dots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, $k = 3$, $\bar{R} = \text{mean}[R_t]$, and $\eta = 0.9$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to 1st March of each year.

Figure 8: Evolution of risky returns, dynamic cushions, and percentages invested in the risky portfolio for daily portfolio evaluations.

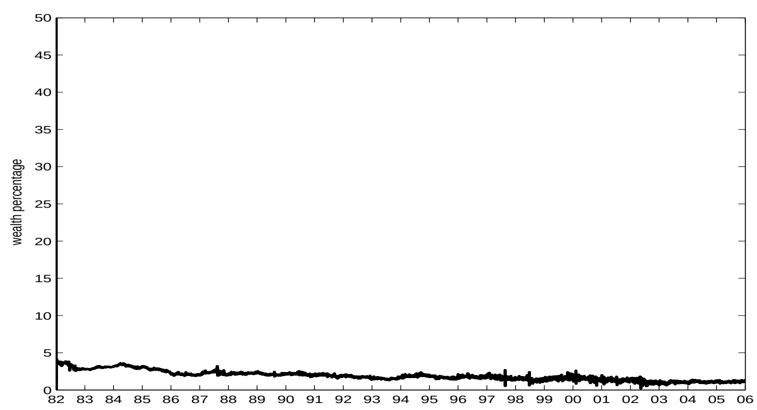
(a) Daily S&P 500 returns.



(b) Daily dynamic cushions.



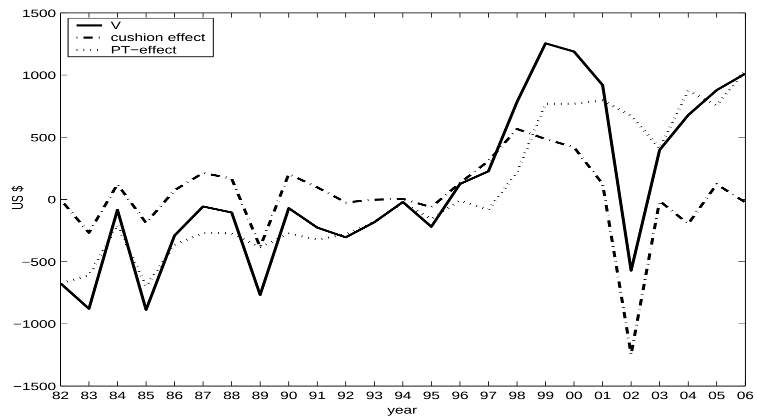
(c) Daily percentage investments in S&P 500.



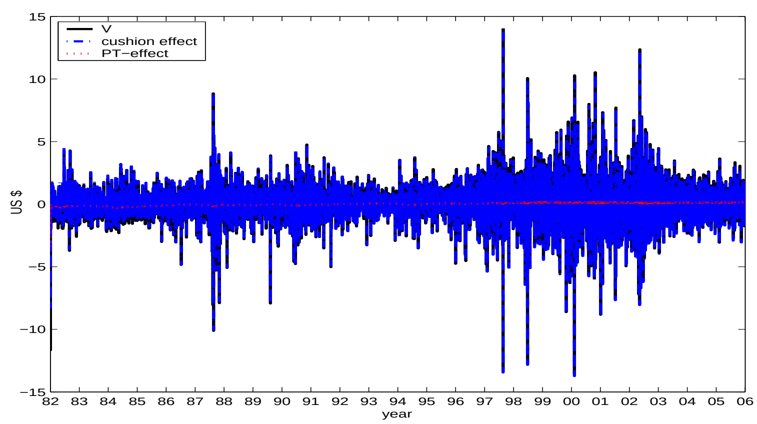
This figure illustrates the daily log-returns R_t of the index S&P 500, the corresponding daily past performance (in US \$) encompassed by the dynamic cushions $\eta(S_t - Z_{t-1}\bar{R})$, and the resulting daily wealth percentages S_t/W_t invested in the risky portfolio. The wealth percentages are obtained from Equation (5), where VaR^{ex} is replaced by the VaR*-values from Equation (11) and the risky investment S_t results from Equation (6). We assume $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\ldots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, $k = 3$, $\bar{R} = \text{mean}[R_t]$, and $\eta = 0.9$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to 1st March of each year.

Figure 9: Prospective value evolution for yearly and daily evaluations.

(a) Yearly evaluations.



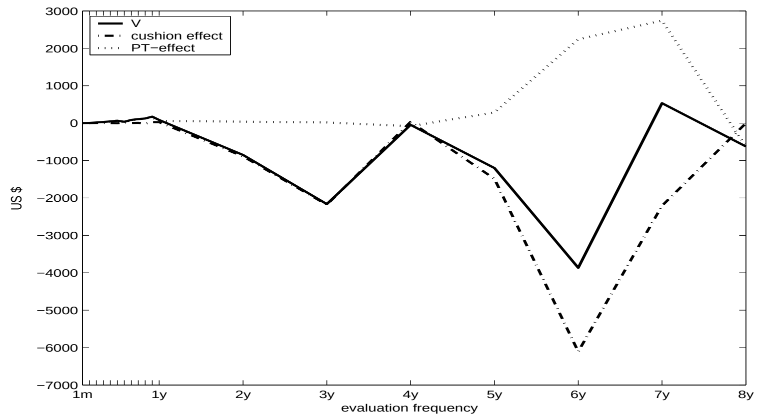
(b) Daily evaluations.



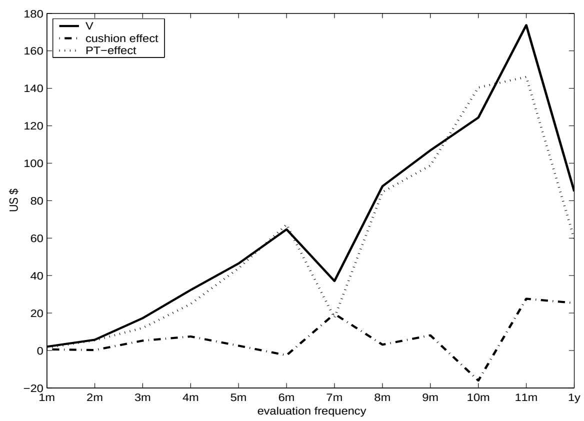
This figure illustrates the yearly and daily evolution of the prospective value V (in US \$) from Equation (13) and of its two components, the PT-effect and the cushion effect, that correspond to the two terms in this equation. The PT-effect corresponds to the representation of V in PT, without accounting for the influence of past performance, which is encompassed by the cushion effect. We assume dynamic cushions $\eta(S_t - Z_{t-1}\bar{R})$, $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\dots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, $k = 3$, $\bar{R} = \text{mean}[R_t]$, and $\eta = 0.9$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to 1st March of each year.

Figure 10: Prospective value evolution in for different evaluation frequencies.

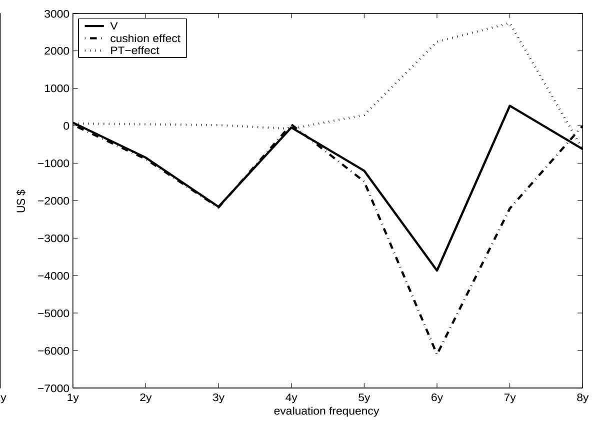
(a) All frequencies.



(b) $\tau \leq 1$ year.



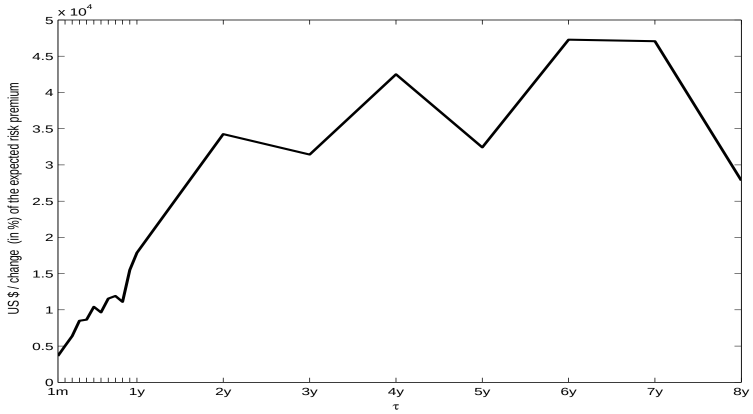
(c) $\tau \geq 1$ year.



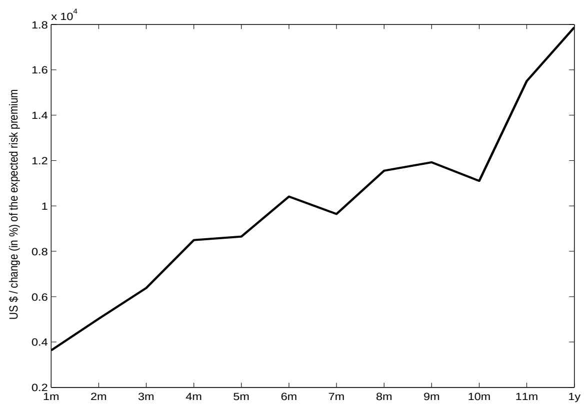
This figure illustrates the prospective value V (in US \$) from Equation (13) and its two components, the PT-effect and the cushion effect, as functions of the portfolio evaluation horizon τ. V reflects the perceived utility of risky investments and captures the total impact of τ on investor behavior (through expected returns and other model variables). The PT-effect corresponds to the representation of V in PT, without accounting for the influence of past performance, which is encompassed by the cushion effect. Higher V-values point to an increased utility of risky investments as perceived by non-professional investors. Panel a depicts the evolution of V for all evaluation horizons, panel b focuses on horizons up to one year (in monthly increments), and panel c on horizons from one to eight years (in yearly increments). We assume dynamic cushions $\eta(Z_{t-1}\bar{R} + S_t)$, $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\ldots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, $k = 3$, $\bar{R} = \text{mean}[R_t]$, and $\eta = 0.9$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006).

Figure 11: Evolution of the global first-order risk aversion for different evaluation frequencies.

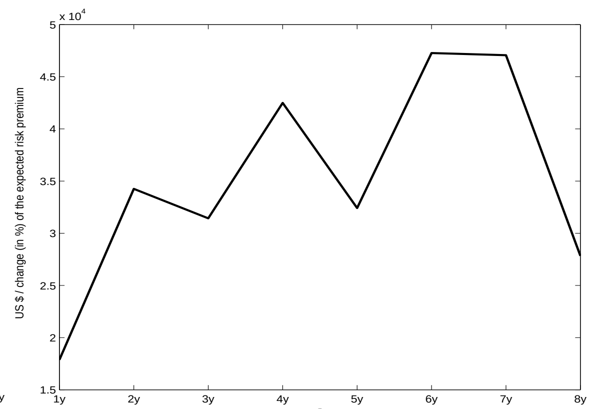
(a) All evaluation frequencies.



(b) $\tau \leq 1$ year.



(c) $\tau \geq 1$ year.



This figure illustrates the evolution of our measure of the loss attitude gRA (in US \$ per percentage change of the expected risk premium) from Equation (14) as function of the portfolio evaluation horizon τ. gRA reflects the sensitivity of the prospective value to the variation of expected returns. It captures merely the indirect impact of τ on investor behavior, i.e. through channels other than expected returns, such as the cushion, the probabilities of past gains and losses, etc. Higher gRA-values point to a more relaxed loss attitude. Panel a depicts gRA for all evaluation horizons, panel b focuses on horizons up to one year (in monthly increments), and panel c on horizons from one to eight years (in yearly increments). We assume dynamic cushions $\eta(Z_{t-1}\bar{R} + S_t)$, $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\ldots,t}{\text{mean}}[R_s]$, $\lambda = 2.25$, $k = 3$, $\bar{R} = \text{mean}[R_t]$, and $\eta = 0.9$. The sample covers 24 years of analysis (from 03/01/1983 to 03/01/2006).

Table 7: Wealth percentages invested in S&P 500 and the average λ^*, for $\alpha = 1\%$.

	Wealth %		λ^*	
Evaluation frequency	**Portfolio returns** Normal	Student-t	**Portfolio returns** Normal	Student-t
1 year	61.00	36.48	1.10	1.04
6 months	59.73	34.63	0.96	0.94
4 months	59.40	34.17	0.81	0.94
3 months	59.30	34.01	0.82	1.39
1 month	59.04	33.65	0.97	1.05
1 week	58.82	33.34	1.03	0.99
1 day	58.70	33.20	1.01	0.98

This table presents the average wealth percentages invested in the risky portfolio (first two columns) and the average loss-aversion coefficients λ^* from Equation (12) (last two columns) equivalent to a portfolio significance level $\alpha = 1\%$ at different evaluation horizons τ up to one year. The 100% level for the wealth percentages invested in risky assets corresponds to $\alpha = 5\%$ and $\lambda = 1$ points to a loss-neutral attitude. We assume dynamic cushions $\eta(S_t - Z_{t-1}\bar{R})$, standard normal and Student-t with 5 degrees of freedom distributed portfolio gross returns R_t, $\lambda = 2.25$, $k = 3$, $\bar{R} = \text{mean}[R_t]$, and $\eta = 0.9$.

Table 8: Wealth percentages invested in S&P 500 and the average λ^*, for $\alpha = 10\%$.

	Wealth %		λ^*	
Evaluation frequency	**Portfolio returns** Normal	**Portfolio returns** Student-t	**Portfolio returns** Normal	**Portfolio returns** Student-t
1 year	120.80	125.37	1.09	1.03
6 months	121.47	126.11	1.01	0.99
4 months	121.64	126.29	1.00	0.99
3 months	121.70	126.36	1.00	1.00
1 month	121.84	126.50	1.00	1.00
1 week	121.96	126.63	1.00	1.00
1 day	122.00	126.67	1.00	1.00

This table presents the average wealth percentages invested in the risky portfolio (first two columns) and the average loss-aversion coefficients λ^* from Equation (12) (last two columns) equivalent to a portfolio significance level $\alpha = 10\%$ at different evaluation horizons τ up to one year. The 100% level for the wealth percentages invested in risky assets corresponds to $\alpha = 5\%$ and $\lambda = 1$ points to a loss-neutral attitude. We assume dynamic cushions $\eta(S_t - Z_{t-1}\bar{R})$, standard normal and Student-t with 5 degrees of freedom distributed portfolio gross returns R_t, $\lambda = 2.25$, $k = 3$, $\bar{R} = \text{mean}[R_t]$, and $\eta = 0.9$.

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