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ABSTRACT:

In this paper we explore the performance of Experience Weighted Attraction (EWA) in two different auction institutions: First Price Sealed Bid, and Becker-DeGroot-Marschak. Our results suggest that learning has some promise as a possible explanation for previously documented cross-institutional choice anomalies usually attributed to risk aversion. Additionally, we present results on the likely econometric (ir)recoverability of EWA parameters in these institutions.

Keywords: Auctions, Risk Aversion, Learning, Experience Weighted Attractions

Do time series data from auctions tell us about risk aversion – or something else? Bidders' risk preference parameters play a key role in the Constant Relative Risk Aversion Model (CRRAM) of bidding in the first price sealed bid auction (FPSB). For that matter, inferring bidders' risk preference parameters is the whole point to the Becker-DeGroot-Marschak procedure (BDM), which can be thought of as a version of the second price sealed bid auction. But when researchers implement these institutions in laboratory experiments, the results from the FPSB and BDM contradict one another: results from the FPSB viewed in conjunction with CRRAM, imply that on average subjects are risk averse; results from BDM imply that on average subjects are risk seeking (Isaac and James (2000), Berg, Dickhaut, and McCabe (2004)). This raises the question of whether there is something instead of or in addition to risk preferences that is influencing these auction data time series.

Other attempts at addressing deviations from risk neutral Nash Equilibrium (RNNE) predictions have been made by other researchers. Cox, Roberson, and Smith (1982) suggest the possibility that the utility of the act of winning an auction – separate and additional to the expected utility of the different possible end-state monetary payoffs – could induce bidding in excess of the RNNE prediction (which CRRAM otherwise interprets as "risk aversion"). Engelbrecht-Wiggins (1989) models the possibility that the opportunity cost represented by foregone payoffs associated with strategies not chosen (dubbed "regret") might induce bidding in excess of the RNNE prediction. Andreoni and Miller (1995) examine the behavior of genetic algorithms in a number of institutional settings, and find some "overbidding". One can also refer to the record of a spirited debate, in which, among other things, the possibility that deviations from the RNNE prediction were "errors" due to insufficient motivation of the laboratory subject bidders was examined (Harrison (1989)). This explanation was pitted against rejoinders including that the deviations did not scatter symmetrically around the RNNE prediction, as might ordinarily be expected of errors (Cox, Smith, and Walker (1992)).

This paper seeks to examine the possibility that a learning model might explain some of the regularities – which are also anomalies – in this area. We choose to use Experience Weighted Attraction (EWA) learning as a possible explanation, and we do so because of its ability to nest a number of other learning models. This means that we can start with a relatively general model, and then investigate which specific parameter combinations within the general model allow for possible matching of experimental regularities or "stylized facts". Thus while some have argued that the various branches of the learning literature are engaged in an exercise which is inherently ad hoc, in choosing a fairly general model that is already in broad use we are minimizing the extent to which our own approach can be considered ad hoc.

On the contrary, this current paper could alternatively be viewed, if the reader is more interested in learning models than in auctions and auction data, as helping to extend and generalize the application of EWA. Additionally, in the course of trying to establish whether EWA can explain stylized facts in FPSB and BDM data, we have also made some findings bearing on the ability of researchers to estimate EWA parameters from FPSB and BDM data that comes from human subjects experiments, rather than the simulation-generated EWA behavior we use in this paper. This last set of results might be of most interest to econometricians, but we believe that it helps outline possible limitations to the application of EWA in these institutions that would be of interest to experimentalists.

The rest of this paper then deals with the behavior of EWA in BDM and the first price auction. Section 2 of this paper will review the related literature. Section 3 examines whether or not EWA bidding in BDM and the FPSB is consistent with the stylized facts about human bidding in those institutions. Section 4 views the performance of EWA in the FPSB and BDM from the standpoint of an econometrician: that is, it examines the recoverability of EWA parameters. Section 5 contains further discussion and concludes.

2. Background

2.1 Related Literature

Auction theory and the experimental testing thereof have come to treat bidder risk preferences as a key parameter in explaining bidding data – without which existing models would do a significantly poorer job of fitting actual bid data. A particularly clear example of this is the research program surrounding the FPSB.

The first model of bidding behavior in the FPSB, due to Vickrey, assumes that each bidder's value is independently and privately drawn from a uniform distribution whose support is common knowledge. Vickrey then derives that a utility-maximizing-risk-neutral bidder's bid, b_i, should be related to her own (private) value, v_i, for the object at auction according to:

$$b_i = [(n - 1)/n]v_i \qquad (1)$$

where n is the total number of bidders participating in the auction. This model has testable implications; in testing this model with data from induced value experiments, Coppinger, Smith, and Titus (1980) found that bids were statistically significantly higher than would be predicted by (1).

As a consequence, Cox, Roberson, and Smith (followed by Cox, Smith, and Walker (1982)) set out to amend Vickrey's original model in such a way as to encompass those empirical findings. They did so by allowing for risk aversion on the part of the bidders. Introducing non-risk-neutral preferences into the theory ultimately leads to a modified bid function:

$$b(v_i,r_i) = v_{lower} + [(N - 1)/(N - 1 + r_i)] (v_i - v_{lower}) \qquad (2)$$

where:

v_{lower} = the lower bound on the support, $[v_{lower}, v_{upper}]$, of a uniform distribution from which object values are drawn

v_i = bidder i's value of the object being auctioned

N = the number of bidders in the auction

r_i = the exponent in bidder i's Constant Relative Risk Aversion utility function.

This modified theory, the Constant Relative Risk Aversion Model (CRRAM), and its associated bid function are better able to fit data from experimental FPSB auctions (Cox, Smith, and Walker (1988). (That is, behavior which might appear as over-bidding in the original Vickrey model can be consistent with optimal bidding under CRRAM.)

The development of theory surrounding the FPSB stands as an example of scientific development within economics: an initial theory is developed, and then tested. When found to predict incorrectly, the initial theory is modified, and the modified theory is then itself tested. In the case of the FPSB, the modified theory (CRRAM) was found to predict more accurately than the initial theory (Vickrey's original model). Risk preferences play a key role in this process of improved modeling of the FPSB, as it is the introduction of heterogeneous and potentially risk averse preferences that allows a predictively more accurate bid function to be derived.

In contrast, BDM was developed at the outset as a way to measure risk preferences. Risk preferences are not added to the BDM as a way to better explain empirical data, but instead are assumed to be representable by expected utility, and recoverable as a consequence of optimal bidding behavior in the second price auction.

Specifically, an individual whose risk preferences have the expected utility property will value a two-state lottery as:

$$\text{Certainty Equivalent} = E[U(w)] = p_{high}[U(w_{high})] + p_{low}[U(w_{low})] \quad (3)$$

Also, an individual has a dominant strategy to bid their value in a second price auction, so their bid function is simply:

$$b_i = v_i = E[U(w)] \quad (4)$$

As such, were an expected utility maximizer to bid for a two-state lottery in a second price auction, we should be able to observe her certainty equivalent over the probabilities and payoffs associated with that lottery. In turn, an economist could then use such certainty equivalents to draw inferences about the utility functions of such expected utility maximizers.

It is at this point that the relationship between this current paper and the learning literature becomes apparent. While the role of risk preferences in the theoretical development of BDM or models of bidding in the FPSB is in each case quite clear, when implementing these institutions experimentally one encounters additional issues relating to how actual human subjects approach the tasks set them in each institution. For instance, what if subjects do not bid optimally as provided for by some bidding model – need one attempt to explain this by recourse to modeling of risk preferences, or are there alternative modeling constructs which might be employed? What if, over the course of multiple trials, subjects seem to change their approach to bidding – how does one explain this, and does this imply that their risk preferences have changed?

These questions suggest that there might be benefits to studying the effect(s) of learning on bidding behavior. Change in behavior over time is obviously something that learning models try to explain. Including an appropriate learning model in a theory of bidding might then possibly allow more accurate prediction *over time* of bidding behavior. That is, appending a learning model to a model of bidding in addition to information about risk preferences might explain previously unexplained variation. Alternatively, a learning model might simply replace information about risk preferences in formulation of a bidding model in an observationally equivalent fashion; in this case however, while learning and risk preferences might be said to be observationally equivalent in a particular institution (i.e., auction), they would presumably have different predictions in other institutions.

If one aims to document the effect of learning on bidding, a key question then becomes: which learning model does one use? There are many learning models from which to choose: we choose Experience Weighted Attraction (EWA), as it nests a number of otherwise competing learning models (Camerer and Ho, 1999).

In EWA, an agent makes decisions according to a difference equation system that includes variables that try to accommodate various cognitive processes. Most notable of these variables are δ, which parameterizes the ex post allure of payoffs to strategies other than the strategy chosen by the agent, and ϕ, which parameterizes the extent to which the agent remembers past play. Higher (lower) values of δ imply a greater (lesser) ex post "awareness" of the payoffs associated with other strategies than the strategy actually chosen by the agent. Higher (lower) values of ϕ are associated with longer (shorter) spans of memory of past "stimuli" (which could be either payoffs to actually chosen strategies or "imagined" foregone payoffs associated with strategies not taken, or both). The system of equations is then:

$$A^j_i(a,t) = \phi N(t-1)A^j_i(a,t-1)/N(t) + [\delta+(1-\delta)I(s^j_i,s_i(t))]\pi_i(s^j_i,s_{-i}(t))/N(t) \quad (5)$$

$$N(t) = (1-\kappa)\phi N(t-1) + 1 \quad \text{for } t = 1 \quad (6)$$

$$P^j_i(a, t+1) = e^{f(A(a,t))}/\Sigma e^{f(A(a,t))} \quad (7)$$

where: A^j : the "attraction" of a particular strategy j

N : a measure of experience

P^j_i = the probability that a particular strategy j is chosen by player i

I = an indicator function; I = 1 if $s^j_i=s_i(t)$, 0 otherwise

$\pi_i(s^j_i,s_{-i}(t))$ = payoff associated with strategy profile $(s^j_i,s_{-i}(t))$

κ : affects the accumulation of attractions

$f(\cdot)$ = a monotonically increasing function scaling the attractions

Once the system of equations is initialized, an EWA agent makes choices in the following manner. First, the agent calculates the attraction, $A^j(t)$, for each strategy j, based on past experience with actual and or foregone payoffs associated with the various strategy profiles. Second, the agent forms a probability measure over the possible strategies by employing the scaling of attractions implied by the equation for P^j_i. Finally, the agent draws a strategy choice from the probability measure so constructed.

One can alter $f(\cdot)$ to give more (or less) weight to strategies with relatively high attractions. For instance, if $f(A(a,t)) = \lambda(A(a,t))$, then as λ goes to infinity the strategy having the maximum attraction is chosen with probability one.[1] We vary the value of λ among 1, 10, and

[1] Proof available upon request.

infinity in order to document EWA's performance across varying levels of randomization over strategies.

One should note that the classic learning models which EWA nests as particular parameterizations have long-run convergence properties which have been characterized analytically. Two notable recent contributions in this area are those of Beggs (2002) and Hopkins (2002). Among their many respective results, the following are of direct interest for our paper. First, Beggs shows that in individual choice problems, reinforcement learning (e.g. $\delta=0$) converges towards expected value maximization. Second, Hopkins shows that versions of reinforcement learning and fictitious play (e.g. $\delta=1$) can both be characterized as transformations of replicator dynamics, and as such that both will converge to similar strategies, only at different speeds. (Fictitious play converges faster than reinforcement learning.) Hence we would expect that in the individual choice problems posed in this paper, that EWA would eventually tend toward expected value maximization. However, EWA's choices in the short run (say, over the length of a typical human subjects experiment) and in particular choice settings (auctions) may exhibit transitory asymmetries which might potentially reproduce broad patterns in human bidding. It is this possibility that is of primary interest in this paper.

2.2 Implementation of EWA

Having outlined the auction theoretic and learning models to be used in this study, we now wish to outline their implementation by computer. We implement FPSB by drawing a value for the EWA agent's opponent from a [$0.00, $2.00] uniform distribution, multiplying this by 1/N (which for two bidders, including the EWA agent, is ½), and comparing this number to the EWA agent's bid. If the EWA agent's bid is higher, it receives its object value minus its bid; otherwise, it receives zero. The EWA agent's object value is varied in cross-section, but not in time series, between $0.50, $1.00, and $1.50. For the n=2 case this implies risk neutral optimal bids of $0.25, $0.50, and $0.75, respectively, which are comparable to the risk neutral optimum in each case for BDM. Thus when reference is made to "holding constant the appropriate random draws", in the case of FPSB we mean that the same sequence of draws for the opponent's value is used.

We implement BDM by first making a draw of the opponent's bid from a uniform distribution on the support [$0, $2.00], and then comparing this value to the EWA agent's bid. If the EWA agent's bid is higher than that of the opponent, the EWA agent receives the results of a draw from a lottery having a 50% probability of the high state payoff (varied in cross-section, but not in time series, between $0.50, $1.00, and $1.50) and a 50% probability of the low state payoff, $0.00 (constant). If the EWA agent's bid is lower than that of the opponent, the EWA agent receives an amount equal to the opponent's bid. Thus for a single round of BDM, there are two random draws to be made. When reference is made to "holding constant the appropriate random draws", in the case of BDM we mean that the same sequence of draws for the opponent's bid *and* for the lottery outcome are used. Note also that when the high state payoff is varied between $0.50, $1.00, and $1.50 the corresponding risk neutral optima will be $0.25, $0.50, and $0.75, respectively.

Finally, we note that the processes just outlined need to be initialized. We vary the method by which we do this, to check for robustness of results. The key items in terms of initialization are initial attractions to strategies (that is, bids), and the initial bids. The initial attractions over strategies are set as either a) uniform (akin to uninformed priors) or b) proportional to expected values. Results using both approaches are reported, and labeled. The initial bid was varied on either side of the risk neutral optimum bid.

3. EWA bidding in auctions and its comparison with human bidding

We now document the “short run” behavior of EWA in two economic institutions in BDM and FPSB, and compare that EWA bidding to the stylized facts concerning bidding by human subjects. Put another way, if over less than asymptotic time sequences some parameterization of EWA fails to bid the risk neutral optimum, does it at least fail to do so in the same way that humans on average fail to do so?

We will address this issue by comparing cross-sectional mean EWA bids in each auction institution at particular points in time with canonical experimental results, or stylized facts, in each auction institution. We do for two reasons. First, there is arguably little to be gained by a more detailed, individual-subject-level analysis if this initial aggregate level comparison does not show a match between EWA bidding and human subject bidding. Second, that more detailed, individual-subject-level analysis strictly speaking requires new experiments using a design different from that employed in the past experiments that serve as the current basis for drawing empirical regularities. Specifically, the existing experimental data all come from experiments that employ non-stationary environments. This makes sense given that previous experimental work with human subjects has focused on issues such as the efficiency properties of different auction institutions, and not on, say, the reinforcement learning properties of those institutions. However, a clean comparison of period-by-period human behavior with period-by-period learning agent behavior should employ a stationary environment. Such human-subjects experimental results do not as yet exist, but we believe that this paper does establish the need for such results.

For now, we concentrate on whether EWA can reproduce the following broad patterns in human bidding data. The first stylized fact concerning FPSB with human subjects is that they tend to start out bidding above the risk neutral optimum bid (as discussed in Section 2). Second, human subjects do not alter their approach to bidding much over the course of a typical experiment (30 to 50 rounds of bidding); that is to say, they continue to bid above the risk neutral optimum over the course of a typical experiment. This is the case both in experiments in which each human subject faces other human subjects (Cox, Smith, and Walker (1988)), and in experiments where human subjects bid against risk neutral "robots" (Isaac and James 2000).

With regard to human subjects' bidding in BDM, we find that human subjects tend in the early rounds of an experiment to bid above the risk neutral optimum (Berg, Dickhaut, and McCabe (2004), Isaac and James (2000), James (2004)). However, unlike the data from FPSB, in BDM human subjects have been observed to alter their approach to bidding between the early and late rounds of an experiment. Specifically, the cross-sectional average of the human bidders' data moves from well above risk neutral within the early rounds of an experiment to close to risk neutral within the late rounds of that same experiment (James (2004)).

Thus we wish to determine whether there is a set of EWA parameters and agent starting bids that generates bidding behavior that is consistent with human subjects' bidding in *both* BDM and the FPSB, in both "early" and "late" rounds. For comparison with human experiments, we will take the 10^{th} round as our early measurement, and the 50^{th} round as our late measurement (though we continue running the simulations past the 500^{th} period).

To aid in subsequent discussion of bidding behavior over time, we now present plots of the time path of bids over 50 periods under different sets of EWA parameters, *but for the same value draws* (for the agent, and for the opponent, as appropriate in each of the FPSB and BDM).

For each graph, the players value is $1.00, therefore the auction theoretic optimal bid is $0.50.

We fix $\kappa=1$ and $\lambda=1$ in this example.

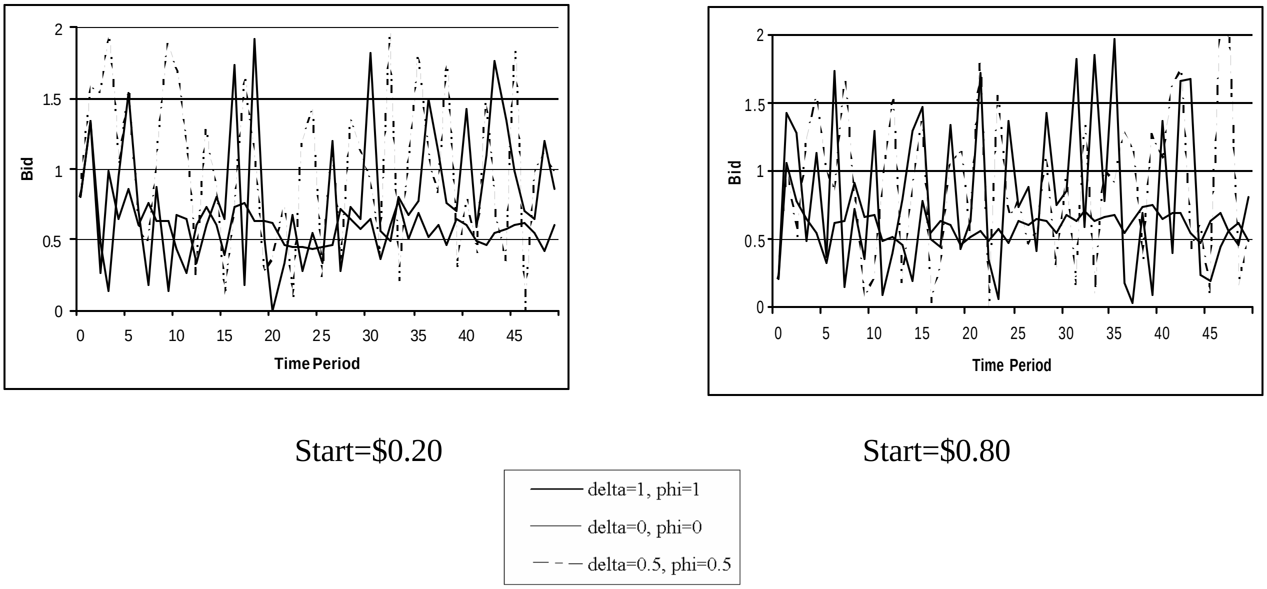


Figure 1: Time Path of Bids, FPSB

With BDM we observe the following examples.

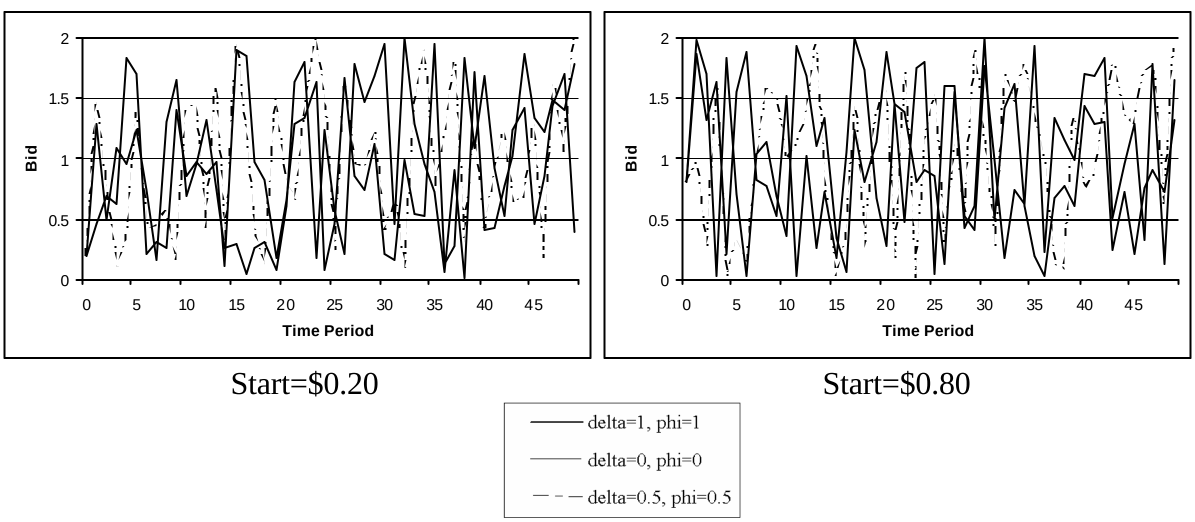


Figure 2: Time Path of Bids, BDM

These plots use 50 periods in order to demonstrate the functioning of the learning model over such a number of rounds as might be encountered in a human-subjects experiment. 50 periods does not give EWA players enough time to display anything like asymptotic behavior. To further examine the effect of the number of periods of bidding on convergence, we run Monte Carlo simulations. Since Camerer and Ho (1999) indicates that ϕ and δ are the most influential parameters on the bidding process, we fix $\kappa=1$ unless indicated.

Table 1 reports the Monte Carlo results for the FPSB for object values \$0.50, \$1.00, and \$1.50, which have the risk neutral optimal bids \$0.25, \$0.50, and \$0.75, respectively. For $\phi=\delta=1$, the (ensemble) mean bid appears to be converging to the relevant risk neutral optimum in each case as the time span gets longer. However, between Periods 2 and 50 we find that parameterizations featuring either $\phi<1.0$ or $\delta<0.5$, or both, "overbid". Furthermore, of those parameterizations, all but ($\phi=1.0,\delta=0$) continue to overbid in Period 500.

Table 1: FPSB, summary statistics of final bid in the respective period in 999 Monte Carlo trials, l =1, N=2

			Object Value=0.50				Object Value=1.00				Object Value=1.5			
Period	ϕ	δ	Start= $0.20		Start= $0.80		Start= $0.20		Start= $0.80		Start= $0.20		Start= $0.80	
2nd	1.0	1.0	0.76	*0.52*	0.72	*0.51*	0.82	*0.51*	0.81	*0.52*	0.87	*0.53*	0.91	*0.53*
	1.0	0.5	0.84	*0.56*	0.86	*0.56*	0.90	*0.55*	0.90	*0.55*	0.93	*0.55*	0.94	*0.55*
	1.0	0.0	1.02	*0.58*	0.98	*0.57*	1.00	*0.58*	1.02	*0.60*	1.01	*0.59*	1.02	*0.59*
	0.5	1.0	0.71	*0.52*	0.74	*0.52*	0.81	*0.52*	0.80	*0.52*	0.90	*0.51*	0.87	*0.51*
	0.5	0.5	0.91	*0.56*	0.86	*0.55*	0.93	*0.57*	0.90	*0.55*	0.97	*0.55*	0.94	*0.55*
	0.5	0.0	0.99	*0.60*	1.03	*0.58*	0.98	*0.58*	0.99	*0.56*	0.97	*0.59*	1.02	*0.59*
	0.0	1.0	0.78	*0.53*	0.73	*0.52*	0.81	*0.53*	0.82	*0.55*	0.92	*0.54*	0.86	*0.52*
	0.0	0.5	0.83	*0.55*	0.89	*0.57*	0.89	*0.56*	0.90	*0.55*	0.94	*0.56*	0.93	*0.54*
	0.0	0.0	1.01	*0.58*	0.99	*0.57*	0.97	*0.59*	1.00	*0.58*	1.01	*0.57*	0.98	*0.57*
10th	1.0	1.0	0.31	*0.19*	0.31	*0.19*	0.50	*0.23*	0.49	*0.23*	0.70	*0.24*	0.70	*0.24*
	1.0	0.5	0.39	*0.27*	0.40	*0.28*	0.53	*0.30*	0.56	*0.32*	0.74	*0.31*	0.74	*0.31*
	1.0	0.0	1.00	*0.58*	1.01	*0.60*	1.00	*0.58*	0.99	*0.58*	1.01	*0.58*	0.98	*0.57*
	0.5	1.0	0.58	*0.43*	0.56	*0.42*	0.71	*0.45*	0.70	*0.44*	0.83	*0.46*	0.80	*0.44*
	0.5	0.5	0.73	*0.49*	0.73	*0.51*	0.80	*0.53*	0.84	*0.51*	0.88	*0.52*	0.90	*0.51*
	0.5	0.0	1.00	*0.58*	1.02	*0.58*	1.00	*0.58*	1.01	*0.60*	0.99	*0.58*	1.01	*0.57*
	0.0	1.0	0.73	*0.52*	0.73	*0.53*	0.79	*0.52*	0.84	*0.54*	0.85	*0.51*	0.87	*0.53*
	0.0	0.5	0.87	*0.57*	0.84	*0.55*	0.89	*0.56*	0.90	*0.55*	0.91	*0.55*	0.94	*0.53*
	0.0	0.0	1.02	*0.57*	1.01	*0.59*	0.98	*0.58*	1.01	*0.58*	1.00	*0.59*	0.98	*0.57*
50th	1.0	1.0	0.24	*0.10*	0.24	*0.10*	0.49	*0.11*	0.49	*0.11*	0.73	*0.13*	0.72	*0.13*
	1.0	0.5	0.26	*0.13*	0.26	*0.12*	0.49	*0.14*	0.49	*0.15*	0.73	*0.14*	0.73	*0.14*
	1.0	0.0	0.94	*0.57*	0.96	*0.58*	0.97	*0.57*	0.99	*0.58*	0.94	*0.56*	0.92	*0.57*
	0.5	1.0	0.56	*0.42*	0.57	*0.42*	0.70	*0.44*	0.68	*0.44*	0.82	*0.45*	0.83	*0.45*
	0.5	0.5	0.72	*0.51*	0.72	*0.52*	0.82	*0.51*	0.80	*0.51*	0.88	*0.51*	0.88	*0.52*
	0.5	0.0	0.97	*0.57*	1.00	*0.58*	1.02	*0.56*	1.01	*0.58*	0.97	*0.58*	1.01	*0.56*
	0.0	1.0	0.73	*0.53*	0.74	*0.52*	0.79	*0.50*	0.82	*0.51*	0.90	*0.52*	0.90	*0.52*
	0.0	0.5	0.87	*0.56*	0.83	*0.56*	0.92	*0.55*	0.89	*0.55*	0.92	*0.53*	0.93	*0.54*
	0.0	0.0	1.02	*0.59*	0.98	*0.59*	1.01	*0.57*	0.98	*0.57*	0.98	*0.58*	1.00	*0.59*
500th	1.0	1.0	0.25	*0.04*	0.25	*0.04*	0.49	*0.04*	0.50	*0.05*	0.74	*0.06*	0.74	*0.06*
	1.0	0.5	0.25	*0.04*	0.25	*0.04*	0.49	*0.06*	0.49	*0.06*	0.73	*0.10*	0.73	*0.10*
	1.0	0.0	0.62	*0.50*	0.64	*0.51*	0.43	*0.15*	0.42	*0.14*	0.61	*0.21*	0.62	*0.20*
	0.5	1.0	0.55	*0.43*	0.57	*0.43*	0.69	*0.45*	0.67	*0.45*	0.82	*0.45*	0.83	*0.44*
	0.5	0.5	0.74	*0.52*	0.73	*0.51*	0.79	*0.50*	0.80	*0.53*	0.92	*0.52*	0.87	*0.52*
	0.5	0.0	1.00	*0.58*	1.01	*0.58*	0.97	*0.57*	0.98	*0.60*	0.99	*0.59*	1.01	*0.57*
	0.0	1.0	0.72	*0.52*	0.74	*0.53*	0.82	*0.53*	0.80	*0.51*	0.87	*0.52*	0.89	*0.51*
	0.0	0.5	0.84	*0.56*	0.86	*0.54*	0.89	*0.56*	0.91	*0.55*	0.94	*0.55*	0.91	*0.55*
	0.0	0.0	1.00	*0.58*	0.99	*0.58*	0.98	*0.58*	1.01	*0.57*	0.99	*0.57*	1.02	*0.58*

Table shows the mean final period bid of 999 draws of an EWA player with the respective parameters. The standard deviation of the 999 final bids is shown in italics.

While Table 1 presumes that $\lambda=1$, the effect(s) of alternative values of λ should be examined. Table 2 presents such results for varying values of λ. In doing so, we find that as λ increases from 1 to 10, the region supporting overbidding *shrinks* to ($\phi=0,\delta=0$), ($\phi=0.5,\delta=0$), ($\phi=0,\delta=0.5$) (notably *not* including ($\phi=0.5,\delta=0.5$)). Then, as λ increases from 10 to 8 , the region *expands* to reclaim ($\phi=0.5,\delta=0.5$) and add ($\phi=1.0,\delta=0$). We also note that as λ approaches infinity, a dependence on initial values develops, as the learning model seizes on early positive payoffs due to overbidding and makes this the basis of its subsequent bidding. One can see this in the difference in mean bids across different starting values, for given values of ϕ and δ, that is apparent for $\lambda=8$, but otherwise generally absent.

Table 2: FPSB, summary statistics of final bid in the respective period in 999 Monte Carlo trials, Object Value=$1.00

			λ=1				λ=10				λ=8			
			Start=		Start=		Start=		Start=		Start=		Start=	
Period	ϕ	δ	$0.20		$0.80		$0.20		$0.80		$0.20		$0.80	
2nd	1.0	1.0	0.82	*0.51*	0.81	*0.52*	0.48	*0.27*	0.48	*0.27*	0.50	*0.29*	0.50	*0.29*
	1.0	0.5	0.90	*0.55*	0.90	*0.55*	0.52	*0.32*	0.54	*0.31*	0.51	*0.28*	0.53	*0.30*
	1.0	0.0	1.00	*0.58*	1.02	*0.60*	0.91	*0.61*	0.98	*0.56*	0.90	*0.61*	0.83	*0.27*
	0.5	1.0	0.81	*0.52*	0.80	*0.52*	0.50	*0.26*	0.48	*0.26*	0.51	*0.30*	0.49	*0.29*
	0.5	0.5	0.93	*0.57*	0.90	*0.55*	0.52	*0.33*	0.53	*0.31*	0.50	*0.28*	0.52	*0.30*
	0.5	0.0	0.98	*0.58*	0.99	*0.56*	0.91	*0.61*	1.02	*0.58*	0.90	*0.61*	0.85	*0.26*
	0.0	1.0	0.81	*0.53*	0.82	*0.55*	0.48	*0.26*	0.48	*0.27*	0.50	*0.29*	0.49	*0.29*
	0.0	0.5	0.89	*0.56*	0.90	*0.55*	0.51	*0.33*	0.54	*0.31*	0.50	*0.28*	0.53	*0.31*
	0.0	0.0	0.97	*0.59*	1.00	*0.58*	0.89	*0.61*	0.99	*0.57*	0.93	*0.61*	0.83	*0.25*
10th	1.0	1.0	0.50	*0.23*	0.49	*0.23*	0.46	*0.15*	0.45	*0.15*	0.44	*0.15*	0.44	*0.16*
	1.0	0.5	0.53	*0.30*	0.56	*0.32*	0.45	*0.16*	0.47	*0.16*	0.48	*0.19*	0.51	*0.19*
	1.0	0.0	1.00	*0.58*	0.99	*0.58*	0.63	*0.51*	0.70	*0.51*	0.63	*0.33*	0.78	*0.13*
	0.5	1.0	0.71	*0.45*	0.70	*0.44*	0.47	*0.21*	0.46	*0.21*	0.42	*0.22*	0.43	*0.22*
	0.5	0.5	0.80	*0.53*	0.84	*0.51*	0.49	*0.23*	0.48	*0.24*	0.51	*0.20*	0.51	*0.19*
	0.5	0.0	1.00	*0.58*	1.01	*0.60*	0.94	*0.59*	0.92	*0.58*	0.65	*0.35*	0.78	*0.14*
	0.0	1.0	0.79	*0.52*	0.84	*0.54*	0.49	*0.27*	0.48	*0.27*	0.52	*0.28*	0.51	*0.29*
	0.0	0.5	0.89	*0.56*	0.90	*0.55*	0.54	*0.32*	0.51	*0.30*	0.55	*0.27*	0.53	*0.27*
	0.0	0.0	0.98	*0.58*	1.01	*0.58*	0.97	*0.60*	0.94	*0.57*	0.88	*0.43*	0.89	*0.43*
50th	1.0	1.0	0.49	*0.11*	0.49	*0.11*	0.47	*0.09*	0.48	*0.09*	0.47	*0.09*	0.48	*0.09*
	1.0	0.5	0.49	*0.14*	0.49	*0.15*	0.49	*0.12*	0.48	*0.13*	0.51	*0.15*	0.52	*0.16*
	1.0	0.0	0.97	*0.57*	0.99	*0.58*	0.42	*0.21*	0.48	*0.21*	0.59	*0.28*	0.77	*0.11*
	0.5	1.0	0.70	*0.44*	0.68	*0.44*	0.46	*0.21*	0.47	*0.21*	0.42	*0.22*	0.42	*0.22*
	0.5	0.5	0.82	*0.51*	0.80	*0.51*	0.48	*0.23*	0.49	*0.24*	0.52	*0.19*	0.52	*0.18*
	0.5	0.0	1.02	*0.56*	1.01	*0.58*	0.94	*0.57*	0.91	*0.59*	0.61	*0.28*	0.78	*0.11*
	0.0	1.0	0.79	*0.50*	0.82	*0.51*	0.48	*0.27*	0.49	*0.27*	0.50	*0.28*	0.49	*0.29*
	0.0	0.5	0.92	*0.55*	0.89	*0.55*	0.53	*0.31*	0.52	*0.32*	0.55	*0.26*	0.56	*0.27*
	0.0	0.0	1.01	*0.57*	0.98	*0.57*	0.95	*0.57*	0.93	*0.59*	0.91	*0.38*	0.89	*0.35*
500th	1.0	1.0	0.49	*0.04*	0.50	*0.05*	0.49	*0.05*	0.49	*0.05*	0.49	*0.04*	0.49	*0.04*
	1.0	0.5	0.49	*0.06*	0.49	*0.06*	0.48	*0.12*	0.48	*0.13*	0.51	*0.14*	0.52	*0.16*
	1.0	0.0	0.43	*0.15*	0.42	*0.14*	0.42	*0.21*	0.50	*0.20*	0.58	*0.29*	0.77	*0.11*
	0.5	1.0	0.69	*0.45*	0.67	*0.45*	0.46	*0.21*	0.46	*0.21*	0.44	*0.22*	0.42	*0.22*
	0.5	0.5	0.79	*0.50*	0.80	*0.53*	0.49	*0.23*	0.48	*0.23*	0.53	*0.18*	0.54	*0.18*
	0.5	0.0	0.97	*0.57*	0.98	*0.60*	0.94	*0.58*	0.91	*0.58*	0.58	*0.29*	0.77	*0.12*
	0.0	1.0	0.82	*0.53*	0.80	*0.51*	0.50	*0.26*	0.48	*0.26*	0.49	*0.29*	0.49	*0.29*
	0.0	0.5	0.89	*0.56*	0.91	*0.55*	0.54	*0.31*	0.54	*0.32*	0.53	*0.26*	0.54	*0.27*
	0.0	0.0	0.98	*0.58*	1.01	*0.57*	0.93	*0.59*	0.94	*0.57*	0.92	*0.32*	0.95	*0.32*

Table shows the mean final period bid of 999 draws of an EWA player with the respective parameters. The standard deviation of the 999 final bids is shown in italics.

There are two considerations in the case of the FPSB, which we will not encounter in the case of BDM, which we now address. First is the possibility of more than 2 bidders (including the EWA agent). One should check that the characterization of bidding extends to larger numbers of bidders, and hence to different risk neutral predictions. Table 3 verifies that our conclusions for N=2 are substantially unchanged when N=3 or N=4. Note that the risk neutral optima for the N=3 case and the N=4 case are ((2/3)Object Value) and ((3/4)Object Value), respectively.

Second, there is some controversy associated with what is allowed as a maximum bid by the agent. With human subjects we typically do not observe bidding more than value in the FPSB. However, were we to rule out bidding more than object value by the EWA agent, we might then add rationality to EWA that is not due to the model itself. This issue has arisen previously with respect to the research program of Gode and Sunder (1993). Gjerstad and Shachat (2004) show that disallowing "zero-intelligence" agents from bidding more than value is essential to achieving convergence in simulations of double auctions such as those conducted by Gode and Sunder, but that enforcing this restriction is equivalent to endowing agents with individual rationality. This in turn invalidates the description of agents as being "zero-intelligence". In our present paper, this issue arises in that we vary the value of the object on which the EWA agent bids. Do we then vary the support of possible attractions, in each case restricting attractions to the interval bounded by zero and the value of the object (that is, ruling out bidding more than object value)? To do so would be to imbue EWA with a rationality not inherent in it. On the other hand, do we allow a wider support than that suggested by object value, and allow at least some obviously irrational behavior like bidding more than object value? We have chosen the latter possibility, but explore an alternative in Table 4. This issue is not as pressing in BDM, as when it is implemented with human subjects, bidding more than maximum possible object value has been both allowed and observed.

Table 3: FPSB, summary statistics of final bid in the respective period in 999 Monte Carlo trials l =1, Object Value=$1.00

			2 Players				3 Players				4 Players			
			Start=		Start=		Start=		Start=		Start=		Start=	
Period	ϕ	δ	$0.20		$0.80		$0.20		$0.80		$0.20		$0.80	
2nd	1.0	1.0	0.82	*0.51*	0.81	*0.52*	0.85	*0.52*	0.84	*0.53*	0.82	*0.52*	0.87	*0.53*
	1.0	0.5	0.90	*0.55*	0.90	*0.55*	0.93	*0.55*	0.91	*0.56*	0.92	*0.55*	0.92	*0.56*
	1.0	0.0	1.00	*0.58*	1.02	*0.60*	0.96	*0.58*	1.01	*0.59*	1.00	*0.57*	0.98	*0.58*
	0.5	1.0	0.81	*0.52*	0.80	*0.52*	0.81	*0.51*	0.80	*0.53*	0.81	*0.52*	0.85	*0.53*
	0.5	0.5	0.93	*0.57*	0.90	*0.55*	0.90	*0.55*	0.92	*0.55*	0.93	*0.57*	0.90	*0.56*
	0.5	0.0	0.98	*0.58*	0.99	*0.56*	0.99	*0.58*	0.96	*0.59*	1.02	*0.58*	1.00	*0.57*
	0.0	1.0	0.81	*0.53*	0.82	*0.55*	0.85	*0.53*	0.85	*0.54*	0.82	*0.51*	0.84	*0.51*
	0.0	0.5	0.89	*0.56*	0.90	*0.55*	0.90	*0.56*	0.89	*0.57*	0.90	*0.56*	0.91	*0.55*
	0.0	0.0	0.97	*0.59*	1.00	*0.58*	1.00	*0.59*	0.97	*0.58*	0.98	*0.58*	1.01	*0.57*
10th	1.0	1.0	0.50	*0.23*	0.49	*0.23*	0.57	*0.26*	0.58	*0.27*	0.60	*0.30*	0.61	*0.28*
	1.0	0.5	0.53	*0.30*	0.56	*0.32*	0.61	*0.34*	0.63	*0.34*	0.64	*0.36*	0.63	*0.36*
	1.0	0.0	1.00	*0.58*	0.99	*0.58*	0.98	*0.58*	1.00	*0.58*	0.98	*0.58*	1.00	*0.59*
	0.5	1.0	0.71	*0.45*	0.70	*0.44*	0.74	*0.46*	0.75	*0.45*	0.76	*0.46*	0.74	*0.45*
	0.5	0.5	0.80	*0.53*	0.84	*0.51*	0.82	*0.52*	0.83	*0.53*	0.84	*0.51*	0.82	*0.52*
	0.5	0.0	1.00	*0.58*	1.01	*0.60*	0.98	*0.59*	0.98	*0.56*	0.99	*0.59*	0.99	*0.58*
	0.0	1.0	0.79	*0.52*	0.84	*0.54*	0.80	*0.52*	0.82	*0.52*	0.84	*0.52*	0.83	*0.52*
	0.0	0.5	0.89	*0.56*	0.90	*0.55*	0.91	*0.54*	0.95	*0.56*	0.92	*0.54*	0.92	*0.56*
	0.0	0.0	0.98	*0.58*	1.01	*0.58*	0.99	*0.58*	1.01	*0.58*	1.02	*0.59*	1.03	*0.57*
50th	1.0	1.0	0.49	*0.11*	0.49	*0.11*	0.64	*0.12*	0.64	*0.11*	0.71	*0.13*	0.71	*0.13*
	1.0	0.5	0.49	*0.14*	0.49	*0.15*	0.61	*0.17*	0.62	*0.17*	0.66	*0.20*	0.67	*0.19*
	1.0	0.0	0.97	*0.57*	0.99	*0.58*	0.94	*0.57*	1.00	*0.57*	0.98	*0.58*	0.97	*0.56*
	0.5	1.0	0.70	*0.44*	0.68	*0.44*	0.75	*0.46*	0.72	*0.45*	0.73	*0.47*	0.80	*0.46*
	0.5	0.5	0.82	*0.51*	0.80	*0.51*	0.83	*0.52*	0.84	*0.51*	0.85	*0.53*	0.84	*0.54*
	0.5	0.0	1.02	*0.56*	1.01	*0.58*	1.01	*0.58*	1.01	*0.58*	0.97	*0.58*	1.00	*0.56*
	0.0	1.0	0.79	*0.50*	0.82	*0.51*	0.82	*0.52*	0.85	*0.52*	0.87	*0.54*	0.84	*0.52*
	0.0	0.5	0.92	*0.55*	0.89	*0.55*	0.91	*0.57*	0.89	*0.55*	0.92	*0.56*	0.90	*0.57*
	0.0	0.0	1.01	*0.57*	0.98	*0.57*	1.00	*0.57*	1.01	*0.58*	0.99	*0.58*	1.03	*0.57*
500th	1.0	1.0	0.49	*0.04*	0.50	*0.05*	0.66	*0.04*	0.66	*0.04*	0.74	*0.04*	0.75	*0.04*
	1.0	0.5	0.49	*0.06*	0.49	*0.06*	0.65	*0.05*	0.66	*0.04*	0.74	*0.04*	0.74	*0.04*
	1.0	0.0	0.43	*0.15*	0.42	*0.14*	0.71	*0.42*	0.70	*0.42*	0.77	*0.48*	0.76	*0.49*
	0.5	1.0	0.69	*0.45*	0.67	*0.45*	0.74	*0.46*	0.74	*0.46*	0.76	*0.48*	0.77	*0.47*
	0.5	0.5	0.79	*0.50*	0.80	*0.53*	0.85	*0.52*	0.85	*0.52*	0.84	*0.52*	0.88	*0.53*
	0.5	0.0	0.97	*0.57*	0.98	*0.60*	1.01	*0.56*	1.00	*0.57*	1.00	*0.59*	0.98	*0.57*
	0.0	1.0	0.82	*0.53*	0.80	*0.51*	0.85	*0.52*	0.82	*0.52*	0.82	*0.51*	0.85	*0.53*
	0.0	0.5	0.89	*0.56*	0.91	*0.55*	0.90	*0.55*	0.95	*0.56*	0.92	*0.57*	0.94	*0.55*
	0.0	0.0	0.98	*0.58*	1.01	*0.57*	1.01	*0.58*	0.96	*0.58*	0.99	*0.58*	0.98	*0.56*

Table shows the mean final period bid of 999 draws of an EWA player with the respective parameters. The standard deviation of the 999 final bids is shown in italics.

Table 4: FPSB, summary statistics of final bid in the respective period in 999 Monte Carlo trials l =1, Object Value=$1.00, Strategies Truncated at $1.00, Median Bids Reported

			2 Players				3 Players				4 Players			
Period	φ	δ	Start= $0.20		Start= $0.80		Start= $0.20		Start= $0.80		Start= $0.20		Start= $0.80	
2nd	1.0	1.0	0.76	*0.33*	0.76	*0.32*	0.74	*0.34*	0.75	*0.33*	0.79	*0.33*	0.81	*0.34*
	1.0	0.5	0.88	*0.33*	0.89	*0.33*	0.89	*0.34*	0.82	*0.33*	0.87	*0.34*	0.84	*0.33*
	1.0	0.0	1.00	*0.33*	1.00	*0.32*	1.00	*0.32*	0.93	*0.33*	0.97	*0.34*	1.00	*0.32*
	0.5	1.0	0.77	*0.33*	0.74	*0.34*	0.78	*0.33*	0.82	*0.32*	0.80	*0.33*	0.81	*0.32*
	0.5	0.5	0.87	*0.33*	0.84	*0.33*	0.89	*0.33*	0.93	*0.32*	0.88	*0.32*	0.91	*0.32*
	0.5	0.0	0.98	*0.34*	1.00	*0.32*	0.99	*0.33*	1.00	*0.33*	0.99	*0.33*	0.94	*0.33*
	0.0	1.0	0.72	*0.33*	0.80	*0.33*	0.77	*0.34*	0.75	*0.34*	0.82	*0.33*	0.80	*0.33*
	0.0	0.5	0.88	*0.33*	0.85	*0.34*	0.87	*0.33*	0.90	*0.33*	0.87	*0.33*	0.84	*0.34*
	0.0	0.0	1.00	*0.31*	1.00	*0.32*	1.00	*0.32*	1.00	*0.31*	0.90	*0.33*	0.97	*0.33*
10th	1.0	1.0	0.48	*0.22*	0.50	*0.23*	0.62	*0.25*	0.60	*0.26*	0.62	*0.27*	0.64	*0.27*
	1.0	0.5	0.51	*0.28*	0.52	*0.28*	0.61	*0.30*	0.60	*0.29*	0.62	*0.30*	0.66	*0.30*
	1.0	0.0	1.00	*0.33*	0.97	*0.33*	0.98	*0.32*	0.99	*0.33*	1.00	*0.32*	0.96	*0.33*
	0.5	1.0	0.61	*0.32*	0.63	*0.32*	0.70	*0.32*	0.68	*0.33*	0.74	*0.32*	0.68	*0.33*
	0.5	0.5	0.75	*0.33*	0.80	*0.33*	0.75	*0.34*	0.81	*0.32*	0.80	*0.33*	0.81	*0.32*
	0.5	0.0	0.96	*0.32*	0.99	*0.33*	0.90	*0.33*	0.99	*0.33*	0.97	*0.33*	1.00	*0.32*
	0.0	1.0	0.77	*0.33*	0.76	*0.33*	0.76	*0.33*	0.83	*0.33*	0.80	*0.34*	0.80	*0.34*
	0.0	0.5	0.85	*0.33*	0.79	*0.34*	0.88	*0.32*	0.83	*0.33*	0.88	*0.33*	0.89	*0.32*
	0.0	0.0	1.00	*0.33*	1.00	*0.32*	1.00	*0.33*	1.00	*0.33*	0.93	*0.33*	0.98	*0.32*
50th	1.0	1.0	0.49	*0.12*	0.49	*0.11*	0.65	*0.12*	0.65	*0.12*	0.73	*0.12*	0.73	*0.12*
	1.0	0.5	0.50	*0.14*	0.49	*0.15*	0.64	*0.16*	0.64	*0.17*	0.69	*0.19*	0.70	*0.18*
	1.0	0.0	0.98	*0.32*	0.91	*0.34*	0.97	*0.33*	1.00	*0.32*	0.97	*0.32*	1.00	*0.32*
	0.5	1.0	0.67	*0.32*	0.64	*0.32*	0.66	*0.32*	0.66	*0.32*	0.72	*0.32*	0.74	*0.32*
	0.5	0.5	0.73	*0.33*	0.73	*0.33*	0.79	*0.33*	0.77	*0.33*	0.79	*0.33*	0.78	*0.34*
	0.5	0.0	1.00	*0.33*	0.99	*0.32*	1.00	*0.32*	1.00	*0.32*	0.96	*0.33*	1.00	*0.32*
	0.0	1.0	0.73	*0.32*	0.72	*0.33*	0.73	*0.33*	0.77	*0.32*	0.76	*0.34*	0.77	*0.33*
	0.0	0.5	0.87	*0.32*	0.85	*0.33*	0.93	*0.33*	0.88	*0.32*	0.87	*0.33*	0.86	*0.33*
	0.0	0.0	1.00	*0.33*	1.00	*0.33*	0.99	*0.32*	1.00	*0.32*	1.00	*0.32*	1.00	*0.32*
500th	1.0	1.0	0.49	*0.04*	0.49	*0.05*	0.66	*0.04*	0.66	*0.04*	0.75	*0.04*	0.74	*0.04*
	1.0	0.5	0.49	*0.06*	0.50	*0.06*	0.66	*0.05*	0.66	*0.05*	0.74	*0.04*	0.74	*0.04*
	1.0	0.0	0.42	*0.17*	0.42	*0.16*	0.75	*0.31*	0.77	*0.30*	0.90	*0.32*	0.91	*0.31*
	0.5	1.0	0.61	*0.32*	0.63	*0.31*	0.68	*0.31*	0.71	*0.32*	0.68	*0.32*	0.72	*0.33*
	0.5	0.5	0.77	*0.33*	0.70	*0.34*	0.82	*0.33*	0.79	*0.33*	0.81	*0.33*	0.78	*0.32*
	0.5	0.0	0.97	*0.33*	0.95	*0.32*	1.00	*0.33*	0.98	*0.32*	1.00	*0.32*	1.00	*0.32*
	0.0	1.0	0.75	*0.33*	0.76	*0.33*	0.80	*0.32*	0.80	*0.33*	0.81	*0.33*	0.79	*0.34*
	0.0	0.5	0.85	*0.33*	0.87	*0.33*	0.85	*0.33*	0.89	*0.33*	0.89	*0.32*	0.88	*0.34*
	0.0	0.0	0.96	*0.33*	1.00	*0.33*	0.97	*0.32*	1.00	*0.32*	1.00	*0.32*	1.00	*0.32*

Table shows the median final period bid of 999 draws of an EWA player with the respective parameters. The standard deviation of the 999 final bids is shown in italics.

Table 5 reports the Monte Carlo results for BDM for a range of "values" (i.e. high state payoffs) while holding λ=1. For ϕ=δ=1, the (ensemble) mean of the bids converges to the optimal bid by Period 500 except when the starting bid is $0.20 and the value is $1.50. Similar to the case of FPSB, we observe overbidding between Periods 2 and 50 for parameterizations featuring either ϕ<1.0 or δ<0.5, or both. However, given that one of the "stylized facts" for BDM is a movement towards the risk neutral optimum between Period 10 and Period 50, the parameterization (ϕ=1,δ=0) is the best match across institutions. (ϕ=1,δ=0) allows initial overbidding in BDM that reduces through time, and also allows sustained overbidding in the FPSB through the 50 period window that approximates the length of a human subjects experiment.

Table 5: BDM, summary statistics of final bid in the respective period in 999 Monte Carlo trials, l =1

			High Payoff =$0.50				High Payoff =$1.00				High Payoff =$1.50			
			Start=		Start=		Start=		Start=		Start=		Start=	
Period	φ	δ	$0.20		$0.80		$0.20		$0.80		$0.20		$0.80	
2nd	1.0	1.0	0.90	*0.56*	0.90	*0.56*	0.94	*0.54*	0.91	*0.58*	0.50	*0.57*	0.97	*0.58*
	1.0	0.5	0.93	*0.57*	0.96	*0.57*	0.93	*0.57*	0.96	*0.57*	0.51	*0.57*	0.99	*0.58*
	1.0	0.0	1.00	*0.59*	1.04	*0.58*	0.98	*0.58*	1.01	*0.57*	0.90	*0.59*	0.96	*0.58*
	0.5	1.0	0.90	*0.57*	0.90	*0.56*	0.91	*0.55*	0.90	*0.56*	0.51	*0.57*	0.98	*0.57*
	0.5	0.5	0.93	*0.58*	0.91	*0.56*	0.96	*0.56*	0.99	*0.57*	0.50	*0.58*	0.96	*0.56*
	0.5	0.0	0.99	*0.59*	1.01	*0.58*	0.98	*0.57*	1.00	*0.58*	0.90	*0.57*	0.99	*0.58*
	0.0	1.0	0.89	*0.56*	0.91	*0.56*	0.93	*0.57*	0.92	*0.56*	0.50	*0.57*	0.97	*0.56*
	0.0	0.5	0.93	*0.58*	0.93	*0.57*	0.96	*0.57*	0.96	*0.58*	0.50	*0.57*	0.98	*0.57*
	0.0	0.0	1.01	*0.58*	1.00	*0.58*	0.95	*0.58*	1.03	*0.58*	0.93	*0.59*	0.98	*0.58*
10th	1.0	1.0	0.52	*0.36*	0.52	*0.37*	0.65	*0.41*	0.64	*0.43*	0.44	*0.47*	0.85	*0.46*
	1.0	0.5	0.63	*0.45*	0.62	*0.45*	0.74	*0.48*	0.75	*0.48*	0.48	*0.51*	0.87	*0.50*
	1.0	0.0	0.97	*0.58*	1.00	*0.57*	0.97	*0.58*	0.98	*0.56*	0.63	*0.59*	1.00	*0.56*
	0.5	1.0	0.80	*0.54*	0.82	*0.54*	0.89	*0.54*	0.84	*0.54*	0.42	*0.54*	0.92	*0.54*
	0.5	0.5	0.86	*0.54*	0.87	*0.56*	0.89	*0.56*	0.93	*0.56*	0.51	*0.56*	0.99	*0.57*
	0.5	0.0	1.02	*0.60*	1.03	*0.57*	0.99	*0.59*	1.01	*0.58*	0.65	*0.59*	1.01	*0.57*
	0.0	1.0	0.89	*0.56*	0.88	*0.56*	0.96	*0.57*	0.98	*0.57*	0.52	*0.58*	0.97	*0.56*
	0.0	0.5	0.96	*0.55*	0.95	*0.57*	0.96	*0.57*	0.96	*0.57*	0.55	*0.58*	0.99	*0.55*
	0.0	0.0	0.99	*0.58*	0.99	*0.58*	1.00	*0.57*	1.01	*0.59*	0.88	*0.57*	0.98	*0.59*
50th	1.0	1.0	0.30	*0.18*	0.30	*0.18*	0.50	*0.23*	0.50	*0.23*	0.47	*0.27*	0.75	*0.27*
	1.0	0.5	0.31	*0.21*	0.32	*0.21*	0.52	*0.26*	0.52	*0.25*	0.51	*0.30*	0.76	*0.29*
	1.0	0.0	0.65	*0.49*	0.74	*0.46*	0.75	*0.52*	0.81	*0.46*	0.59	*0.54*	0.92	*0.48*
	0.5	1.0	0.79	*0.52*	0.78	*0.53*	0.85	*0.54*	0.87	*0.54*	0.42	*0.56*	0.91	*0.55*
	0.5	0.5	0.87	*0.56*	0.89	*0.56*	0.92	*0.55*	0.91	*0.57*	0.52	*0.55*	0.97	*0.57*
	0.5	0.0	1.01	*0.58*	1.02	*0.57*	1.03	*0.57*	1.01	*0.58*	0.61	*0.59*	1.01	*0.58*
	0.0	1.0	0.92	*0.55*	0.88	*0.56*	0.93	*0.57*	0.93	*0.56*	0.50	*0.58*	0.97	*0.56*
	0.0	0.5	0.97	*0.58*	0.98	*0.57*	0.96	*0.58*	0.96	*0.58*	0.55	*0.58*	1.00	*0.58*
	0.0	0.0	1.02	*0.59*	1.00	*0.57*	1.02	*0.58*	0.98	*0.59*	0.91	*0.58*	1.03	*0.59*
500th	1.0	1.0	0.24	*0.07*	0.25	*0.07*	0.50	*0.09*	0.50	*0.09*	0.49	*0.11*	0.74	*0.11*
	1.0	0.5	0.26	*0.16*	0.28	*0.16*	0.49	*0.21*	0.51	*0.21*	0.51	*0.26*	0.75	*0.24*
	1.0	0.0	0.61	*0.45*	0.69	*0.41*	0.73	*0.50*	0.78	*0.44*	0.58	*0.54*	0.94	*0.50*
	0.5	1.0	0.80	*0.53*	0.77	*0.51*	0.85	*0.54*	0.85	*0.54*	0.44	*0.54*	0.92	*0.54*
	0.5	0.5	0.88	*0.56*	0.91	*0.56*	0.92	*0.56*	0.91	*0.55*	0.53	*0.56*	0.94	*0.55*
	0.5	0.0	0.99	*0.59*	1.03	*0.57*	0.99	*0.58*	1.02	*0.58*	0.58	*0.57*	1.03	*0.59*
	0.0	1.0	0.90	*0.56*	0.88	*0.55*	0.92	*0.54*	0.93	*0.55*	0.49	*0.57*	0.97	*0.57*
	0.0	0.5	0.94	*0.57*	0.93	*0.56*	0.94	*0.58*	1.00	*0.57*	0.53	*0.58*	1.00	*0.58*
	0.0	0.0	0.98	*0.58*	1.01	*0.58*	0.98	*0.58*	1.00	*0.58*	0.92	*0.58*	1.01	*0.58*

Table shows the mean final period bid of 999 draws of an EWA player with the respective parameters. The standard deviation of the 999 final bids is shown in italics.

In examining the effect of varying λ on EWA bidding in BDM, we note that as λ increases, the importance of the initial bid in determining the path of subsequent bidding increases in what appears to be a monotonic manner. The effect of high λ is such that either extreme underbidding or extreme overbidding could be observed for the same parameterization of ϕ and δ, depending upon whether the initial bid is low or high, respectively. Furthermore, for high values of λ, bidding behavior does not converge towards the risk neutral optimum, but rather sticks near the initial bid. This suggests that λ=1, along with (ϕ=1,δ=0), supplies the best starting point for matching human data. The effect of varying λ for EWA within BDM is illustrated in Table 6.

Table 6: BDM, summary statistics of final bid in the respective period in 999 Monte Carlo High Payoff =$1.00

			λ=1				λ=10				λ=8			
			Start=		Start=		Start=		Start=		Start=		Start=	
Period	ϕ	δ	$0.20		$0.80		$0.20		$0.80		$0.20		$0.80	
2nd	1.0	1.0	0.48	*0.27*	0.48	*0.27*	0.80	*0.53*	0.80	*0.55*	0.91	*0.20*	0.91	*0.21*
	1.0	0.5	0.52	*0.32*	0.54	*0.31*	0.46	*0.55*	0.79	*0.38*	0.28	*0.36*	0.78	*0.13*
	1.0	0.0	0.91	*0.61*	0.98	*0.56*	0.33	*0.49*	0.84	*0.28*	0.17	*0.25*	0.84	*0.08*
	0.5	1.0	0.50	*0.26*	0.48	*0.26*	0.82	*0.53*	0.79	*0.55*	0.91	*0.21*	0.90	*0.22*
	0.5	0.5	0.52	*0.33*	0.53	*0.31*	0.48	*0.57*	0.82	*0.40*	0.23	*0.31*	0.76	*0.15*
	0.5	0.0	0.91	*0.61*	1.02	*0.58*	0.34	*0.49*	0.84	*0.25*	0.12	*0.14*	0.84	*0.08*
	0.0	1.0	0.48	*0.26*	0.48	*0.27*	0.80	*0.54*	0.80	*0.54*	0.76	*0.55*	0.74	*0.55*
	0.0	0.5	0.51	*0.33*	0.54	*0.31*	0.50	*0.58*	0.81	*0.39*	0.21	*0.36*	0.66	*0.27*
	0.0	0.0	0.89	*0.61*	0.99	*0.57*	0.30	*0.47*	0.83	*0.28*	0.13	*0.17*	0.84	*0.27*
10th	1.0	1.0	0.46	*0.15*	0.45	*0.15*	0.53	*0.32*	0.54	*0.32*	0.66	*0.28*	0.67	*0.28*
	1.0	0.5	0.45	*0.16*	0.47	*0.16*	0.34	*0.40*	0.70	*0.30*	0.28	*0.35*	0.77	*0.13*
	1.0	0.0	0.63	*0.51*	0.70	*0.51*	0.29	*0.44*	0.83	*0.25*	0.16	*0.22*	0.84	*0.08*
	0.5	1.0	0.47	*0.21*	0.46	*0.21*	0.62	*0.42*	0.64	*0.43*	0.61	*0.33*	0.61	*0.32*
	0.5	0.5	0.49	*0.23*	0.48	*0.24*	0.53	*0.44*	0.60	*0.38*	0.26	*0.32*	0.68	*0.22*
	0.5	0.0	0.94	*0.59*	0.92	*0.58*	0.31	*0.44*	0.82	*0.32*	0.12	*0.14*	0.84	*0.08*
	0.0	1.0	0.49	*0.27*	0.48	*0.27*	0.79	*0.54*	0.82	*0.54*	0.74	*0.53*	0.73	*0.53*
	0.0	0.5	0.54	*0.32*	0.51	*0.30*	0.77	*0.53*	0.76	*0.53*	0.38	*0.46*	0.41	*0.42*
	0.0	0.0	0.97	*0.60*	0.94	*0.57*	0.81	*0.55*	0.88	*0.50*	0.26	*0.41*	0.75	*0.54*
50th	1.0	1.0	0.47	*0.09*	0.48	*0.09*	0.50	*0.18*	0.50	*0.18*	0.53	*0.19*	0.52	*0.18*
	1.0	0.5	0.49	*0.12*	0.48	*0.13*	0.35	*0.38*	0.71	*0.30*	0.27	*0.34*	0.77	*0.13*
	1.0	0.0	0.42	*0.21*	0.48	*0.21*	0.26	*0.40*	0.83	*0.27*	0.17	*0.24*	0.84	*0.08*
	0.5	1.0	0.46	*0.21*	0.47	*0.21*	0.61	*0.42*	0.65	*0.43*	0.49	*0.33*	0.50	*0.33*
	0.5	0.5	0.48	*0.23*	0.49	*0.24*	0.51	*0.38*	0.51	*0.35*	0.28	*0.26*	0.46	*0.27*
	0.5	0.0	0.94	*0.57*	0.91	*0.59*	0.32	*0.40*	0.68	*0.38*	0.13	*0.15*	0.84	*0.08*
	0.0	1.0	0.48	*0.27*	0.49	*0.27*	0.78	*0.53*	0.79	*0.54*	0.73	*0.54*	0.76	*0.55*
	0.0	0.5	0.53	*0.31*	0.52	*0.32*	0.78	*0.52*	0.75	*0.52*	0.37	*0.45*	0.39	*0.45*
	0.0	0.0	0.95	*0.57*	0.93	*0.59*	0.84	*0.54*	0.81	*0.54*	0.41	*0.47*	0.50	*0.53*
500th	1.0	1.0	0.49	*0.05*	0.49	*0.05*	0.49	*0.09*	0.50	*0.08*	0.49	*0.08*	0.49	*0.08*
	1.0	0.5	0.48	*0.12*	0.48	*0.13*	0.32	*0.37*	0.69	*0.29*	0.28	*0.35*	0.76	*0.13*
	1.0	0.0	0.42	*0.21*	0.50	*0.20*	0.29	*0.44*	0.82	*0.26*	0.16	*0.23*	0.84	*0.08*
	0.5	1.0	0.46	*0.21*	0.46	*0.21*	0.64	*0.41*	0.64	*0.43*	0.50	*0.33*	0.50	*0.32*
	0.5	0.5	0.49	*0.23*	0.48	*0.23*	0.52	*0.37*	0.52	*0.37*	0.35	*0.16*	0.34	*0.16*
	0.5	0.0	0.94	*0.58*	0.91	*0.58*	0.46	*0.39*	0.47	*0.39*	0.12	*0.15*	0.84	*0.08*
	0.0	1.0	0.50	*0.26*	0.48	*0.26*	0.80	*0.55*	0.81	*0.55*	0.75	*0.55*	0.74	*0.53*
	0.0	0.5	0.54	*0.31*	0.54	*0.32*	0.77	*0.52*	0.79	*0.54*	0.39	*0.46*	0.40	*0.47*
	0.0	0.0	0.93	*0.59*	0.94	*0.57*	0.83	*0.52*	0.83	*0.55*	0.36	*0.49*	0.37	*0.48*

Table shows the mean final period bid of 999 draws of an EWA player with the respective parameters. The standard deviation of the 999 final bids is shown in italics.

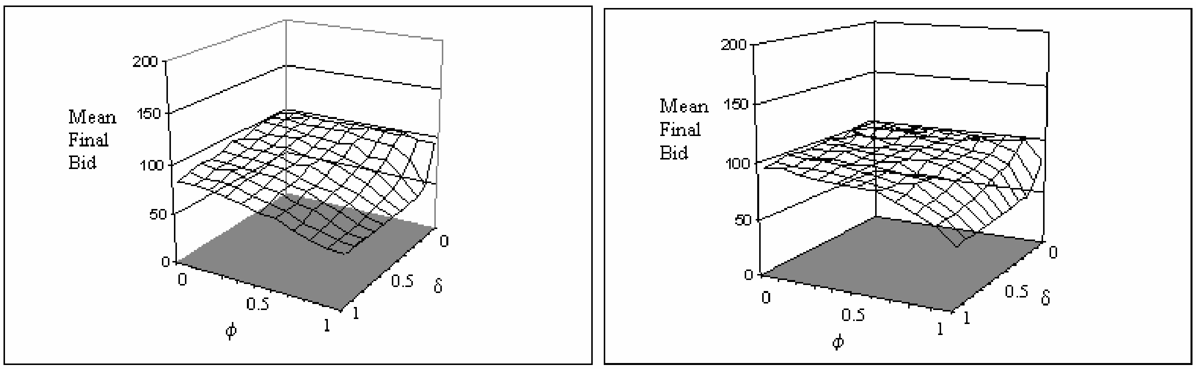
In the results so far, we have initialized the first-period attractions of the EWA agent in such a way as to assign equal attractiveness to all strategies. This seems like the most parsimonious a priori solution for a player entering the game. In much of the EWA literature, however, the relative frequency of each outcome is used to initialize the attractions. Since we are dealing with a game with a closed-form solution, for robustness we re-ran the trials using the expected payoff of each strategy to initialize the attractions. These results are reported in Table 7. This should swing the results in favor of the EWA player since they start the game with attractions that have a maximum at the optimum bid. As Table 7 shows, however, the 50th period results are virtually identical to those obtained using the uniform initialized attractions. This shows that players revert to sub-optimal strategies even when given a head start by means of "better informed" initial attractions to strategies.

Table 7: FPSB and BDM, summary statistics for 50th period under EV initial attraction 999 Monte Carlo trials, Object Value (High Payoff) =$1.00, λ =1

			E(V) initial attraction				κ=0			
Institution	φ	δ	Start= $0.10		Start= $0.80		Start= $0.20		Start= $0.80	
First Price	1.0	1.0	0.51	*0.11*	0.50	*0.11*	0.83	*0.54*	0.79	*0.52*
	1.0	0.5	0.51	*0.15*	0.51	*0.14*	0.89	*0.55*	0.91	*0.55*
	1.0	0.0	0.80	*0.50*	0.85	*0.52*	0.98	*0.57*	0.96	*0.58*
	0.5	1.0	0.69	*0.44*	0.69	*0.44*	0.80	*0.53*	0.80	*0.52*
	0.5	0.5	0.82	*0.52*	0.83	*0.52*	0.92	*0.55*	0.90	*0.55*
	0.5	0.0	1.03	*0.56*	1.01	*0.57*	0.96	*0.58*	1.02	*0.57*
	0.0	1.0	0.80	*0.51*	0.82	*0.53*	0.81	*0.52*	0.83	*0.53*
	0.0	0.5	0.90	*0.55*	0.89	*0.56*	0.88	*0.55*	0.90	*0.54*
	0.0	0.0	1.01	*0.58*	1.00	*0.58*	0.99	*0.58*	0.98	*0.59*
BDM	1.0	1.0	0.53	*0.23*	0.51	*0.22*	0.90	*0.54*	0.91	*0.55*
	1.0	0.5	0.54	*0.27*	0.53	*0.25*	0.97	*0.58*	0.93	*0.57*
	1.0	0.0	0.80	*0.49*	0.81	*0.42*	0.98	*0.57*	1.03	*0.57*
	0.5	1.0	0.88	*0.55*	0.88	*0.55*	0.95	*0.55*	0.92	*0.56*
	0.5	0.5	0.91	*0.55*	0.92	*0.55*	0.98	*0.57*	0.99	*0.57*
	0.5	0.0	1.00	*0.57*	1.01	*0.58*	1.02	*0.58*	1.01	*0.57*
	0.0	1.0	0.94	*0.56*	0.90	*0.56*	0.93	*0.57*	0.94	*0.57*
	0.0	0.5	0.99	*0.58*	0.96	*0.59*	0.96	*0.58*	0.98	*0.57*
	0.0	0.0	1.03	*0.58*	1.00	*0.58*	1.01	*0.58*	0.99	*0.57*

Table shows the mean final period bid of 999 draws of an EWA player with the respective parameters. The standard deviation of the 999 final bids is shown in italics.

As a check on and further illustration of the effect of low δ in both institutions, we present two further types of graphs. Figure 3 shows how mean 50th period bids vary as a function of ϕ and δ, given a starting bid of \$0.80, for each institution.



FPSB BDM

Figure 3: Effect of d and f on Final Bid Conditional on Starting Bid=\$0.80

One can see that δ close to 0 is necessary to observe overbidding in period 50 in FPSB if one also maintains that ϕ is close to 1. That ϕ would need to be close to 1 is established by the convergence in mean in BDM data toward risk neutral bidding that takes place between period 10 and period 50 when ϕ is close to 1. This is needed to at least broadly match the data on human bidding over similar intervals in BDM from James (2004). (In contrast, for lower ϕ, EWA bidding in BDM does not start out above the risk neutral optimum and then converge towards it, but instead tends to stick near the (exogenously determined) initial bid.) To summarize then, the desire to match human BDM data would rule out all parameter combinations including ϕ much less than 1. In turn, given this, the desire to match human FPSB data would rule out all parameter combinations with δ much greater than 0. Both of these results in turn work better with λ=1, rather than $\lambda=10$, or $\lambda=\infty$. Hence we would describe the region of EWA parameters that holds most promise for generating simulated data not inconsistent with that generated by human subjects as ($\delta\approx0$, $\phi\approx1$,$\lambda\approx1$).

4. Recoverability of EWA parameters

4.1 Effects on attractions of varying EWA parameters while holding value draws constant

Our next objective is to assess how much each of the parameters in EWA affects the agent's attraction to each strategy. After all, the attractions are the determinant of agent behavior for any given choice.

We use the Kolmogorov-Smirnov (K-S)[2] goodness of fit test as a way to compare different sets of probability cdf's obtained from the pdf on strategies from equation (7) rewritten below.

$$P^j_i(a, t+1) = e^{f(A(a,t))}/\Sigma e^{f(A(a,t))} \qquad (7)$$

We do so for sets of attractions built up after the agent has participated in a sequence of 50 auctions. In this way, we compare sets of attractions for agents having different underlying values of the EWA parameters, holding constant the random draws necessitated by each institution. This allows us to judge if the graph of attractions to strategies is significantly affected by changing the EWA parameters. We do this for both the FPSB, and BDM.

[2] We opt for the K-S test instead of the Likelihood Ratio (LR) test due to the additional data available with the Monte Carlo simulations. We are working with the entire cdf on all strategies for a single point in time rather than a time series of observed bids used in the LR test.

4.1a First Price Sealed Bid Auction

For the FPSB our findings are as follows. Table 8 reports the K-S test statistic for combinations of δ, ϕ, and κ versus a null of $\delta=\phi=\kappa=1$.[3,4] The areas of insignificance suggest difficulty in estimating parameters of EWA using maximum likelihood since there are large regions of the strategy space where the parameters do not significantly affect the graph of attractions. For example, $\delta=0$, $\phi=.4$, and $\kappa=0$ is statistically indistinguishable from $\delta=\phi=\kappa=1$.

[3] The K-S results are reported for an initial bid of $0.80. We calculated the results for other initial bids. Bids farther away from optimal produced comparable results, while bids closer to optimal showed even less effect of parameter changes on the graph of attractions.

[4] The same set of random draws (as appropriate to the institution) is used throughout each table of Kolmogorov-Smirnoff results. As such the K-S results can be said to reflect differences in combinations of model parameters, and not just sampling variation in the draws.

Table 8: FPSB, l =1, Value=1, K-S goodness of fit test of listed parameters versus d =1, f =1

δ↓	ϕ→0	0.2	0.4	0.6	0.8	1	kappa=0.0
0.0	0.1492*	0.1147*	0.0810	0.0484	0.0307	0.0319	
0.2	0.1486*	0.1060*	0.0648	0.0284	0.0295	0.0485	
0.4	0.1476*	0.0921	0.0398	0.0238	0.0540	0.0943	
0.6	0.1465*	0.0645	0.0126	0.0718	0.1247*	0.1693*	
0.8	0.1416*	0.0239	0.1354*	0.2125*	0.2664*	0.3048*	
1.0	0.1505*	0.6607*	0.6394*	0.5464*	0.4921*	0.4970*	
0.0	0.1503*	0.1225*	0.0951	0.0684	0.0426	0.0316	kappa=0.2
0.2	0.1498*	0.1154*	0.0819	0.0495	0.0298	0.0306	
0.4	0.1490*	0.1041*	0.0612	0.0254	0.0254	0.0533	
0.6	0.1481*	0.0817	0.0211	0.0340	0.0819	0.1238*	
0.8	0.1447*	0.0145	0.0908	0.1667*	0.2230*	0.2649*	
1.0	0.0759	0.6536*	0.5529*	0.4928*	0.4796*	0.4863*	
0.0	0.1512*	0.1301*	0.1094*	0.0890	0.0689	0.0493	kappa=0.4
0.2	0.1508*	0.1248*	0.0993*	0.0743	0.0501	0.0308	
0.4	0.1503*	0.1163*	0.0833	0.0517	0.0268	0.0265	
0.6	0.1495*	0.0991*	0.0519	0.0172	0.0328	0.0695	
0.8	0.1475*	0.0476	0.0400	0.1092*	0.1646*	0.2087*	
1.0	0.1055*	0.3495*	0.4261*	0.4455*	0.4598*	0.4698*	
0.0	0.1519*	0.1378*	0.1238*	0.1099*	0.0962*	0.0827	kappa=0.6
0.2	0.1517*	0.1342*	0.1169*	0.0999*	0.0832	0.0667	
0.4	0.1513*	0.1285*	0.1061*	0.0842	0.0628	0.0421	
0.6	0.1509*	0.1169*	0.0842	0.0529	0.0232	0.0173	
0.8	0.1493*	0.0812	0.0191	0.0373	0.0854	0.1269*	
1.0	0.1257*	0.2639*	0.3638*	0.4043*	0.4263*	0.4416*	
0.0	0.1525*	0.1454*	0.1384*	0.1313*	0.1243*	0.1173*	kappa=0.8
0.2	0.1524*	0.1436*	0.1349*	0.1262*	0.1176*	0.1090*	
0.4	0.1522*	0.1407*	0.1293*	0.1180*	0.1068*	0.0958	
0.6	0.1519*	0.1348*	0.1180*	0.1015*	0.0853	0.0694	
0.8	0.1512*	0.1165*	0.0830	0.0511	0.0209	0.0109	
1.0	0.1413*	0.1226*	0.2516*	0.3161*	0.3531*	0.3777*	
0.0	0.1530*	0.1523*	0.1516*	0.1508*	0.1501*	0.1494*	kappa=1.0
0.2	0.1530*	0.1521*	0.1512*	0.1503*	0.1494*	0.1486*	
0.4	0.1530*	0.1518*	0.1506*	0.1495*	0.1483*	0.1472*	
0.6	0.1529*	0.1512*	0.1495*	0.1478*	0.1460*	0.1443*	
0.8	0.1528*	0.1493*	0.1458*	0.1423*	0.1388*	0.1353*	
1.0	0.1519*	0.1187*	0.0867	0.0562	0.0273	0.0000	

*significant at the 0.05 level

In order aid the reader in developing an intuitive idea of the attractions, Figure 4 shows the graph of the attractions after the 50^{th} period of bidding under three different parameter specifications, with a starting bid of $0.80.

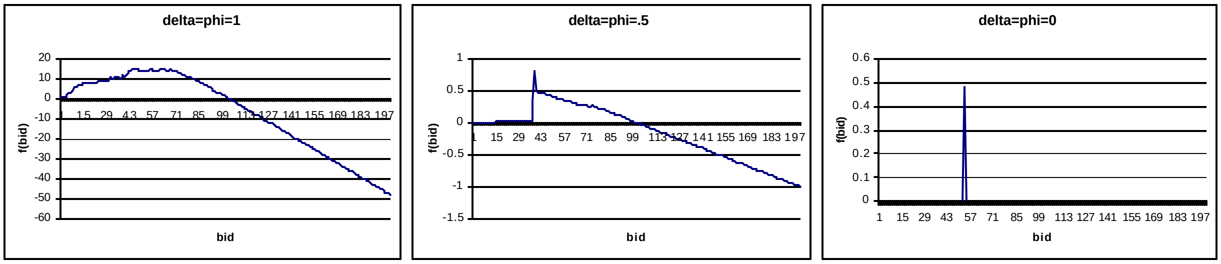


Figure 4: FPSB, Graph of Attractions

For ϕ=δ=1, the right-hand half of the pdf reflects the expected payoffs relatively well. The left-hand half deviates from expectations based on low realizations of the computer's value where low bids would temporarily pay off. As the parameters get farther from 1, information is ignored, and we see a spike in attraction for strategies that are used more often. Finally, as ϕ=δ=0, all other strategies are ignored, and only the initial bid shows positive attraction.

4.1b Becker-DeGroot-Marschak

Our findings regarding the recoverability of EWA parameters in BDM are even less promising than the findings from the case of the FPSB. Large parameter changes are needed for any change in the cdf to register as statistically significant. In fact, ϕ=1, δ=0 (reinforcement learning) is not statistically distinguishable from ϕ=δ=1 (fictitious play) for κ from 1 to 0.2. For κ=1, there is no combination of ϕ and δ that are statistically significant from ϕ=δ=1.

Table 9: BDM, l =1, value=1, K-S goodness of fit test of listed parameters versus d =1, f =1

		φ→0.0	0.2	0.4	0.6	0.8	1.0	kappa=0.0
δ↓	0.0	0.0763	0.0562	0.0388	0.0479	0.0844	0.1208*	
	0.2	0.0759	0.0547	0.0353	0.0468	0.0818	0.1167*	
	0.4	0.0752	0.0461	0.0197	0.0576	0.0955	0.1332*	
	0.6	0.0757	0.0255	0.0423	0.0889	0.1343*	0.1773*	
	0.8	0.0772	0.0287	0.1034*	0.1610*	0.2159*	0.2602*	
	1.0	0.3633*	0.3954*	0.4573*	0.4778*	0.4945*	0.5067*	
	0.0	0.0769	0.0604	0.0456	0.0314	0.0552	0.0844	kappa=0.2
	0.2	0.0765	0.0594	0.0435	0.0280	0.0531	0.0811	
	0.4	0.0759	0.0526	0.0299	0.0340	0.0644	0.0949	
	0.6	0.0762	0.0359	0.0222	0.0602	0.0976*	0.1338*	
	0.8	0.0772	0.0114	0.0750	0.1259*	0.1724*	0.2146*	
	1.0	0.1494*	0.3365*	0.4192*	0.4549*	0.4777*	0.4928*	
	0.0	0.0773	0.0649	0.0534	0.0426	0.0314	0.0479	kappa=0.4
	0.2	0.0771	0.0641	0.0517	0.0400	0.0283	0.0455	
	0.4	0.0766	0.0590	0.0417	0.0250	0.0333	0.0562	
	0.6	0.0766	0.0464	0.0181	0.0310	0.0596	0.0878	
	0.8	0.0772	0.0178	0.0429	0.0881	0.1251*	0.1607*	
	1.0	0.0666	0.2775*	0.3765*	0.4189*	0.4488*	0.4688*	
	0.0	0.0778	0.0694	0.0612	0.0537	0.0464	0.0388	kappa=0.6
	0.2	0.0776	0.0689	0.0603	0.0521	0.0443	0.0365	
	0.4	0.0773	0.0655	0.0538	0.0423	0.0309	0.0201	
	0.6	0.0769	0.0570	0.0377	0.0189	0.0207	0.0399	
	0.8	0.0773	0.0369	0.0091	0.0418	0.0730	0.1004*	
	1.0	0.0703	0.1872*	0.3034*	0.3640*	0.3992*	0.4237*	
	0.0	0.0781	0.0739	0.0697	0.0656	0.0615	0.0576	kappa=0.8
	0.2	0.0780	0.0737	0.0693	0.0650	0.0607	0.0564	
	0.4	0.0779	0.0720	0.0661	0.0602	0.0543	0.0485	
	0.6	0.0777	0.0677	0.0578	0.0481	0.0385	0.0290	
	0.8	0.0774	0.0568	0.0371	0.0183	0.0084	0.0248	
	1.0	0.0749	0.0728	0.1767*	0.2465*	0.2932*	0.3285*	
	0.0	0.0784	0.0780	0.0776	0.0771	0.0767	0.0763	kappa=1.0
	0.2	0.0784	0.0780	0.0775	0.0771	0.0767	0.0762	
	0.4	0.0784	0.0778	0.0772	0.0766	0.0760	0.0754	
	0.6	0.0784	0.0774	0.0764	0.0754	0.0744	0.0734	
	0.8	0.0783	0.0762	0.0742	0.0721	0.0700	0.0680	
	1.0	0.0778	0.0616	0.0456	0.0300	0.0148	0.0000	

* significant at the 0.05 level

We now show examples of EWA attractions in BDM for different parameter sets.

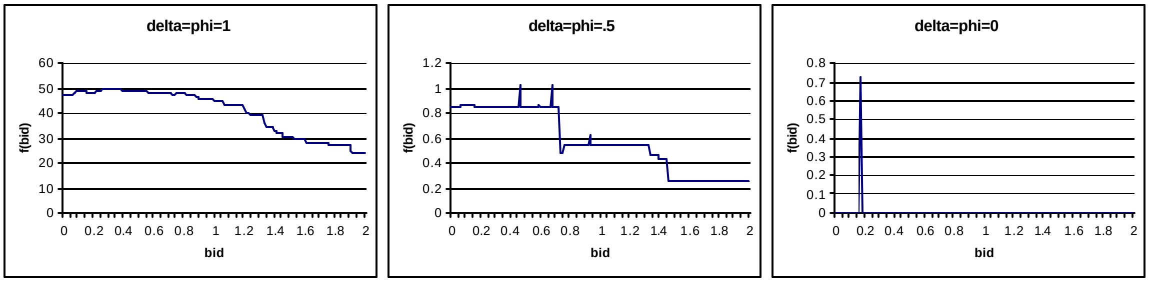


Figure 5: BDM, Graph of Attractions

These graphs of attractions illustrate that changes in EWA parameterization appear to make a difference to the attractions, which in turn leads to different behavior, but that fairly extreme changes in parameters are needed to guarantee that the resulting cdf can be distinguished statistically once passed through the logit transformation.

4.2 Effects on attractions of varying value draws while holding EWA parameters constant

One way of further examining the sources of potential difficulties in estimating EWA parameters is to see whether a K-S test rejects the similarity of cdf's that are all generated by the same underlying EWA parameters. To do this, we ran a Monte Carlo simulation of 999 trials, wherein the relevant random draws for each institution are redrawn at each trial, but the EWA parameters stay constant throughout all 999 trials. Probabilities over 5% indicate a greater Type I error than specified by K-S test, and that the cdf on strategies is not unique for given parameter values.

The results for each institution are as follows in Tables 10 and 11, respectively.

Table 10: FPSB, K-S goodness of fit test of listed parameters versus 999 Monte Carlo simulations

		$\phi\rightarrow$0.0	0.2	0.4	0.6	0.8	1.0	kappa=0.0
$\delta\downarrow$	0.0	0%	0%	0%	0%	5%	11%	
	0.2	0%	0%	0%	0%	4%	10%	
	0.4	0%	0%	0%	0%	0%	4%	
	0.6	0%	0%	0%	0%	0%	1%	
	0.8	0%	0%	0%	0%	0%	0%	
	1.0	0%	0%	0%	0%	0%	0%	
	0.0	0%	0%	0%	0%	2%	18%	kappa=0.2
	0.2	0%	0%	0%	0%	0%	15%	
	0.4	0%	0%	0%	0%	0%	8%	
	0.6	0%	0%	0%	0%	0%	6%	
	0.8	0%	0%	0%	0%	0%	0%	
	1.0	0%	0%	0%	17%	39%	54%	
	0.0	0%	0%	0%	0%	11%	15%	kappa=0.4
	0.2	0%	0%	0%	0%	9%	12%	
	0.4	0%	0%	0%	0%	6%	7%	
	0.6	0%	0%	0%	0%	1%	9%	
	0.8	0%	0%	0%	0%	1%	10%	
	1.0	0%	5%	42%	67%	88%	87%	
	0.0	0%	0%	0%	0%	9%	18%	kappa=0.6
	0.2	0%	0%	0%	0%	8%	15%	
	0.4	0%	0%	0%	0%	7%	8%	
	0.6	0%	0%	0%	1%	6%	19%	
	0.8	0%	0%	0%	0%	9%	16%	
	1.0	0%	32%	72%	84%	90%	89%	
	0.0	0%	0%	0%	0%	8%	12%	kappa=0.8
	0.2	0%	0%	0%	0%	9%	19%	
	0.4	0%	0%	0%	2%	9%	19%	
	0.6	0%	0%	0%	2%	11%	19%	
	0.8	0%	0%	0%	3%	22%	42%	
	1.0	0%	47%	80%	97%	93%	97%	
	0.0	0%	0%	0%	0%	9%	14%	kappa=1.0
	0.2	0%	0%	0%	1%	6%	17%	
	0.4	0%	0%	0%	2%	18%	16%	
	0.6	0%	0%	0%	4%	19%	31%	
	0.8	0%	0%	0%	8%	31%	54%	
	1.0	0%	75%	96%	95%	96%	99%	

Table 11: BDM, K-S goodness of fit test of listed parameters versus 999 Monte Carlo simulations

		ϕ→0.0	0.2	0.4	0.6	0.8	1.0	kappa=0.0
δ↓	0.0	100%	100%	100%	100%	81%	59%	
	0.2	100%	100%	100%	100%	100%	55%	
	0.4	100%	100%	100%	100%	100%	49%	
	0.6	100%	100%	100%	100%	100%	27%	
	0.8	100%	100%	100%	100%	100%	11%	
	1.0	100%	100%	100%	100%	100%	0%	
	0.0	100%	100%	100%	100%	77%	62%	kappa=0.2
	0.2	100%	100%	100%	100%	100%	55%	
	0.4	100%	100%	100%	100%	100%	55%	
	0.6	100%	100%	100%	100%	100%	43%	
	0.8	100%	100%	100%	100%	100%	35%	
	1.0	100%	100%	100%	100%	100%	99%	
	0.0	100%	100%	100%	100%	77%	72%	kappa=0.4
	0.2	100%	100%	100%	100%	100%	74%	
	0.4	100%	100%	100%	100%	100%	54%	
	0.6	100%	100%	100%	100%	100%	62%	
	0.8	100%	100%	100%	100%	100%	58%	
	1.0	100%	97%	100%	100%	100%	100%	
	0.0	100%	100%	100%	100%	100%	63%	kappa=0.6
	0.2	100%	100%	100%	100%	100%	68%	
	0.4	100%	100%	100%	100%	100%	64%	
	0.6	100%	100%	100%	100%	100%	69%	
	0.8	100%	100%	100%	100%	100%	78%	
	1.0	77%	19%	100%	39%	100%	100%	
	0.0	100%	100%	100%	100%	100%	56%	kappa=0.8
	0.2	100%	100%	100%	100%	100%	62%	
	0.4	100%	100%	100%	100%	100%	67%	
	0.6	100%	100%	100%	100%	100%	63%	
	0.8	100%	100%	100%	100%	100%	89%	
	1.0	91%	63%	100%	99%	100%	100%	
	0.0	100%	100%	100%	100%	70%	66%	kappa=1.0
	0.2	100%	100%	100%	100%	100%	64%	
	0.4	100%	100%	100%	100%	100%	70%	
	0.6	100%	100%	100%	100%	100%	80%	
	0.8	100%	100%	100%	100%	100%	88%	
	1.0	4%	100%	96%	86%	100%	100%	

For BDM, we see that except for a handful of parameter sets with ϕ or δ equal to1, K-S rejects similarity of graphs of attractions even though those attractions were generated subject to the same EWA parameters. Put another way, the random draws dominate the EWA parameters in shaping the cdf. This suggests that estimating EWA parameters from BDM data generated in an environment involving use of random draws might be problematic. FPSB shows similar concerns, but in another region of the parameter set where ϕ or δ equals1.

Indeed, the overall results from the various sub-sections of Section 4 of this paper support the conclusion reached by Salmon (2001) that identifying which learning model parameters underlie the generation of a particular data set can be problematic. Our results from 4.1 demonstrate that attractions are often unresponsive to changes in EWA parameters, conditional on the agent facing the same set of random draws (appropriate to the institution). Put another way, different parameter sets could map to much the same behavior. Conversely, our results from section 4.2 show that quite different probability distributions on strategies can evolve from the same EWA parameter sets, provided that sufficiently different sets of random draws (appropriate to the institution) confront the agent. That is, the same parameter sets could map to very different behavior. Clearly parameter estimation is a daunting task under such conditions.

5. Conclusion

On the basis of what we have learned in this paper, EWA learning would seem to have both promise and practical limitations in further application to auctions. On the one hand, at an aggregate level, EWA is able to match human behavior across institutions in a way that models intended to explain behavior in a single institution fail to do. On the other hand, there would seem to be tremendous difficulties awaiting the researcher who tried to estimate EWA parameters for data from a single institution should the actual EWA parameters she is trying to estimate happen to lie in one of the regions of the parameter space where the attractions (and by extension, the behavior based on those attractions) associated with different parameter sets simply cannot be distinguished statistically. On the other hand, looking at data from multiple institutions, as we have done in this paper, may in addition to helping resolve empirical anomalies also help matters econometrically. For example, while there is a fairly large parameter region that generates behavior not inconsistent with, say, stylized facts about the FPSB, the intersection of this region with the region generating behavior not inconsistent with behavior in BDM is much smaller.

In future research we aim to explore the correspondence between EWA and human subject behavior at the individual subject level. In order to do this we must conduct new human subjects experiments, the design of which will be informed by the results of this current paper.

REFERENCES

Andreoni, James and John H. Miller "Auctions with Artificial Adaptive Agents." *Games and Economic Behavior*, 1995, 10, 39-64.

Becker, Gordon M., Morris H. DeGroot, and Jacob Marschak. "Measuring Utility by a Single-Response Sequential Method." *Behavioral Science*, 1964, 9, 226-232.

Beggs, Alan. "On the Convergence of Reinforcement Learning." *Oxford University Discussion Paper*, 2002, no. 96.

Berg, Joyce, John Dickhaut, and Kevin McCabe. "Risk Preference Instability Across Institutions: A Dilemma." *Proceedings of the National Academy of Sciences,* forthcoming.

Camerer, Colin and T. H. Ho. "Experience-Weighted Attraction Learning in Normal Form Games." *Econometrica*, 1999, 67, 827-874.

Camerer, Colin, T.H. Ho, J.K. Chong. "Sophisticated Experience-Weighted Attraction Learning and Strategic Teaching in Repeated Games." *Journal of Economic Theory*, 2002, 104, 137-188.

Coppinger, Vicki M., Vernon L. Smith, and Jon A. Titus. "Incentives and Behavior in English, Dutch, and Sealed-Bid Auctions." *Economic Inquiry*, 1980, 18,1-22.

Cox, James C., Bruce Roberson, and Vernon L. Smith. "Theory and Behavior of Single

Object Auctions." In: Vernon L. Smith, ed., *Research in Experimental Economics*, Vol. 2, Greenwich, CT: JAI Press, 1982.

Cox, James C., Vernon L. Smith, and James M. Walker. "Auction Market Theory of Heterogeneous Bidders." *Economics Letters*, 1982, 9, 319-325.

Cox, James C., Vernon L. Smith, and James M. Walker. "Theory and Individual Behavior of First-Price Auctions." *Journal of Risk and Uncertainty*, 1988, 1, 61-99.

Cox, James C., Vernon L. Smith and James M. Walker. "Theory and Misbehavior of First-Price Auctions: Comment." *American Economic Review*, 1992, 82, 1392-1412.

Engelbrecht-Wiggins, John. "The Effect of Regret on Optimal Bidding in Auctions." *Management Science*, 1989, 685-692.

Gjerstad, Steven and Jason Shachat. "Individual Rationality and Market Efficiency." Typescript, 2004.

Gode, D. K. and Shyam Sunder. "Allocative Efficiency of Markets with Zero Intelligence Traders: Market as a Partial Substitute for Rationality." *Journal of Political Economy,* 1993, 101, 119-137.

Harrison, Glenn. "Theory and Misbehavior of First-Price Auctions." *American Economic Review*, 1989, 74, 749-762.

Hopkins, Ed. "Two Competing Models of How People Learn in Games." *Econometrica*, 2002, 70, 2141-2166.

Isaac, R. Mark and Duncan James. "Just Who Are You Calling Risk Averse?" *Journal of Risk and Uncertainty*, 2000, 20, 177-187.

James, Duncan. "Stability of Risk Preference Parameter Estimates Within the Becker-DeGroot-Marschak Procedure." Working paper, 2004.

Salmon, Timothy C. "An Evaluation of Econometric Models of Adaptive Learning." *Econometrica*, 2001, 69, 1597-1638.

Vickrey, William. "Counterspeculation, Auctions, and Competitive Sealed Tenders." *Journal of Finance*, 1961, 16, 8-37.