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Abstract

A central issue for managers or investors in portfolio management of assets is to select the assets to be included and to predict the value of the portfolio, given a variety of historical and concurrent information regarding each asset in the portfolio. There exist several criteria or models to predict asset returns, which in turn are sensitive to underlying probability distributions, their unknown parameters, whether it is a bull, bear or flat period subject to further uncertainty regarding switch times between bull and bear periods. It is possible to treat various portfolio-choice criteria as multiple criterion systems in the uncertain world of asset markets from historical market data. This paper develops the initial framework for the selection of assets using information fusion to combine these multiple criterion systems. These MCS' are combined, using the recently developed Combinatorial Fusion Analysis (CFA) to enhance the portfolio performance. We demonstrate with an example using US stock market data that combination of multiple criteria (or models) systems does indeed improve the portfolio performance.

JEL classification G11; C14; D81
Keywords: Rank-score function, combinatorial fusion, stock performance, return of equity

1.Introduction

In managing a portfolio system, investors (or managers) aim to assemble a portfolio which can achieve the highest possible (optimal) return. However, perfect optimality is an elusive goal in the uncertain world of asset markets based on past data, since the past data cannot reveal what the future might hold. Based on information such as historical performance of each of the assets, the investor uses different criteria or models to select assets to be included in the portfolio. A variety of criteria have been used such as: price/earning (P/E ratio), earnings per share (EPS), price/book value (P/BV) ratio, net margin, (net income/net revenue) ratio, and many others. The two most popular models for portfolio management are the Capital Asset Pricing

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Model (CAPM) and the Arbitrage Pricing Theory (APT). Both rely on the mean-variance interrelationship among the assets in the portfolio. It is possible to incorporate utility theory into risk management, some non-linear structures such as neural networks have also been used to forecast returns, Vinod and Reagle (2005).

If the market uncertainty could be characterized by the bell-shaped normal distribution, mean-variance models do indeed yield optimal portfolios. It is possible to view the market uncertainty as a matter of picking the right model for a probability distribution. For example, the Pearson family of distributions can yield a very large variety of shapes based on a handful of parameters. More generally, there are lognormal, inverse-gaussian, Azzalini skew-normal, Pareto-Levy-type stable distributions. Again, if one knew the correct parameters of the correct probability distribution describing the future market, the optimal portfolio can be obtained, Vinod and Reagle (2005). Unfortunately, there remains uncertainty regarding the choice of the correct distribution. Asset market professionals often distinguish between bull, bear and flat markets for various assets at various times, noting that different kinds of uncertainty (probability distributions) apply for bull versus bear markets, while they obsess about the switch from a bull to bear market and vice versa. Even if we know the right probability distribution, one needs to contend with estimation uncertainty, (See Vinod and Reagle (2005), Vinod and Morey (2001, 2002)) about the parameters of the probability distribution based on limited historical data. In short, the "information" contained in historical market data is difficult to use as the number of model choices and underlying uncertainty increases. Let us view the stock market data as huge and diverse, ready to be exploratory "mined." Following computer scientists, we can also abandon the search for optimality and view portfolio choice in the absence of fully known probability models as a guided application of a computer algorithm.

In this paper, we propose to use the Combinatorial Fusion Algorithm (CFA) in the selection of assets to be included in a portfolio (Hsu, Chung and Kristal (2006)). More specifically, the criteria (or models) to select an asset are considered a set of multiple criterion or scoring systems (MCS or MSS), each of which is a function on the set of assets. These functions are in the form of score functions and/or rank functions. When the set of MSS' is too big, it needs to be reduced according to certain criteria. Once it is small enough (five is considered a manageable size), the systems are combined using a mathematical combinatorial algorithm, where both 'rank combination' and 'score combination' are considered. When there are n MSS', we consider $2^n - 1 - n$ combinations for rank and $2^n - 1 - n$ combinations for score combination. Our method differs from other combination methods, e.g., those stated in Hazarika and Taylor (2001), in at least three aspects: (1) for each system, we use both score function and rank function, (2) for an n-size MSS' with a large n, we reduce the MSS' into smaller size. Then for smaller n-size MSS', we consider all $2(2^n-n-1)$ possible combinations in order to search for the ones in which the combined system performs better than or equal to the individual system, and (3) we use the concept of a rank-score function to compute diversity between these MSS'.

In Section 2, we describe the method of combinatorial fusion analysis in the context of portfolio management. Then in Section 3, we describe the experiment including the data set, the criteria used, and the results. Section 4 concludes the paper with possible future work.

2. Combinatorial Fusion Analysis

Let A be set of n assets $a_1, a_2, \ldots, a_n$. Let $M_1, M_2, \ldots, M_p$ be a set of p multiple criteria (or models). For each model *M,* we define a score function s_M on *A* such that $s_M(a)$ is a real number in *R*. By looking at each model criteria we must first make sure that the scores themselves have monotonic similarities. For example, the criterion price earnings (PE) ratio, (where lower the PE ratio the better) is monotonically dissimilar to the criterion of earnings per share (EPS, where higher is better). One can achieve monotonic similarity by simply changing the signs of all scores under the PE ratio criterion to negative.

Treating $s_M(a)$, $a \in A$, as an array of *n* numbers and sorting the array would lead to a rank function r_M on *A* such that $r_M(a)$ is a natural number in $N = [1,. ., n]$. In order for all the score functions s_M to be comparable, especially when they will be combined later, the function s_M is normalized to take values in [0,1].

For a set of *p* multiple models $\{M_1, M_2, \ldots, M_p\}$, Hsu, Chung and Kristal (2006) describes several ways of combination. In this paper, we use only the average combination because we emphasize more on comparing rank and score combinations Hsu and Taksa (2005). For the set of *p* models $M_1, M_2, \ldots, M_p$, we define the score function of the score combined model *SC* as:

$$s_{SC}(a) = \left(\sum_{i=1}^{p} s_{Mi}(a) \right) / p \tag{1}$$

Sorting the array $s_{SC}(a)$ into decreasing order would give rise to the rank function of the score combined model *SC,* written as r_{SC}. Similarly, we define the score function of the rank combined model *RC* as:

$$s_{RC}(a) == \left(\sum_{i=1}^{p} r_{Mi}(a) \right) / p. \tag{2}$$

Sorting this array $s_{RC}(a)$ into increasing order gives rise to the rank functions of the rank combined model *RC*, written as r_{RC}.

For each criterion *M*, let *P(M)* be the performance of *M.* We are most interested in the combination *C,* where $C = C(M_1, M_2, \ldots M_p)$ so that $P(C) \geq \max \{P(M_i)\})$. We will call these as **positive cases.** If $P(C) > \max \{P(M_i)\}$ will call these as **strictly positive cases.** If $P(C) < \max \{P(M_i)\}$ will call these as **strictly negative cases.** Obviously, our approach is successful if we find portfolio combinations leading to definitive performance improvements as revealed by strictly positive cases.

Combinatorial fusion analysis has been used in information retrieval and virtual screening (See Hsu and Taska, (2005), Ng and Kantor (2000) and Yang et al (2005)) with several applications in natural sciences. The framework of CFA and a survey is given in Hsu, Chung and Kristal (2006). We can exploit some established results listed in following remarks to help choose a good portfolio. The established results (stated here without proof or further explanation) exhibit the following phenomena for multiple scoring systems:

Remark A:
(1) Combining multiple scoring systems would improve the performance only if:
(a) individual systems have relatively good performance, and
(b) individual systems are **diverse**, and
(2) rank combination performs better than score combination under certain conditions.

The **diversity between systems M_1 and M_2, $d(M_1,M_d)$,** used above in Remark A(1)(b) is defined as follows:

Remark B: The diversity between systems M_1 and M_2, $d(M_1, M_2)$, is defined as either:
(1) $d(M_1, M_2) = d(s_{M1}, s_{M2})$ = correlation between s_{M1} and s_{M2},
(2) $d(M_1, M_2) = d(r_{M1}, r_{M2})$ = rank correlation between r_{M1} and r_{M2}, *or*
(3) $d(M_1, M_2) = d(f_{M1}, f_{M2})$, where

f_M is the **rank-score function** defined by $f_M : N = |A| \longrightarrow [0,1]$ so that

$$f_M(i) = s_M(r_M^{-1}(i)) = (s_M \circ r_M^{-1})(i) \tag{3}$$

The graph of the rank-score function f_M is the graph f_M with rank as the x-coordinate and score as the y-coordinate. In Remark B(2), the rank correlation $d(r_{M1}, r_{M2})$ can have different definitions including Kendall's τ (tau) coefficient or Spearman's ρ (rho) coefficient. Likewise, the difference (or diversity) between rank-score functions f_{M1} and f_{M2} can exist in several different forms. In Yang et al (2005) they used the formula:

$$d(f_{M1}, f_{M2}) = \left(\sum (1/n)^n{}_{i=1} (f_{M1}(i) - f_{M2}(i))^2\right)^{1/2} \tag{4}$$

Let P (A) denote the performance of criterion A. The pair wise performance ratio of low to high is defined as $PR(A, B) = Pl / Ph = \min\{P(A), P(B)\} / \max\{P(A), P(B)\}$. A graphical insight is gained in this literature by a **diversity-performance graph,** which plots Pl / Ph on the horizontal axis and suitably defined pair wise distance or diversity on the vertical axis. The strictly positive cases where fusions lead to strictly superior performance are indicated by circles ("o") and negative cases indicated by ('x') graphic symbols. Past experience and experiments suggests that circles are usually toward the North East area of the diversity-performance graph and x's are found in the South West area. This seems to support the statement in Remark (A)(1) for necessary conditions for improving the performance of the combination.

3. An Illustrative Example as an Experiment

A multi-step algorithm begins with defining the data set, chooses performance measure (return on equity or ROE) to be used for comparing portfolios. Admittedly, we do not expect universal agreement on the choice of ROE as ultimate performance criterion. However, for the purpose of our experiment, the reader should accept as a reasonable choice. In all, we have nine criteria. The algorithm uses the union of potential criteria to focus on 5 out of 9 criteria using Remark A(1)(a). We have $2^5 - 1 - 5 = 26$ rank combinations and 26 score combinations for 126 (9 choose 5) groups. The notion of groups is new and will be explained later with the help of our illustrative example when we describe the algorithm. The results lead to explicit choice of stocks. We try to gain insights from rank-score function and diversity-performance graphs which are similar to those in related literature. The details of the algorithm using the open source R package for this purpose are given in Section 3.2. It was implemented almost immediately on a PC with a 2Ghz processor.

3.1 Description of the Data Set:

Our data are from Prof. *Aswath Damodaran*'s website at the Stern Business School of New York University.
http://pages.stern.nyu.edu/~adamodar/New_Home_Page/data.html
The original data source is Value Line Inc. The site reports data for 7113 stocks (identified by ticker symbols and row numbers) along 7113 rows of an EXCEL workbook having 72 columns. The workbook columns are potential stock selection criteria involving the usual financial statistics including the PE ratio obtained by dividing the company's share price by its earnings per share (EPS), or Price to book value (PBV) ratio as the ratio of market value of equity to book value of equity. The PBV is a measure of shareholders' equity in the balance sheet of a company. For our illustration we select data from the following **nine criteria** with following names and associated symbolic abbreviations used in our discussion below:

[A]=Trailing PE, [B]=Forward EPS, [C]=Forward PE, [D]=PBV Ratio, [E]=Ratio of Enterprise value (EV) to Invested Capital, [F]=Value to B.V. of Capital, [G]=Growth in EPS during the last 5 years, [H]=Growth in Revenue last year, and finally, [K]=Net Margin.

Recall that we have selected Return on equity (ROE) available along the 39^{th} column in the workbook as our performance measure associated with each stock. When we construct an abridged data matrix we place the nine criteria along nine columns and the performance ROE as the tenth column. We have considered other performance measures with similar results, not reported here for brevity.

Many rows in the original workbook had missing data (or NA's) or zeros for the relevant ten columns. We clean out all those rows (remove stocks) from the workbook. Although we start with 7113 rows in the workbook, we end up with only 1129 rows in the abridged workbook focusing on nine criteria and ROE. Note that we now have 1129 stocks as candidates for inclusion in our ideal portfolio.

In order to ensure that each of the multiple systems satisfy 'monotonic similarity', that is, they satisfy the same increasing or decreasing norm, i.e. the bigger the better, we multiply values in these nine columns (A,..H, K) by the vector c(-1,1,-1,-1,1,1,1,1,1), where (-1) means it is desirable to have smaller values. For example, since it is desirable to have a small price earnings ratio, we multiply the column for A=Trailing PE by -1, as indicated. Similarly we change the sign of values in columns for C=Forward PE and D=PBV Ratio. In the end, we make sure that all columns have numbers so that bigger values are preferred by investors.

3.2 Description of the Steps in Our Algorithm:

(1) Defining scores and computing their ranks

The 1129 data values in the nine columns for (A,..H, K) are called a vector of "scores" achieved by that stock under that criterion. Next, we rank these values from the smallest to the largest in an ascending order so that larger rank has a larger score number to yield a similar 1129 $\times$1 vector of "ranks".

(2) Choosing the best 113 stocks implicitly recommended by each criterion
We now have best stocks along the bottom rows for each criterion in terms of scores as well as ranks. Of course, at this point the rank contains no additional information beyond what is contained in the score vector. However, this will change later. Among our 1129 (cleaned up rows) of stocks we now choose 10 % or 113 stocks situated along the bottom rows (bigger the better!) for both scores and ranks. These 113 stocks are possible candidates to be selected in our final portfolio of stocks.

(3) Union of all potentially desirable stocks.
Next step is to consider a union of the 113 stocks recommended by each of the nine criteria with possibly 113*9=1017 stocks. Of course, even if the criteria (A,..H, K) are distinct, same stocks are implicitly recommended (i.e., among the top 113) by more than one criteria. In our example the union contains n=584 stocks and 433 repetitions.

(4) Scaled Scores.
In the sequel, it is important to make the nine criteria numerically comparable to each other in suitable units before any fusion can take place. For this purpose, we normalize the scores by each criterion to the closed interval [0,1], by using the standard linear interpolation formula. If we start with score numbers x and denote LO=min(x) and UP=max(x), then the normalized (rescaled) values of x are given by: y=(x-LO)/(UP-LO). After this rescaling, the units are comparable and simple averages of these numbers in columns A to H can be considered for the purpose of potential combinations of criteria. Despite rescalings, it is convenient to refer to the rescaled scores as simply scores.

(5) Procedure for the one-criterion-at-a-time case.
Next, we bring in the data on ROE performance in the tenth column for each stock into the memory. Assuming we are using only one criterion at a time (A, B, ..H, K), we compute the ROE performance for each stock recommended by each criterion. Towards this end, we create a matrix M_3 with 3 columns. The first column has numbers 1 to n (=584), the second column has either the score values or the ranks and the last column has the ROE. We sort this entire M3 matrix on the second column, so that the best stocks will again be at the bottom of the matrix. Now we select the best 10% stocks by each criterion and compute the 'average ROE' for these chosen stocks along columns using the abbreviations (A,.., H, K).

When the second column of M_3 is the score, we have one number for each of the nine criteria representing the 'average ROE' if the investor relied solely on that criterion to choose her stocks. We do the same when the second column of M_3 contains ranks. A check on our programming is that the average ROE should be exactly the same whether we use scores or ranks in this case when we have only one criterion at a time (before we combine them by a fusion algorithm).

In traditional method of portfolio selection this is the end of story. It is, however, only the beginning under our proposal. Instead of being satisfied with choosing only one criterion at a time, we consider combinations of two or more criteria. For simplicity the combinations considered in this paper are simple averages, but weighted averages can be considered without loss of generality.

(6) Procedure for the two-criteria-at-a-time case.
Recall that we are working with a set of 584 chosen stocks from the union such that each row has a unique ticker symbol and a unique performance measure ROE. Assuming we are going to combine two criteria at a time, we compute the ROE performance for each stock recommended by each pair. There are (9 Choose 2) or 36 possible pairs of criteria from (A, B, C, D, E, F, G, H, K). For example, AB, AC, AD, AE, etc. For each such pair we create a matrix M3 with 3 columns similar to Step 5. The first column has numbers 1 to n (=584), the second column has either the average of two score values (or the average of two ranks) and the last column still has the ROE for that stock. We sort this entire M_3 matrix on the second column, so that the best ones will again be at the bottom of the matrix. Now we select the best 10% stocks for each paired criterion (residing along the bottom 10% of the rows) and compute the average ROE for these chosen stocks.

Each pair (out of the 36 possible pairs of criteria) yields one ROE number when the second column of the M3 matrix has the *average score* of two criteria (e.g., simply average the score for A and B). In all, with 36 pairs we have 36 ROE numbers and additional 36 ROE numbers when the second column of M3 is the *average rank* based on two criteria (AB, AC, AD, …). It is perhaps not obvious that, unlike the one-at-a-time case above, the ROE numbers (hence recommended stocks) are different when the second column of M3 contains average scores instead of ranks.

Compared to the traditional method of choosing one criterion at a time, the fusion algorithm is ahead of the game if we have the *strictly* higher ROE for any combination of two criteria at a time. For our example, the combination of criteria A and E is often found to be superior to A or E alone.

(7) Procedure for the general k-criteria-at-a-time case.
We are still working with the set of 584 chosen stocks from the union such that each row has a unique ticker symbol and unique performance measure ROE. From the set (A, B, C, D, E, F, G, H, K), if we have k=3 criteria at a time we must enumerate all 84 choices similar to ABC, ACD, ADE, etc. Similarly, for both k=4 and 5 we enumerate 126 possible combinations like (ABCD, ACDE, ADEF, etc) and 126 combinations like (ABCDE, ABCDF, ABCDG, ..) In general, there are (9 Choose k, or '9Ck')=(84, 126, 126) possible criteria for k=3,4,5, respectively. For each k-at-a-time fusion set we again create a matrix M3 with 3 columns. The first column has numbers 1 to n (=584), the second column has either the average of k score values or the average of k ranks (belonging to that set) and the last column still has the ROE for that stock. We always sort this entire M3 matrix on the second column, so that the best will again be at the bottom of the matrix. Now we select the best 10% stocks for each k-at-a-time fusion set and compute the average ROE for these chosen stocks.

Each of the '9Ck' possible k-at-a-time fusion sets yields '9Ck' ROE numbers when the second column of M_3 is the *average score* of the included k criteria and additional '9Ck' ROE numbers when the second column of M_3 matrix contains the *average rank* of the k criteria included in that fusion.

After finishing this for k=1 to 5 we will have 2*381=762 ROE numbers for each stock associated with the nine criteria (A to H and K) and their fusions involving k at a time, where the doubling is needed because we have score-based numbers as well as rank-based numbers. Of the 762, we can ignore the k=1 case leading to 744 relevant ROE numbers to be compared.

(8) The total of 126 groups:
Even if we have 9 criteria we have determined that it is impractical to use a criterion for the choice of stocks based on a fusion of more than five stock-picking criteria at a time in our context. This means we are not allowing a grand fusion of all nine criteria (ABCDEFGHK). We are also disallowing fusions containing 6, 7, or 8 criteria at a time. However we must consider a complete listing of choices for all possible sets of 5 out of 9 leading to (9 choose 5) or 126 distinct choices to be considered separately. We refer to these as *groups* for the purpose of discussion. Since the portfolio of best stocks recommended for one group will not, in general, coincide with the best portfolio for another group, we need to study all of them.

(9) Final ranking of all stock choices for all 126 groups
Compared to the traditional method of choosing one criterion at a time, the fusion method is much ahead of the game, since we have noticed that a great many cases exist where combined criteria using averages of scaled scores have *strictly* higher ROE performance than the ROE of their individual components taken one at a time. Now we will consider $2*(2^5 - 1 - 5) = 52$ ROE numbers for each of the 126 groups. However, we fully expect to have many duplicate fusions among these groups. It turns out that we need not be concerned with explicitly separating the duplicates, because we can simply rank order with respect to 52*126=6552 ROE numbers from the lowest ROE to the largest ROE, and eventually pick the best rows based on the highest ROE. If there are duplicates they will simply become identical rows, easily omitted by a computer algorithm. Identical rows do not affect the value of ROE or the ranking.

4. The Results

For our example, the best fusion is the rank combination of criteria A, B, E, and F. Having found this fact, our next task is to identify the stock id numbers (ticker symbols) for the top 50 stocks recommended by this ABEF combination. Although the calculation was already done, it is not efficient to keep such results in computer memory. Instead, we suggest going back to the ranks for A, B, E and F after the choice is decided. It is a simple matter to find the associated stock symbols after sorting the ranks. This will finally yield the best portfolio of 50 stocks suggested by our method. Ticker symbols of 50 stocks in the ideal portfolio which is the result of rank combination of MCS' A,B,E and F with highest normalized ROE = 0.77762:

BOBE, WY, AAI, FTD, ORFR, SNHY, TCBI, MXM, TKCRF, SSYS, CPF, GBTB, WTAI, CFR, XLACF, BWINB, NPO, FCBQ, PNS, KV/A, AMKR, FNCB, NVSL, MGP, RGX, RTC, BHS, KPCG, FCNCA, BKF, MTZ,, MLAB, BKLYY, KMA, TDSC, NFX, TFN, WDC, ELRN, BLSC, ZOOM, TTMI, MSFT, BDOG, BSQR, CNBKA, PCES, COSN, TESS, INFA.

Table 1 illustrates some intermediate calculations for the combination of the five MCS' A,B,C,E and F, where we have the number 1 along the last two columns whenever we have strictly positive (superior) outcome from a fusion algorithm in terms of the ROE performance compared to the outcome achievable by individual criteria. In table 1 with 31 rows we have 36 strictly positive outcomes out of the 62 possible rank- and score- combinations, [A]=Trailing PE, [B]=Forward EPS, [C]=Forward PE, [E]=Ratio of Enterprise value (EV) to Invested Capital, [F]=Value to B.V. of Capital.

Table1
Intermediate Calculations

list		rank-comb.	score-comb.	score superiority	rank superiority
31	ABCEF	0.77217	0.7308	1	1
27	ABCF	0.731	0.72832	1	1
30	BCEF	0.74041	0.71989	1	1
29	ACEF	0.77273	0.7196	1	1
20	ACF	0.75248	0.71865	1	1
23	BCF	0.72213	0.71629	1	1
13	CE	0.73999	0.71593	1	1
15	EF	0.70281	0.71532	0	1
24	BEF	0.73222	0.71517	1	1
28	ABEF	0.77762	0.71486	1	1
12	BF	0.7374	0.71364	1	1
25	CEF	0.74697	0.71296	1	1
21	AEF	0.74846	0.71226	1	1
5	F	0.71218	0.71218	0	0
3	C	0.7109	0.7109	0	0
18	ABF	0.7517	0.71039	1	0
14	CF	0.73806	0.71019	1	0
9	AF	0.77523	0.70949	1	0
19	ACE	0.75123	0.70734	1	0
8	AE	0.76542	0.70064	1	1
7	AC	0.70254	0.70037	0	0
1	A	0.69907	0.69907	0	0
26	ABCE	0.72075	0.69226	1	0
17	ABE	0.75688	0.69208	1	0
16	ABC	0.69961	0.68889	1	0

6	AB	0.69365	0.68749	0	0
22	BCE	0.71751	0.68692	1	0
4	E	0.68494	0.68494	0	0
10	BC	0.69981	0.68233	0	0
11	BE	0.73489	0.6781	1	0
2	B	0.67423	0.67423	0	0

The first column of Table 2 reports the criterion elements in the fusion, such as ABEF. The second column reports the normalized ROE for the rank-combination identified by the letters in the first column. The third column of Table 2 reports the normalized ROE for score-combination identified by the letters in the first column. The table has 60 rows for the top ranked 60 out of 6552 possibilities with respect to this ROE value. A “1” or “0” in the parenthesis following the ROE values in second and third columns indicate whether the ROE value gives a strictly positive or 'greater than or equal to' result. In Table 2 with 60 rows we have 80 strictly positive outcomes and 40 remaining outcomes. The bold numbers in Table 2 indicate the chosen combination (rank or score) along each row. It is interesting that top 39 are all rank combinations. Only along row numbers 40, 45, 50 and 59 we have bold (superior) ROE achievement in the last column.

Table 2

	Fusion	Rank Combination	Score Combination
1	ABEF	**0.77762(1)**	0.71486(1)
2	AF	**0.77523(1)**	0.70949(0)
3	ACEF	**0.77273(1)**	0.7196(1)
4	ABCEF	**0.77217(1)**	0.7308(1)
5	AE	**0.76542(1)**	0.70064(1)
6	ABE	**0.75688(1)**	0.69208(0)
7	ACEFH	**0.75318(1)**	0.67352(0)
8	ACEFG	**0.75283(1)**	0.69005(0)
9	ACF	**0.75248(1)**	0.71865(1)
10	BCEFK	**0.75209(1)**	0.72389(1)
11	ABF	**0.7517(1)**	0.71039(0)
12	ACE	**0.75123(1)**	0.70734(0)
13	ABEFK	**0.75037(1)**	0.71695(1)
14	AEF	**0.74846(1)**	0.71226(1)
15	ACEFK	**0.74782(1)**	0.69338(0)
16	ABEFH	**0.74732(1)**	0.69434(0)
17	CEF	**0.74697(1)**	0.71296(1)
18	ABEFG	**0.74266(1)**	0.69284(0)
19	ABEH	**0.74158(1)**	0.65763(0)

20	AEFG	**0.74108(1)**	0.68565(0)
21	BCEF	**0.74041(1)**	0.71989(1)
22	AFG	**0.74033(1)**	0.6852(0)
23	CE	**0.73999(1)**	0.71593(1)
24	BCEFH	**0.73851(1)**	0.71256(1)
25	CF	**0.73806(1)**	0.71019(0)
26	AEFH	**0.73773(1)**	0.68948(0)
27	BF	**0.7374(1)**	0.71364(1)
28	CEFG	**0.73651(1)**	0.69044(0)
29	ABCFH	**0.73635(1)**	0.68809(0)
30	BE	**0.73489(1)**	0.6781(0)
31	ABEGH	**0.73456(1)**	0.70075(1)
32	ABFHK	**0.73423(1)**	0.68101(0)
33	CEFGK	**0.73401(1)**	0.7055(0)
34	AEFHK	**0.73278(1)**	0.68085(0)
35	BEF	**0.73222(1)**	0.71517(1)
36	ABFH	**0.73216(1)**	0.69015(0)
37	ABEHK	**0.73214(1)**	0.65405(0)
38	ABCEH	**0.73128(1)**	0.65459(0)
39	ABCF	**0.731(1)**	0.72832(1)
40	ABCEF	0.77217(1)	**0.7308(1)**
41	ABFGH	**0.7307(1)**	0.70617(0)
42	ACFHK	**0.72966(1)**	0.6648(0)
43	ACEGH	**0.72948(1)**	0.68705(0)
44	ACFK	**0.72932(1)**	0.6838(0)
45	ABCDF	0.69842(0)	**0.7292(1)**
46	CEFHK	**0.72919(1)**	0.67887(0)
47	AEG	**0.72904(1)**	0.67584(0)
48	AEFGK	**0.72893(1)**	0.71587(1)
49	ACFH	**0.72872(1)**	0.67509(0)
50	ABCF	0.731(1)	**0.72832(1)**
51	EFHK	**0.72685(1)**	0.68104(0)
52	ABFK	**0.72668(1)**	0.70738(0)
53	ACFGH	**0.72647(1)**	0.69966(0)
54	ACEK	**0.72552(1)**	0.67973(0)
55	CFH	**0.72536(1)**	0.68714(0)

56	ADEF	**0.72474(1)**	0.7131(0)
57	ABEG	**0.72438(1)**	0.67483(0)
58	CEFK	**0.72397(1)**	0.69178(0)
59	BCEFK	0.75209(1)	**0.72389(1)**
60	BEFK	**0.7232(1)**	0.70541(0)

Figure 1 illustrates a rank-score function for each of the five criteria in the combination ABCEF (Fig1 (a)) and ABEFH (Fig1 (b)). It is a graph of scaled score (forced into the closed interval range 0 to 1) for a given rank. In our case the ranks are from 1 to 584, with low ranks representing desirable portfolios for the particular fusion of A, B, C, E and F.

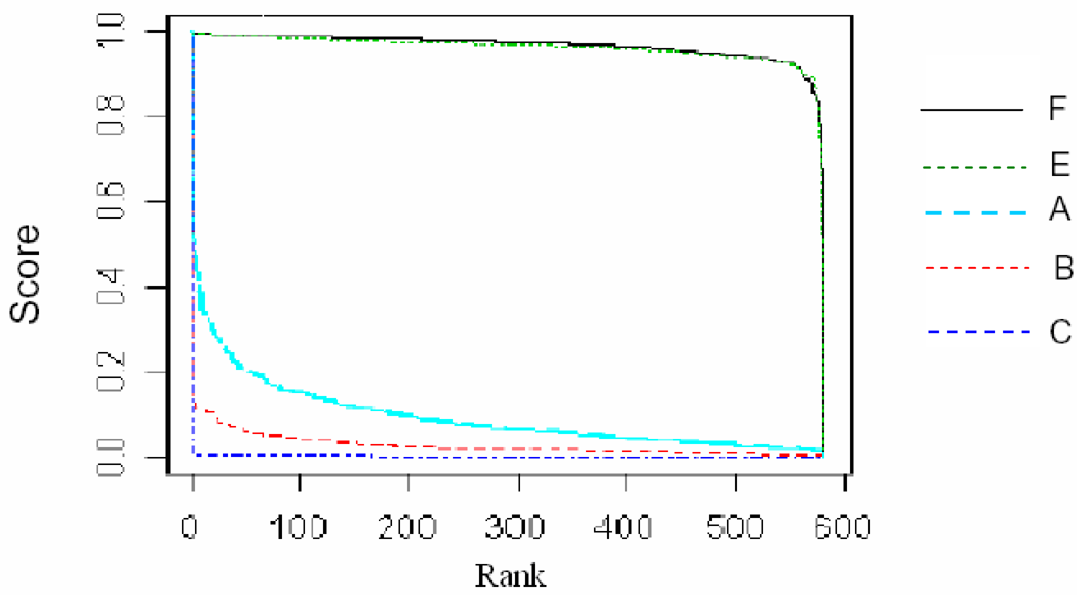

Figure 1(a): Plot of rank-score functions(A,B,C,E,F)

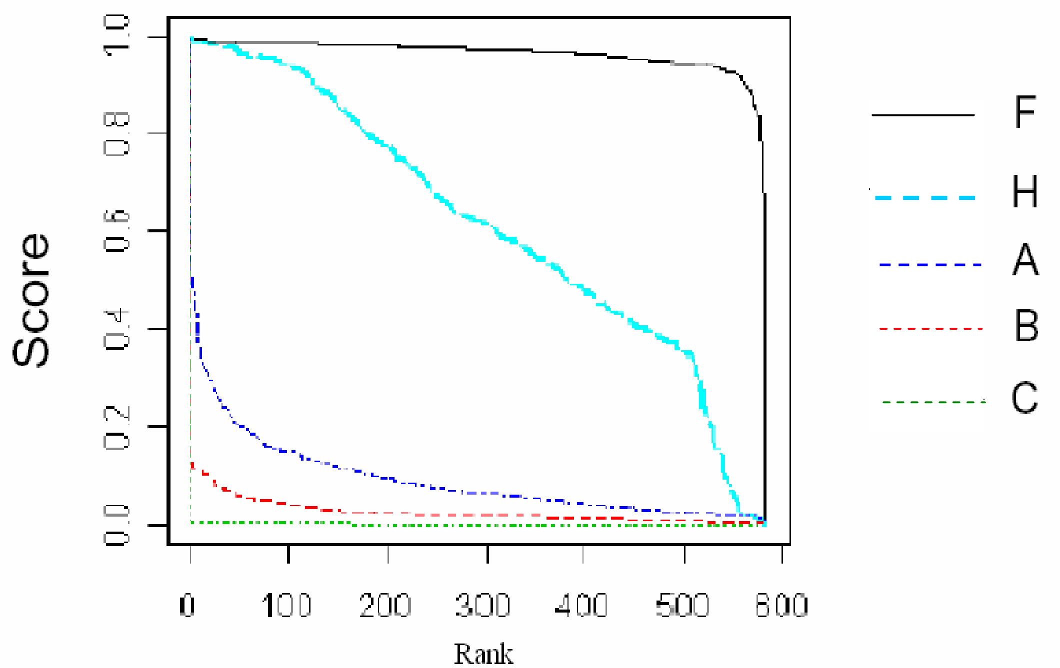

Figure 1(b): Plot of rank-score functions(A,B,C,F,H)

The top ranked ABEF criteria are worth special attention. We draw the rank-score graph for the fusion of A, B, E and F criteria in Figure 2. Vertical axis has average of scaled scores for the four criteria for a given rank. When the rank is 1 it means the best stock and rank 584 is the worst stock by this criterion.

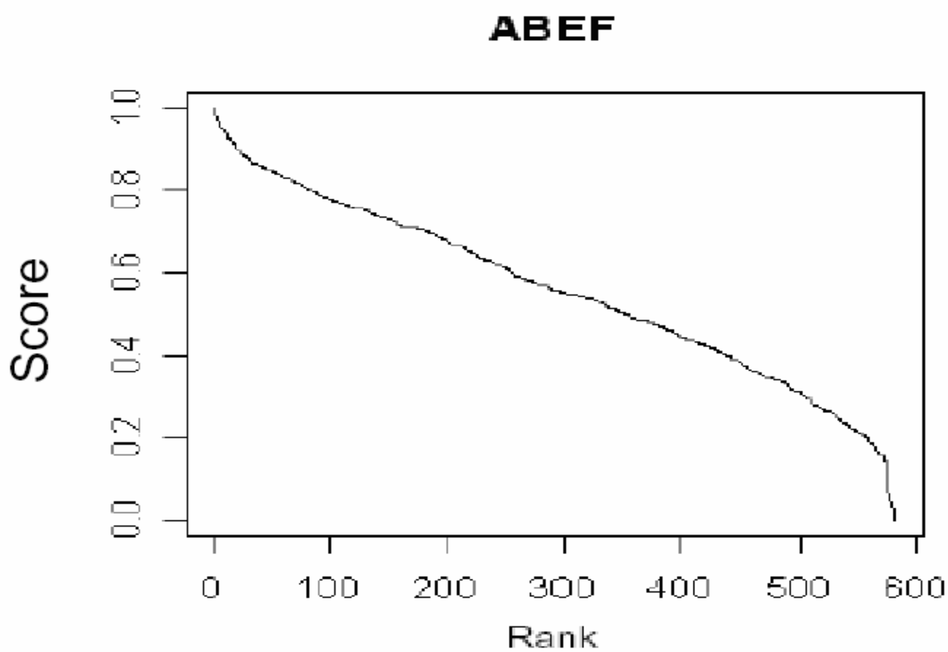

Figure 2

The **diversity d(X,Y)** represents the pair-wise diversity or distance between the rank-score functions of the two criteria x and y. Performance ratio between the lower performance and the higher performance of x and y is indicated a *Pl/Ph.* Figure 3 shows positive cases indicated by "o" symbol and negative cases indicated by 'x' symbols. The total number of points is (9 * 8/1 * 2) * 2 = 72.

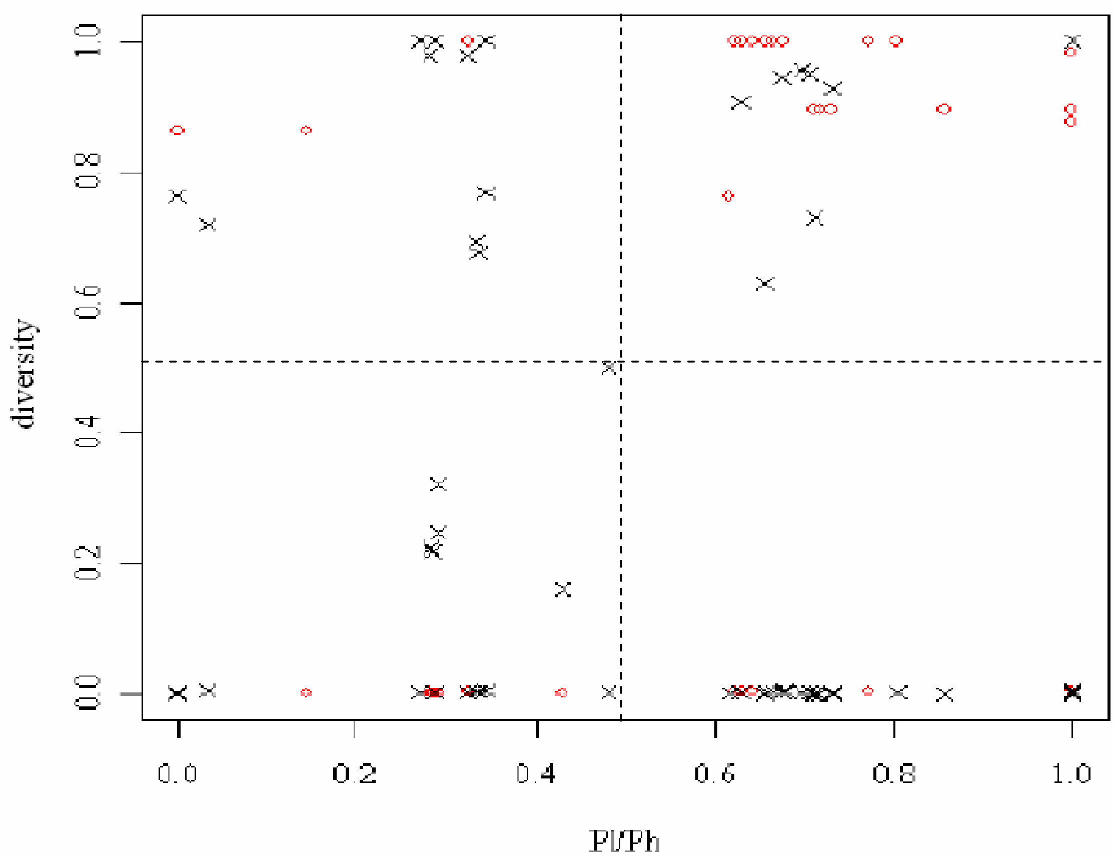

Figure 3

5. Conclusion and Comments on Possible Future Work

This paper considers several novel tools and proposes new ways of thinking about combining different and often competing criteria available in the portfolio selection literature. We show that the portfolio selection problem can be solved by this method and further extensions of the method are possible. Generally financial data sets are huge. We have chosen a well known data set with over 7000 stocks. It is not surprising that tools developed by computer scientists can readily handle large data sets. In fact we are able to come up with a specific recommendation of a portfolio consisting of 50 stocks and list their ticker symbols on the New York Stock exchange. Even though the data we use here in the example are old, the framework and algorithm proposed in this paper can be applied to a more current and general setting.

As we stated in Introduction (aspect (1)) and we can see from Tables 1 and 2, considering both rank and score combination has the distinctive advantage of choosing the better performance combination. Distinctive aspect (2) stated in Introduction allows us to examine all the $2x(2^5-1-5)=52$ combinatorial combinations of the $C(9,5) = 126$ groups when the size of the

MCS' is relatively small. This strategy is better when compared to the linear (or linearly weighted) combination approach. This is evidenced by the fact that the best-case ABEF is the result of the combinatorial combination among A, B, C, E, and F in Table 1. The rank-score graph of each of the MSS' depicts the rank (or scoring) behavior of that criterion (or model) system. Figure (1)(a)(b) and (2) give the rank-score graphs of A,B,C,E,F, of A,B,E,F,H, and of ABEF, respectively.

The current work develops the initial framework for the selection of assets utilizing information fusion (CFA) to combine multiple criterion systems. It has generated several issues for further work:

(a) We will compare rank combination versus score combination and explore the reason (or condition) why one is better than the other. This finding will be compared with Remark (A)(2) which was studied by Hsu and Taksa (2005) in the information retrieval context.

(b) Remark (B) lists three possibilities to define diversity between two criteria X and Y, d(X,Y). In this paper, we use d(X,Y)= $d(f_X,f_Y)$ (see Figures 1 and 2). In the future we may also incorporate $d(s_X,s_Y)$ and/or $d(r_X,r_Y)$ and a comparison among these three diversity measures.

(c) Figure 3 plots positive cases and negative cases of combinations of X and Y on x-y plane where x-axis is the performance ratio P_l/P_h and y-axis is the diversity measure $d(f_X,f_Y)$ for 72 cases. The result is relatively similar to those found and stated in Remark (A)(a) in other application domains such as information retrieval in Hsu et al (2005, 2006), and Ng and Kantor (2000), virtual screening and drug discovery in Yang et al (2005) and more recently multiple classifier systems in Chung et al (2007). We will produce much more points for combination of the criteria X and Y.

(d) We will apply the framework to a variety of other large data sets in the portfolio selection process. We will also extend our multiple criterion systems to multiple model systems where each scoring system is a model such as neural network, support vector machine, or singular value decomposition.

By way of extension, it is also quite possible to use time series data for each stock and myriad other choices of performance and stock-picking criteria. For example, Vinod. et.al. (2005) and Vinod (2004) discusses four orders of stochastic dominance based on different empirical probability distributions suggested by the past data for each stock. With the use of fusion algorithm we are not restricted to focus only one stochastic order of dominance at a time. Thus, the CFA approach can have a great potential and a bright future in the process and the arts and sciences of portfolio selections.

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