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Sophie Mitra
Fordham University, Department of Economics
Jean-Marc Boussard
Académie d'Agriculture de France

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Department of Economics
Fordham University
441 E Fordham Rd, Dealy Hall
Bronx, NY 10458
(718) 817-4048

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Sophie Mitra
Department of Economics,
Fordham University,
441 East Fordham Road,
Bronx, NY 10458, USA

Jean-Marc Boussard
Académie d'Agriculture de France
3 quai Malaquais
75006 Paris, France

Corresponding Author: Sophie Mitra, Ph: 718 817 5337, Fax: 718 817 3518
Email: mitra@fordham.edu

Abstract
A debate has been raging for centuries regarding the effects of inter-annual storage on commodity prices. Most analysts consider storage to function as a price stabilizer, while others place it at the core of an explanation of intriguing features of commodity price series, such as skewed distributions. Most studies have been developed in the context of the theory of competitive storage where random shocks affect supply or demand. Recently, the endogenous chaotic behavior of markets has become another possible hypothesis regarding the origin of commodity price fluctuations. We develop a nonlinear cobweb model with intra- and inter-annual storage, risk averse agents and adaptive expectations. Like the theory of competitive storage, this nonlinear cobweb model with storage can reproduce some of the stylized facts of agricultural commodity prices (autocorrelation of first rank, low kurtosis and skewness). In addition, the effects of storage on price variation are mixed. In the presence of inter-annual storage, chaotic price series show less variation compared to a situation without inter-annual storage but we find that storage contributes to the endogenous volatility of prices by making chaotic dynamics more likely.

Keywords: Agricultural Prices; Nonlinear Cobweb Model; Endogenous Fluctuations; Storage

JEL classification: Q11, E39, D84

1. Introduction

It is well established that agricultural price fluctuations have negative effects on welfare, in particular on children's health (Jensen 2001), food security and growth (Myers 2006) in developing countries. However, there has been much debate around the food policies that can be used to reduce fluctuations. The sharp increase in the prices of agricultural commodities in 2007/2008 has led to a renewed interest in agricultural price fluctuations and how they can be reduced. The design of a policy response to agricultural price fluctuations requires an understanding of how fluctuations arise in the first place.

Two main types of explanations have been proposed. One is exogenous (Williams 1936; Gustafson 1958; Gardner 1979; Scheinkman and Schechtman, 1983, Cafiero and Wright 2006; Deaton and Laroque 1996, 1992): weather shocks or any other factor outside the economic environment perturbs supply. A thorough presentation of this explanation is in Scheinkman and Schechtman (1983). The other explanation is endogenous (e.g., Ezekiel 1938; Day and Hanson 1991, Finkenstadt and Kuhbier 1992, Boussard 1996, Hommes 1998, Athanasiou et al 2008): erroneous expectations lead to over- or under-supply. In both cases, an inelastic demand magnifies imbalances and creates large and detrimental price fluctuations. In the exogenous case, all methods based on the law of large numbers (e.g., insurance schemes, stockpiling, futures markets, widening markets) are efficient to alleviate price volatility. Under the endogenous explanation, on the contrary, price stabilization strategies may be in the form of government interventions such as production quotas and national supply management.

In the ‘exogenous fluctuation’ theory context, storage has been the subject of much None of these theories are satisfactory in their ability to mimic reality. In particular, time series of agricultural commodity prices typically exhibit irregular fluctuations (Cochrane 1958), and in particular small kurtosis, positive autocorrelation and positive skewness (occasional spikes) (Tomek and Robinson 1972; Deaton and Laroque 1992; Tomek 2000). Such characteristics have not been reproduced through series with random shocks, nor have they been encountered with simple endogenous fluctuations models such as those quoted above. We explain each of these two theories in more detail below.

attention. This can be explained by two reasons: first, if fluctuations are of random origin, and liable to the law of large numbers--at least throughout time if not across space--then storage could be a solution to stabilize markets and avoid large fluctuations. Secondly, storage creates a link between two successive periods of time, which makes it a candidate for explaining the serial autocorrelation of price series.

Among non-economists, the inventory holder, or more generally the ‘middleman’, is often considered to be the ‘bad guy’ whose storing activity artificially creates the scarcity which will allow him/her to get large profits. In contrast, in the theory of competitive storage (Gustafson 1958; Cafiero and Wright 2006; Wright and Williams 1982; 1984), the inventory holder, in order to make money, must buy at a low price and sell at a high price. This has a stabilizing impact by increasing demand when prices have to be increased, and supply when they should be lowered. Thus, in the face of random shocks affecting supply, storage should lead to prices that are less volatile.

Still in the context of the theory of competitive storage, storage has been studied in how it may help model commodity markets so as to reproduce stylized facts of actual prices. Deaton and Laroque (D&L thereafter) (1992, 1996) develop a model which may cast doubt on this conclusion. Assuming production is subject to identically and independently distributed (i.i.d.) random shocks, D&L relate the average inventory holder's expected profit with the quantity stored. The most striking conclusion of this model is that storage can transform a stationary i.i.d. Gaussian exogenous process into an irregular time series, with positive skewness, low kurtosis, and autocorrelation. As a consequence, storage is central to understanding the dynamics of agricultural commodity prices. The strength of the D&L model is its parsimony in the assumptions it uses: operators are risk neutral, and their price expectations are rational. Like in other competitive storage models, it relies upon an exogenous shock as the basic source of supply and price variability. While this is a common assumption in agricultural economics, it cannot be taken for granted.

In the endogenous explanation, fluctuations are endogenous to agricultural markets and result from the behavior of agents or characteristics of the market (e.g., Voituriez 2001; Chavas and Holt 1993). The cobweb model, originally popularized by Ezekiel (1938), is often used to illustrate this explanation: fluctuations arise primarily due to erroneous expectations that lead to over- or under-supply. The original cobweb model leads to three simple price trajectories (converging, diverging and periodic) which are simply not realistic, except for the "converging" one. A periodic motion is impossible, since at least some operators would notice the periodicity, organize production in order to take advantage of the periodicity, and, in doing so, would break it. The diverging situation, which implies negative prices and quantities after a while, is not possible either. However, the cobweb model has been refined over the past three decades. While keeping the two basic ingredients of Ezekiel's model (the role of imperfect expectations, and the genuine instability of the 'repelling' fixed equilibrium point), by adding mechanisms which call back the system toward equilibrium whenever it is "far" from it, authors such as Day and Hanson (1991) and Hommes (1998) obtain chaotic price trajectories that are irregular and thus more realistic than those of the original cobweb model. As explained in Werndl (2009), "chaotic systems are deterministic systems showing *irregular*, or even *random*, behavior and sensitive dependence to initial conditions", which "means that small errors in initial conditions lead to totally different solutions" (emphasis in original).

Chaotic cobweb models of agricultural markets (Athanasiou et al 2008; Boussard 1996; Finkenstadt and Kuhbier 1992) have been built along these lines, and exhibit the same apparently random pattern of price fluctuations. Yet, they lead to a dynamic that often exhibits negative skewness and low first order autocorrelation (e.g., Brock *et al* 2007; Hommes 1998) and thus greatly differs from that of actual agricultural prices. As mentioned above, time series of agricultural commodity prices typically exhibit small kurtosis, positive autocorrelation and positive skewness (occasional spikes). So far, chaotic cobweb models of agricultural markets have not been able to reproduce the stylized facts of agricultural commodity prices.

This paper seeks to investigate price dynamics in a nonlinear cobweb model with private storage. It attempts to contribute to the existing literature in three ways. First, with seasonality and storage, our model incorporates standard features of agricultural markets and is more realistic than past cobweb models. Second, we test whether our nonlinear cobweb model reproduces the stylized facts of agricultural price series. Third, we investigate the effect of private inter-annual storage on prices and production.

The rest of this paper is organized as follows. Section 2 provides a motivation for the model of endogenous price fluctuations with storage, seasonality, adaptive expectations and risk aversion, and Section 3 presents the model. Section 4 includes simulation results and an empirical test of the model. Section 5 concludes.

2. Background

We present the main features of the nonlinear cobweb model developed in this paper below. Compared to earlier cobweb models, our model is novel in that it incorporates seasonality and private storage. Like some earlier cobweb models, we use assumptions of risk aversion and adaptive expectations. We argue that these assumptions are as plausible as those of risk neutrality and rational expectations used in the theory of competitive storage.

2.1. Feature 1: Seasonality and Private Storage

We extend the cobweb model by including seasonality, an important feature for annual crops. We study the case of a crop year with two seasons: a season when the crop is harvested, "summer" or season 1, and a season when the crop is planted, hereafter, "winter" or season 2. Private storage (i.e. storage by the inventory holder) within and between crop years is made possible to allow for consumption throughout the year. The theory of competitive storage has already developed models including intra- and inter-annual storage (e.g., Lowry et al 1987). However, prior Cobweb models--including recent chaotic ones--do not incorporate private storage. In the case of seasonal nonperishable crops, private storage becomes a technical necessity since production takes place only at certain times and consumption occurs all the time.

In addition, storage has been at the center of the understanding of the dynamics of commodity markets as part of the theory of competitive storage. The famous papers by D & L (1992, 1996) stand as some of the most impressive achievements in this direction[1], showing that storage can indeed take place within a stationary random production setting and contributes to explain the dynamics of agricultural prices. Their model accounts for sudden large jumps at irregular intervals in the price series as they are observed in actual commodity price series. The central reason that their model with storage reproduces the large spikes of prices at irregular intervals is due to the non-negativity constraint of inventory (D&L 1992; p. 4). Results from the theory of competitive storage thus suggest that storage plays a central role in explaining the dynamics of commodity prices. Does storage help model commodity price dynamics in an entirely different theoretical framework, i.e. a cobweb model where fluctuations are entirely endogenous? We attempt to answer this question by developing a cobweb model with storage and by testing it empirically.

2.2. Feature 2: Segmented Markets

[1] It should be noted that the fact that we cite D&L (1992) repeatedly does not in any way imply that we assess other papers of the theory of competitive storage to be less significant. Instead, it is due to the fact that we follow here a similar empirical approach.

We model an agricultural commodity market that is segmented into a local market where farmers sell the commodity to storage firms and a central market where storage firms sell the commodity to consumers. This scenario is especially relevant for developing economies where production and consumption are geographically dispersed over wide areas and where transport infrastructure is limited. It is also plausible for some regions of developed countries (Bobenrieth et al 2006).

2.3. Feature 3: Risk Aversion

Planting and storage decisions are made under uncertainty; attitudes toward risk hence become an essential part of the model. Boussard (1996) found that adding risk aversion to an otherwise linear cobweb model can lead to a chaotic dynamics for prices. We start from the model of Boussard (1996) with risk averse agents, to which we add seasonality and private storage and nonlinear supply and demand curves[2]. Thus our analysis differs from the theory of competitive storage, where agents are risk neutral. It is important to note that attitudes toward risk are notoriously difficult to model and measure. We use the mean-standard deviation specification of preferences, which can be justified as a Taylor's expansion of order two of the mean of a concave utility function (Newbery and Stiglitz, 1981). Agents are assumed to be risk averse, which is debatable. The possibility of farmers being risk lovers has been considered (Binswanger 1980), and, over the last two decades, it has been shown that there are yet other possible behaviors with respect to risk

[2] Like Finkelstadt and Khubier but unlike Boussard (1996), we use a nonlinear demand curve and adaptive expectations. However, in another version of the model (Mitra 2001) we use, like Boussard (1996), a linear demand curve and naive expectations and reach results that are qualitatively similar.

such as prudence (Kimball, 1990). In the context of agricultural commodity markets, where large shares of the revenues of farmers and middle men are at stake in planting and storage decisions, our assumption of risk aversion appears to be no less plausible than the assumption of risk neutrality of the theory of competitive storage. Finally, whatever the representation of attitudes toward risk, a difficulty arises while aggregating heterogeneous decision makers: for instance, some farmers could be risk averse, and others risk lovers. In this paper, we stick to the crudest representation with a representative farmer and a representative inventory holder, assuming for each that increasing mean income is "good", and increasing variance is "bad". We do not seek a fine-tuning of the model with heterogeneous agents, which could be the subject of further research.

2.4. Feature 4: Expectations

The theory of competitive storage assumes, sometimes without any justification, that agents have “rational expectations”. Since the formulation by Muth (1961), rational expectations have been the dominating hypothesis for expectation formation in economics. According to this hypothesis, decision makers process all available information. Their subjective expectation is equal to the mathematical expectation conditional on available information. In a model like D&L's, with production engendered by a zero mean stationary process, decision makers know the probability distribution of shocks (and therefore, of prices), conditional on the current volume of the stock (which they are supposed to know, an assumption certainly not always justified). This is a way of representing rational expectation based on abundant yet incomplete information (next year's actual price is not known with certainty): decision makers know at least the mean value of the price they will get. Although perhaps debatable, the rational expectation hypothesis is within a well established tradition of economic representations of the world, and its relevance is hard to deny. However, it has little empirical support and, when considering specific applications, the problem is to know what information is available to decision makers. From this point of view, assuming the only sure knowledge available to producers is past prices series is tenable. With the additional assumption that memory decays with time (Allais 1966), the Nerlovian adaptive expectation scheme (Nerlove, 1958) becomes relevant. In this paper, we make use of the Nerlovian adaptive expectation scheme involving systematic forecast errors, which lead to over- or under-supply. These errors are what lead to endogenous fluctuations in the cobweb model. Recently, the adaptive expectation scheme has received more empirical support following controlled laboratory experiments (Heemeijer *et al.*, 2009).

Expectations pertaining to mean prices are not sufficient. As in Boussard (1996), we assume that the perceived probability distribution is defined by both mean and variance. While the mean price is defined by an adaptive expectation scheme, the expected variance of prices is defined through a naive expectation scheme that is the squared difference of the observed price and the expected price for the relevant season of the previous year. For example, the expected price variance for next winter is the squared difference of the expected and actual price of the last winter. Expected price variance is thus assumed to be seasonal. The seasonality of price variance has been empirically demonstrated on the futures markets of agricultural commodities (Hauser *et al.* 1987) but is yet to receive attention on spot markets. It should be noted that the model was also formulated using alternate variance expectation schemes, for instance where the expected price is derived from a naive scheme. We did not reach significantly different outcomes: of course, especially when the outcome is chaotic, changing the variance expectation scheme results in a different series, but the dynamics of the model remains qualitatively unchanged.

3. A Nonlinear Cobweb Model with Private Storage

The agricultural commodity under consideration is harvested annually, and consumed during two seasons: summer (i=1) and winter (i=2). The representative farmer does not have the capacity to store, but the representative storage firm (also called inventory holder thereafter) can store. The storage decision is made by the storage firm in both seasons, while the production decision is made by a representative farmer in winter. The quantity planted by the farmer in winter is available for sale to the storage firm during the summer of the following year.

3.1 Step 1: Modeling segmented markets and a nonlinear demand

Let $p^c_{t,i}$ be the commodity's price during season *i* of year *t*. How much is produced in summer $q_{t,1}$ is decided during the previous winter. There is no production in winter: $q_{t,2}$ =0. In both seasons, $i \in \{1,2\}$, the price is determined by the inverse demand function:

$$p^c_{t,i} = \beta_i (C_{t,i} + \varepsilon)^{-\alpha_i} \qquad (1)$$

where $C_{t,i}$ is the quantity consumed in season *i* of year *t*, $\alpha_i \geq 0$ and $\beta_i \geq 0$ are constant scalar coefficients, and $\varepsilon \geq 0$ is a small constant[3]. $C_{t,i}$, $q_{t,i}$ and $s_{t,i}$, the volume of the inventory decided during season *i* of year *t*, are related by the recursive equations:

$$s_{t,1} = s_{t-1,2} + q_{t,1} - C_{t,1} \qquad (2a)$$

$$s_{t,2} = s_{t,1} - C_{t,2} \qquad (2b)$$

During summer, production is sold by the farmer to the inventory holder at local market price $p^l_{t,1}$, which is assumed to be a linear function of the central market price $p^c_{t,1}$:

$$p^l_{t,1} = p^c_{t,1} - T \qquad (3)$$

with $T>0$ standing for transportation and handling costs. During winter, no exchange takes place on the local market between the farmer and the inventory holder.

3.3 Step 2: The storage decision

For the storage decision, the question arises as to how the inventory holder forms price expectations. Because the inventory holder operates in the market during both seasons, it is logical that he/she is aware of the seasonality of the market. The Nerlovian expectation scheme discussed above gives an expectation for season 1 of year *t* as follows:

$$\hat{p}^c_{t,1} = \hat{p}^c_{t-1,1} + \lambda_s (p^c_{t-1,1} - \hat{p}^c_{t-1,1}) \qquad (4a)$$

and for season 2 of year *t*:

$$\hat{p}^c_{t,2} = \hat{p}^c_{t-1,2} + \lambda_s (p^c_{t-1,2} - \hat{p}^c_{t-1,2}) \qquad (4b)$$

[3] The introduction of ε prevents prices from going to infinity. Such a restriction could be considered as an unnecessary constraint imposed on the model, especially because, food being an absolute necessity, consumers are ready to spend all their income to get the least quantity, thus allowing prices to go to infinity, if supply is close to zero . Yet, such a situation is not very realistic given that consumers would not be able to survive. With ε , we avoid such a situation.

In (4a) and (4b), the hat over a variable indicates an expectation and λ_s is the inventory holder's expectation correction coefficient. This way, during the summer (winter), the inventory holder remembers the price level last winter (summer) to make expectations regarding next winter (summer). This is certainly better than to start from this summer (winter) to guess the next winter (summer) price. In addition, the risk averse inventory holder also forms an expectation regarding the future variance of prices. As in Boussard (1996), we adopt a seasonal expected variance scheme as follows:

$$\hat{\sigma}_{t,1}^2 = \frac{1}{4}(\hat{p}_{t-1,1}^c - p_{t-1,1}^c)^2 \qquad (5a)$$

$$\hat{\sigma}_{t,2}^2 = \frac{1}{4}(\hat{p}_{t-1,2}^c - p_{t-1,2}^c)^2 \qquad (5b)$$

Like in the theory of competitive storage, we assume that the inventory holding market is perfectly competitive.[4] In the decision of how much to store, the inventory holder maximizes expected utility from sales revenues net of storage costs during the next period. In season 1, the risk averse inventory holder equates the certainty equivalent of the expected price for next season with the cost of storage, that is, the cost of buying the commodity this season, plus the physical unit cost of storage, with an adjustment for the interest rate as follows:

$$\hat{p}_{t,2}^c - A_s s_{t,2} \hat{\sigma}_{t,2}^2 = p_{t,1}^c (1+r) + c \qquad (6a)$$

where A_s is the inventory holder's absolute risk aversion coefficient (A_s >0)[5], *r* the inter-seasonal rate of interest, c is the cost of storage between two seasons, and $s_{t,i}$ is the size

[4] There is, however, some evidence that suggests that this may not be a realistic assumption and that non-competitive storage behavior may lead to reduced stock fluctuations and increased price fluctuations (Chavas 2008). Further research is needed to incorporate a non-competitive market structure in a cobweb model with storage and assess its impact on price fluctuations.

[5] When there is no risk aversion in the model, i.e. the absolute risk aversion coefficient is nil for the inventory holder and the producer, the model's dynamic is the same as with a standard cobweb, either exploding, converging or periodic. The risk aversion here is the only mechanism by which the system is

of the stock during season *i* of year *t*. Details on how equations (6a) and (6b) below are derived are in the Appendix. Similarly, for the storage decision made in season *2* of year *t* for season *1* of year *t+1*, we have:

$$\hat{p}^c_{t+1,1} - A_s s_{t+1,1} \hat{\sigma}^2_{t+1,1} = p^c_{t,2}(1+r) + c \qquad \text{(6b)}$$

Equations (6a) and (6b) implicitly determine the level of storage the inventory holder decides on. One should note that the current price $p^c_{t,i}$ of season *i* plays a role in equations (6a) and (6b), given that the demand for storage competes with the demand for immediate consumption. In season *i*, the decision to store is simultaneous with the decision to sell (or consume) and the price $p^c_{t,i}$ is a market clearing price, resulting from the arbitrage between storing or selling. For that reason, in summer (season 1), according to (1), given $q_{t,1}$, the harvest available , and the stock carried over from the previous year $s_{t-1,2}$, $p^c_{t,1}$ is given by:

$$p^c_{t,1} = \beta_1 (q_{t,1} + s_{t-1,2} - s_{t,1})^{-\alpha_1} \qquad \text{(7)}$$

In winter (season 2) of year t, when there is no harvest, the price $p^c_{t,2}$ is given by:

$$p^c_{t,2} = \beta_2 (s_{t,1} - s_{t,2})^{-\alpha_2} \qquad \text{(8)}$$

Equations (7) and (8) are both subject to ($0 \le s_{t,i} \le S_{\max}$) where $S_{\max}$ is the maximum storage capacity.

3.2 Step 3: the production decision

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constrained to come back toward the repelling equilibrium point whenever it is far away from it. See Boussard (1996).

In the winter (season 2 of year *t*), the utility-maximizing farmer makes the production decision for the following summer (season 1 of year t+1). The producer is assumed to have a non-linear marginal cost $K_{t+1,1}$ given by:

$$K_{t+1,1} = bq^{a}_{t+1,1} \quad (9)$$

where $a > 0$.

The producer's expectation regarding the commodity price follows the standard Nerlovian scheme:

$$\hat{p}^{l}_{t+1,1} = \hat{p}^{l}_{t,1} + \lambda_p (p^{l}_{t,1} - \hat{p}^{l}_{t,1}) \quad (10)$$

where λ_p is the producer's expectation correction coefficient. It should be noted that the producer's expectation is based on the price of the previous summer. There is no exchange between producer and inventory holder during the winter, and hence no local winter price then. The risk averse farmer's expectation of the price variance is given by:

$$\hat{\sigma}^{2}_{t+1,1} = \frac{1}{4}(\hat{p}^{l}_{t,1} - p^{l}_{t,1})^2 \quad (11)$$

Then, the utility-maximizing producer decides how much to produce by equating the marginal cost with the marginal benefit of production. The producer's marginal benefit of production is the certainty equivalent of the expected price, which is derived in a similar fashion as the marginal benefit of storage was derived for the inventory holder for equations (6a) and (6b) (Appendix). The equality of the marginal cost and the marginal benefit is translated into equation (12) below, which can be solved for $q_{t+1,1}$:

$$bq^{a}_{t+1,1} = \hat{p}^{l}_{t+1,1} - A_p \hat{\sigma}^{2}_{t+1,1} q_{t+1,1} \quad (12)$$

The set of equations from (4) to (12) fully determine the variables $p^c_{t,i}$, $p^l_{t,1}$, $s_{t,i}$, and $q_{t+1,1}$. It can be reduced to two sets of equations involving two categories of variables, z_t and x_t: $g(z_t, x_t) = 0$, and $f(z_t, x_t, x_{t-1}) = 0$. The variables belonging to the vector z_t such as the consumption levels in each season, are linked with other variables of the same year, while those belonging to the x_t vector, such as the expected prices, are linked to variables of the previous year as well as some of the current year t. Solving $g(z_t, x_t) = 0$ for z_t and reporting the result in f allows us to write the system as $h(x_t, x_{t-1}) = 0$. The dimension of h is only 5 rows, for 5 elements in x_t. It is then possible to linearize h in the vicinity of a point as $Mx_t = Nx_{t-1}$, and to write :

$$x_t = M^{-1}Nx_{t-1} = Hx_{t-1}. \qquad (13)$$

The behavior of the system is dependant upon the eigenvalues of the matrix H at the fixed point(s). Clearly, there is at least one fixed point if $p_{1,t} = p_{1,t-1} = \tilde{p}$, and $p_{t,2} = p_{t-1,2} = \tilde{p}(1+r)+c$. Then, $\hat{\sigma}_{1,t} = \hat{\sigma}_{2,t} = 0$, and, with $q_{t,1} = C_{t,1} + C_{t,2}$, neglecting ε for simplicity, one gets[6]:

$$\left(\frac{\tilde{p}}{b}\right)^{1/\alpha} = \left[\frac{\tilde{p}(1+r+c)}{\beta}\right]^{-1/\alpha} + \left(\frac{\tilde{p}}{\beta}\right)^{-1/\alpha} \qquad (14)$$

which determines $\tilde{p}$. The quantities $C_{t,1}$ and $C_{t,2}$ follow easily, with $s_{t,1} = C_{t,2}$ and $s_{t,2} = 0$. The current price alternates between summer ($\tilde{p}$) and winter ($p_{t,2} = \tilde{p}(1+r)+c$). This is the standard result of the periodic behavior of prices in competitive seasonal storage equilibrium. Perhaps, other fixed points do exist: we have not been able to demonstrate the

[6] For simplicity, we assume that $\alpha_1 = \alpha_2 = \alpha$.

existence (or non existence) of other fixed points. Similarly, the existence of periodic points is likely, but not demonstrated. We concentrated our attention on the stability of the fixed point defined by equation (14), for which we tried to solve equation (13).

Using Maple software, we could determine that the characteristic polynomial of the matrix *H* is a very complicated function of parameters, which would be tedious to display here. Yet, we could show that it has five roots, including two zero roots, and three non-zero roots that are not simple, and can only be numerically calculated for specific values of the parameters. They are sometimes real, sometimes complex. Figure 1 shows the evolution of the logarithm of modulus of the non-zero roots for various values of the supply and demand elasticity, all other parameters remaining constant[7]. Its magnitude is often larger than one, which, together with the existence of the zero roots, means that the equilibrium point is an unstable saddle point for a wide range of parameters (Alligood et al 1997). It must be noted that in this case (contrary to what happens with the standard cobweb model), high (just as well as low) demand elasticity can make the system unstable: thus this model is even more pessimistic than for instance the risky cobweb of Boussard (1996) with respect to the possibility of stabilizing markets by increasing the elasticity of demand. It should also be noted in Figure 1 that "stable" regions (with root magnitudes less than 1) also occur in some areas of the parameter plane, without any apparent reason.

<INSERT FIGURE 1 AROUND HERE>

4. Numerical Simulations

[7] $\alpha = 3; \beta = 10.648; a = 1; b = 1; A_p = 0.05; A_s = 0.001; \lambda_p = \lambda_s = 0.5; c = 0.1; S_{\max} = 2.5; r = 0.1;$.

We proceed with simulations with plausible values for the key parameters of the model developed in section 2. We vary the values of the parameters which are expected to influence the dynamics of the model. Based on prior research on the cobweb model, this is the case for the supply and demand elasticities (Ezekiel 1938; Boussard 1996), the coefficient of expectation correction (Hommes 1994) and the coefficient of absolute risk aversion (Boussard 1996). Using the literature on the theory of competitive storage (Lowry et al 1987), the cost of storage and the storage capacity are also important parameters in the model so we vary these parameters.

4.1 Calibration

For the demand parameters, we have kept the price elasticity of demand ($\eta_D = 1/\alpha_i$) below 1 given that the demand for food commodities is known to be inelastic. For other parameters, in the absence of conclusive empirical measures, we have picked parameter value ranges that are deemed plausible and are given in Table 1. It should be noted that the farmer is assumed to be more risk averse than the storage firm, which seems plausible on the basis that a storage firm is expected to be wealthier than a farmer (Newbery and Stiglitz 1981; p. 164).

Finally, the constants β, b, and ε in the inverse demand and marginal cost functions have been calibrated in such a way as to maintain prices and quantities within reasonable limits, that is between 0 and 10. Especially, ε has been chosen so as to have $\varepsilon^{\alpha} = 10$.

For b , the problem is to have $\left(\frac{p}{b}\right)^{a} \leq 10$ when p reaches its maximum value, 10: this is done by taking $b = 10^{(1-a)}$. In addition, the model requires an initial history function over two years to formulate the expected price variance. Initial values are important given that chaotic dynamics is by definition sensitive to initial conditions. However, many runs of the model showed that the selection of initial values does not influence the qualitative results of this paper for parameter value ranges as in Table 1. Finally, it should be noted that, in order to wipe out a possible transient effect, all statistics presented below are computed over the last 100 "years" (200 "seasons") of a 200 "years" (400 "seasons") series[8].

<INSERT TABLE 1 AROUND HERE>

4.2 Price Dynamics

<INSERT FIGURES 2, 3 & 4 AROUND HERE>

The above model has been subject to hundreds of simulations by changing parameter values. We illustrate graphically that the nonlinear cobweb model with private storage can lead to a variety of price dynamics, including converging, quasi-periodic[9] or chaotic time series[10]. In figure 2, for the parameter values given above, the system converges. Production remains indeed constant at about 1.21, as well as prices in each of the two seasons (1.61 and 2.0 during seasons 1 and 2 respectively, the difference corresponding to the cost of storage and discounting, the latter being high: 20%). In figure 3, the values of a, α, β and λ are changed. This makes the dynamics switch from converging to a quasi-periodic time series. Although the time series in figure 3 looks periodic, in fact, the price values are always different, albeit close. The regular price time series of the model is not realistic: as noted earlier, as soon as any periodicity is detectable in a price series,

[8] By contrast, figures picture series from season 0 to season 400, thus including possible transient effects.
[9] Quasi-periodic series are series that look periodic on graphs, although a careful inspection of values of the series over time shows that they do not repeat themselves and are thus not periodic.
[10] Notice all figures here picture the last 100 years of a 300 years simulation:

speculators will try to seize the opportunity, *ipso facto* breaking down the periodicity. In figure 4, we show simulations results with exactly the same parameters as for figure 2, except the interest rate, now 5% instead of 20%. We obtain a time series of prices that is now irregular, and looks random, although it is the consequence of a purely deterministic model. This is at least a sign of chaotic dynamics. The formal proof of the chaotic nature of a series is difficult to do, if for any other reason, because there does not exist any universally accepted formal definition of chaos. Yet, the chaotic character can be assessed through the sensitivity to initial conditions or by plotting the distribution of prices (Alligood et al, 1997). Figure 5 illustrates the sensitivity to initial conditions for one particular set of parameter values.

<INSERT FIGURE 5 AROUND HERE>

The nonlinear cobweb model developed earlier can thus lead to irregular price time series. Figure 6 shows another example of a chaotic series obtained from the model using different values for most parameters. Like in figure 4, we get an irregular pattern for price series, but unlike in figure 4, the price series shows frequent peaks and is thus asymmetric, as actual price series are. It could be argued though that the simulated series tend to have too many peaks and to look far more periodic than actual series. Figure 7 shows yet another example of chaotic motion several parameters are changed compared to figure 6, including a higher storage capacity. It has fewer peaks, but again looks more regular than an actual price series.

<INSERT FIGURES 6 & 7 AROUND HERE>

4*.3 Empirical Test*

We put the model above to a test by following the method used by D & L (1992). For the five specifications of the model presented in the figures above, Table 2 shows coefficient of variation, autocorrelation of first rank, skewness, and kurtosis, in order to compare our results with both actual commodity price series and the results of the D&L (1992) model. For actual series, only the mean, minimum and maximum of the 13 commodities is presented for each statistic. Statistics for each commodity can be found in D & L (1992).

<INSERT TABLE 2 AROUND HERE>

With coefficients of variation in the [0;0.78] interval, simulated series tend to be slightly more volatile than actual series, except sugar and cocoa, for which volatility is similar (D&L 1992). The first order autocorrelation coefficients are in the [-0.01;1.18] range for simulated series compared to [0.62;0.91] for actual series: they tend to be lower for simulated annual prices[11] than for actual series. Unlike the traditional cobweb model and the risky cobweb of Boussard (1996)[12], where the first order autocorrelation coefficient of prices is negative, this risky cobweb model with seasonality and storage gives autocorrelation coefficients that are negative or positive, and when positive, can be as high as those of actual price series.

Most of the time, simulated series have significant positive skewness, thus exhibiting, like actual series, asymmetric fluctuations. Similarly, the kurtosis for simulated series is in general smaller than those of actual series, but is close to zero or negative which is

[11] Since there are two seasons in the model, we get two observations per year. An annual mean is computed, weighing prices by consumed quantities. The autocorrelation coefficient is then computed between theses annual means – as it was, actually, for the time series observed by D&L.
[12] In Boussard (1996)'s model, the first order autocorrelation for prices is most often negative. In the traditional cobweb model, it is always negative.

consistent with the "heavy tails", a well-known feature of sugar and several other commodity price probability distributions[13]. The above results also held when simulations were run over a shorter time span (100 years) in order to be consistent with actual series. Additional simulation results are presented in the upper panel of Table 3, using two values for α, two values for *a*, allowing or not allowing inter-annual storage. Because combining all these parameter values with each other results in a large number of situations, each of which corresponding to one run of the model, Table 3 does not list indicators for each of them. Instead, it gives the mean of selected indicators over the runs corresponding to the listed values of the selected parameters that led to a chaotic dynamics[14].

Overall, results from the test of this endogenous price fluctuation model are similar to those of the theory of competitive storage. D & L (1992) could not reproduce the high autocorrelation of actual series and had a maximum autocorrelation coefficient of first order of 0.48. Cafiero and Wright (2006) tried to improve this result of D & L (1992) and found an autocorrelation coefficient of 0.69 after reducing the cost of storage. Here we get a similar level of autocorrelation of first order, sometime rather high, for some values of the parameters. More generally, it must be stressed that our results regarding the main characteristics of the series generated by the model do depend upon the existence of storage: without storage, prices series often become periodic or converge.

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[13] It should be noted that kurtosis is 3 and skewness is zero for a Gaussian variable, a kurtosis less than 3 means a "peaked, fat tail" distribution, a positive skewness means most of the distribution's weight is on the left of the mean.
[14] More explanations are in the notes of Table 3 and intervals for parameter values are given in Table 1.

4.4 The Effect of inter-annual storage on prices and production

Careful analyses of the effects of storage on prices, production and welfare have been conducted in the context of the theory of competitive storage (e.g., Wright and William 1982; 1984). We can use the above nonlinear cobweb model to start to assess the effects of storage on prices and production in this different theoretical framework. The lower panel of Table 3 includes simulation results when we do not allow inter-annual storage, which can be compared to the upper panel with results when we allow inter-annual storage. Table 3 shows that inter-annual storage has no clear effect on average prices – sometimes increasing, then sometimes decreasing them- but it clearly reduces the coefficient of variation of prices. Inter-annual storage therefore leads to less volatility in this context, at least whenever the system leads to a chaotic behavior: it should indeed be noted that the means in Table 3 are computed over chaotic series only.

<INSERT TABLES 3 AND 4 AROUND HERE>

However, the share of chaotic series is generally higher with than without inter-annual storage. This result is consistent with the more general finding, mentioned earlier, that without storage, when the storage capacity is reduced to zero, the model gives rise to prices series that often become periodic or converge. These results suggest that storage in general, and inter-annual storage in particular, may contribute to the endogenous volatility of agricultural commodity prices by increasing the likelihood that prices become chaotic.

This finding is different from that of Lowry *et al* (1987) who conducted a similar analysis of the effects of inter-annual storage but in a seasonal model of the theory of competitive storage. In contrast with our results, they find that inter-annual storage makes prices less volatile. They also reach the conclusion that inter-annual storage leads to a gain in efficiency, in the form of a benefit to the consumer through lower prices, which we do not find here.

In addition, Lowry *et al* (1987) investigated how the cost of storage may influence prices. This is what we do in Table 4, where interest rates vary from 5 to 20%. We find that increasing the rate of interest does not significantly change the results for this range of parameters. Finally, in Table 5, we vary the storage capacity of the storage firm. When storage capacity is increased four-fold, we find a slight increase in producer prices and production. Importantly, we also find a large effect on autocorrelation. When storage capacity increases, the first order autocorrelation coefficient increases from 0.17 to 0.62. The introduction of storage, and the storage capacity in particular, thus seems to drive the ability of this model to reproduce the first order autocorrelation of agricultural commodity prices. This is not surprising, since, for long, storage has been held responsible for autocorrelation in commodity price series. [15]

<INSERT TABLE 5 AROUND HERE>

This first assessment of the effects of storage on prices and production in the context of a nonlinear cobweb model with private storage leads to different results compared to those of several studies of the theory of competitive storage based on random shocks (Wright and Williams 1982; Lowry *et al* 1987, D&L 1992). These studies found that storage leads to lower prices and makes prices less volatile although it can occasionally lead to price

[15] Indeed, although not explicitly stated, a major motivation of D&L was to give evidence that storage was capable of transforming an i.i.d. process into an auto-correlated series.

peaks when stocks are empty. In a nonlinear cobweb model, storage does not seem to increase the efficiency of the system, in the form of lower average prices to the benefit of the consumer. In addition, the effects of storage on price variation are mixed: In the presence of inter-annual storage, chaotic price series show less variation compared to a situation without inter-annual storage but price series are also more likely to be chaotic. Without inter-annual storage, the model often leads to converging or periodic price series. Mathematically, this result makes sense: storage, and its non-negativity constraint, introduces a non-linearity in the model, which is a necessary but not a sufficient condition in order to have a chaotic dynamics. This result is also consistent with results from other lines of research. In economic theory, MacKey (1989) finds that price dependent storage make their price adjustment model move away from equilibrium and leads to oscillatory price series. In economic history, Fogel (1989) finds that in England, between 1500 and 1800, famines resulted from an extremely inelastic demand for food inventories and not from weather shocks. Clearly, more research is needed on the impact of private storage on prices.

5. Concluding remarks

We developed a dynamic agricultural commodity model with storage where no production shocks are needed to generate price fluctuations. The nonlinear cobweb model with storage empirically performs as well as the theory of competitive storage: it can reproduce the variability and asymmetry of actual commodity price series as well as a positive autocorrelation of first rank. Assumptions in this nonlinear cobweb model (i.e. risk aversion, adaptive expectations) stand in contrast with those used in the theory of competitive storage, and lead to different results on the impact of storage on prices. Inter-annual storage reduces the coefficient of variation of prices when price series are chaotic but contributes to the endogenous volatility of prices by making chaotic dynamics more likely. Without storage, the model more often leads to converging or periodic price series.

Our study findings need to be interpreted in the light of its limitations, some of which could be addressed in further research. It could be argued that the results of the nonlinear cobweb model are tied to one set of empirical tests only and to one particular version of the cobweb model. We have elsewhere (Mitra 2001) conducted similar empirical tests on prices at half-year intervals and reached the same results. Further empirical tests need to be conducted though to reproduce other stylized facts such as the characteristics of production series that were beyond the scope of this paper. More work is also needed modeling private storage in a nonlinear cobweb model. The model presented in this paper could be refined in many different ways: for instance with a similar model but without seasonal production, with a non-competitive inventory holding market and perhaps with a household model of on-farm storage. However, it should be noted that we have studied elsewhere other versions of the nonlinear cobweb model with private storage, for example with naive instead of adaptive expectations[16], with one agent producer-inventory holder (Mitra 2001) and with a linear instead of a nonlinear demand curve (Mitra and Boussard, 2011). These different versions of the model could also lead to irregular price movements and reproduce some of the empirical characteristics of actual price series. In addition, further analysis is required on the impact of inter-annual storage on welfare in

[16] The use of naive expectations is supported by Chavas (2000): in an econometric model of US beef producers, close to half of producers behave in a way that is consistent with naive expectations.

the context of the cobweb model. Finally, while expectation errors are plausible, random disturbances undoubtedly exist in agricultural markets, and interact with other factors. Further research is needed to model agricultural price fluctuations by developing models which include *both* exogenous sources of fluctuations in the form of shocks as in the theory of competitive storage, and endogenous sources in the form of erroneous expectations and other behavioral or market characteristics (e.g. risk aversion) as in the cobweb model above.

Despite these limitations, our study contributes to the literature on agricultural commodity price fluctuations. Our results demonstrate the importance of assumptions regarding the information available to decision makers in deriving practical conclusions regarding the role of storage in price fluctuations. The nonlinear cobweb model with private storage developed in this paper performs as well as the theory of competitive storage in reproducing the empirical characteristics of agricultural commodity prices. Results from this paper suggest that the hypothesis of endogenous price volatility cannot be ignored, especially given that it leads to different implications in terms of policies and courses of action, both public and private.

Alongside models based on the theory of competitive storage, and despite the fact that all results are highly sensitive to parameters values, nonlinear cobweb models could therefore be used for the assessment of the impact of policy decisions such as trade reforms or price stabilization programs[17]. Because of the sensitivity of the results to

[17] In fact, nonlinear dynamic models have already been used for the analysis of the impact of trade reforms (Ayouz et al 2006), but not for the impact of price or storage policies so far. We are not arguing that

parameter values, more research is needed in this direction. Indeed, from a purely mathematical point of view, chaotic series, although unpredictable with precision, by definition, stay on some attractor – a finite, not necessarily connected domain in R^n. Therefore, for any practical purpose, the important thing is not the series itself, but the properties of the attractor. This is an area where the mathematical theory of attractors, although in its infancy, could have practical implications.

nonlinear cobweb models need to provide the only framework in which such policies are assessed. Instead, we are arguing that they can provide useful tools in addition to models based on the theory of competitive storage.

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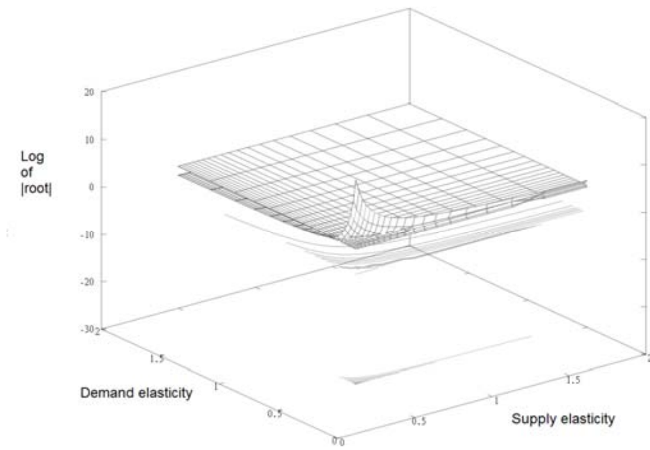

Figure 1
Logarithm of characteristic polynomial roots modulus at the fixed point as functions of supply and demand elasticities

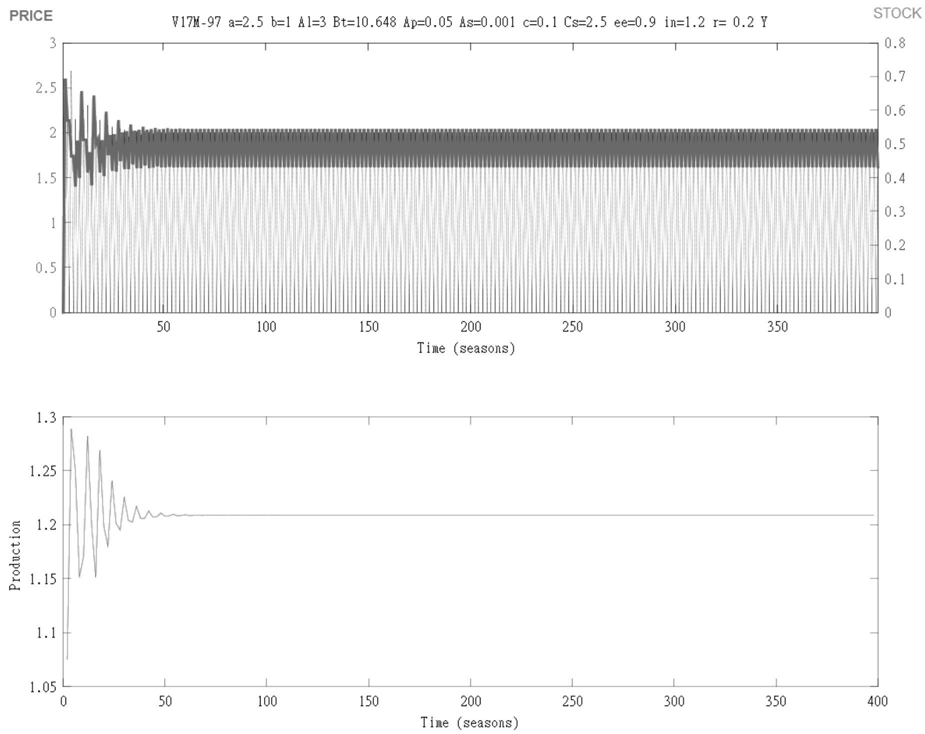

Figure 2: Converging seasonal equilibrium

$$\alpha = 3; \beta = 10.648; a = 2.5; b = 1; A_p = 0.05; A_s = 0.001; \lambda_p = \lambda_s = 0.5; c = 0.01; S_{\max} = 2.5; r = 0.2; T = 0.01$$

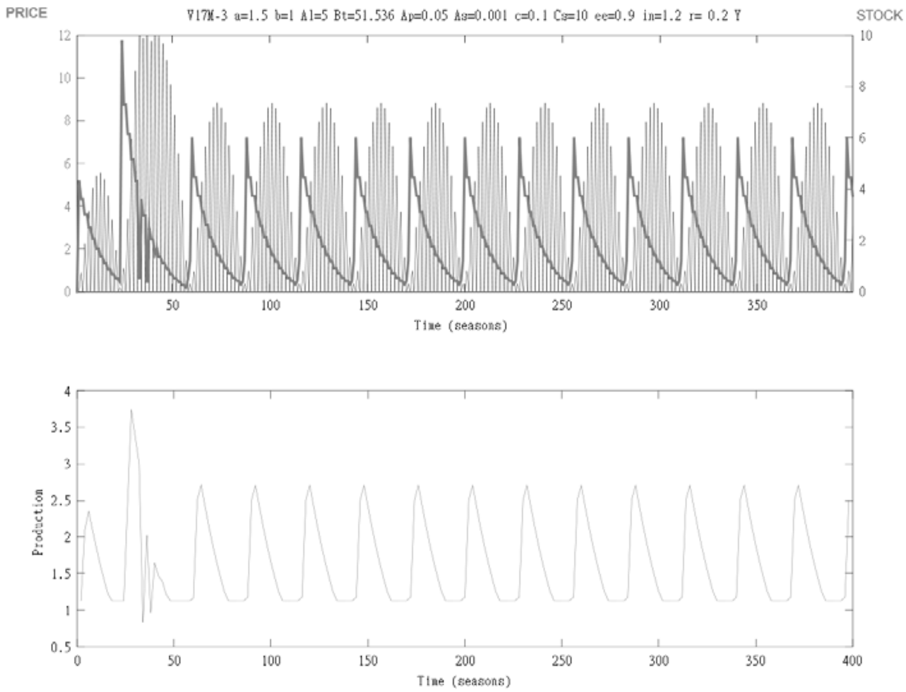

Figure 3: Quasi-periodic motion

$\alpha = 5; \beta = 51.54; a = 1.5; b = 1; A_p = 0.05; A_s = 0.001; \lambda_p = \lambda_s = 0.9; c = 0.1; S_{\max} = 2.5; r = 0.2; T = 0.01$

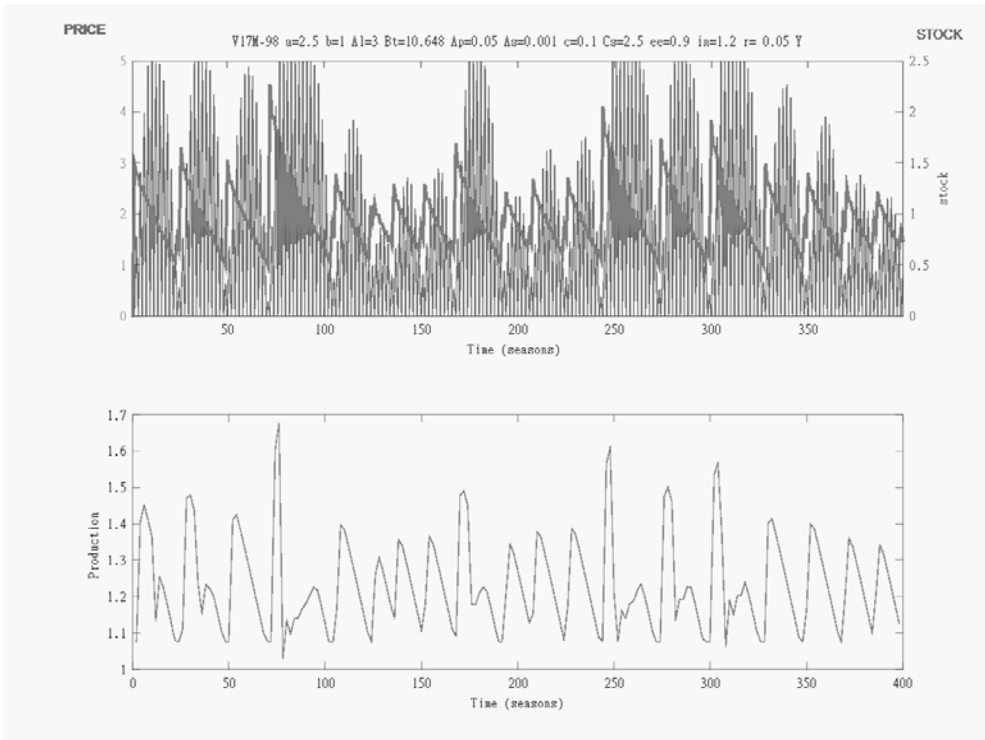

Figure 4: Example of chaotic motion

$$\alpha = 3; \beta = 10.648; a = 2.5; b = 1; A_p = 0.05; A_s = 0.001; \lambda_p = \lambda_s = 0.5; c = 0.1; S_{\max} = 2.5; r = 0.05; T = 0.01$$

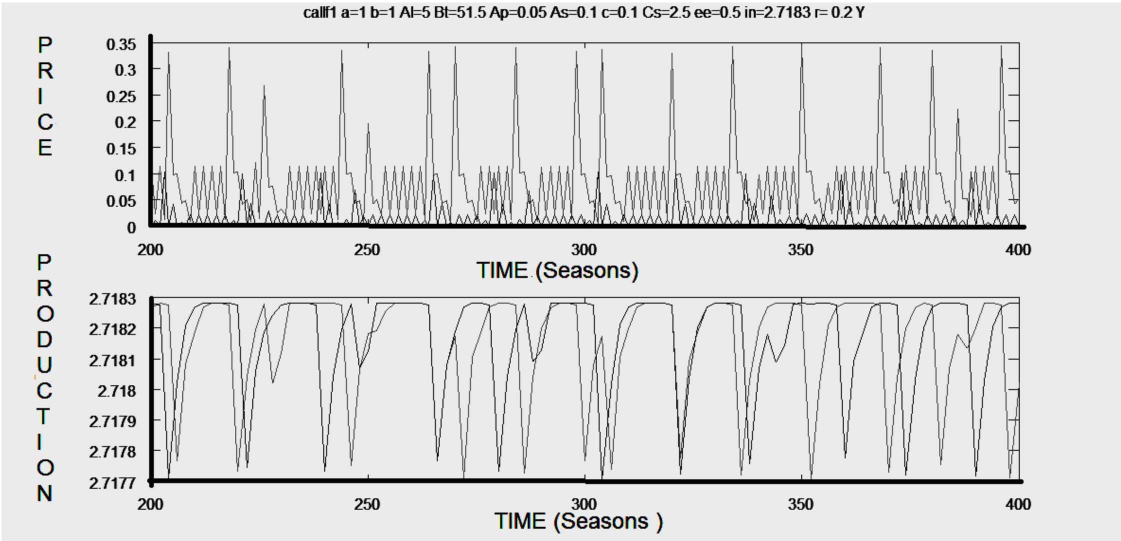

Figure 5 : Sensitivity to initial conditions

$\alpha_1 = 5; \beta_1 = 51.54; a = 1.; b = 1; A_p = 0.05; A_s = 0.1; \lambda_p = \lambda_s = 0.5; c = 0.1; S_{\max} = 2.5; r = 0.2; T = 0.01$

Note : As all other curves presented in this paper, one of the two sets of curves on this figure starts from q_{t0-1}=1.7, q_{t0-2}=2.4, s_{t0-1} season 1 = 0.4, s_{t0-1} season 2=0.5, s_{t0-2} season 2 = 0.8, and the other initial conditions are derived from these. The second set of curves starts from the same point, except that q_{t0-1}, q_{t0-2}, and s_{t0-1} season 1 are increased by 0.001.

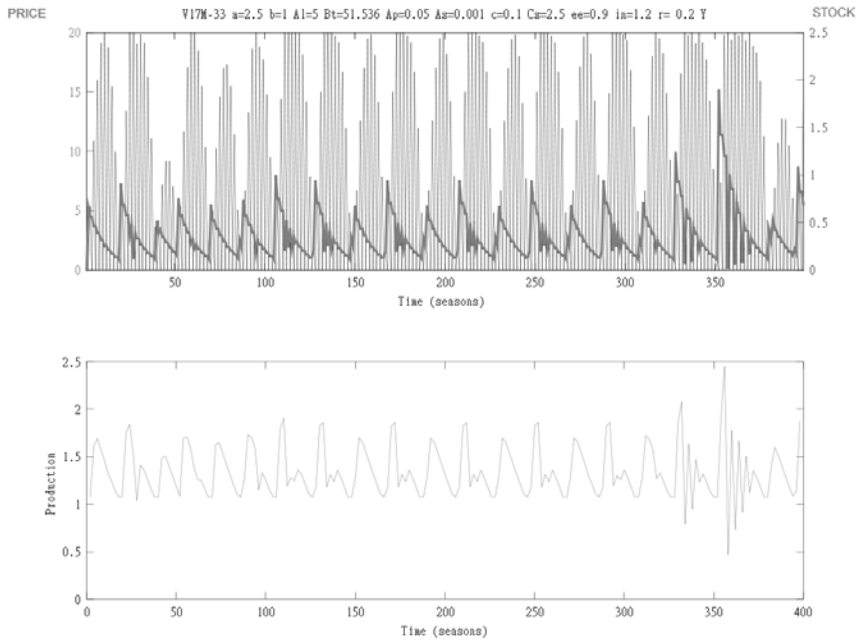

Figure 6: Another example of chaotic motion

$$\alpha = 5; \beta = 51.54; a = 2.5; b = 1; A_p = 0.05; A_s = 0.001; \lambda_p = \lambda_s = 0.9; c = 0.1; S_{\max} = 2.5; r = 0.2; T = 0.01$$

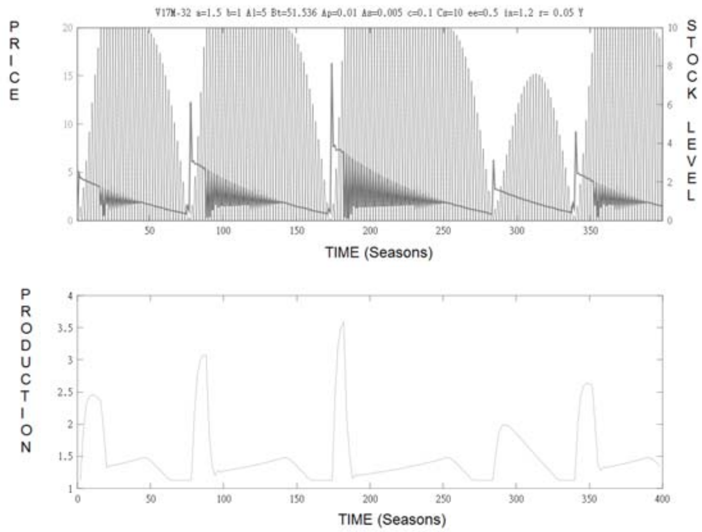

Figure 7: Chaotic motion in the case of increased storage capacity

$\alpha_1 = 5; \beta_1 = 51.54; a = 1.5; b = 1; A_p = 0.01; A_s = 0.005; \lambda_p = \lambda_s = 0.5; c = 0.01; S_{\max} = 10; r = 0.05; T = 0.01$

Table 1: Parameter Specifications

Variable	Definition	Value or interval of values
α_i	Log of the demand slope in season *i*	[1 -5]
β_i	Log of the demand intercept in season *i*	[0.00001 -0.1]
a	Log of the supply slope	[0.5 - 2.5]
b	Log of the supply intercept	[2.08 - 3.16]
A_p	Coefficient of absolute risk aversion (producer)	[0.01-0.05]
A_s	Coefficient of absolute risk aversion (storage firm)	[0.001 - 0.005]
λ_{p}	Coefficient of expectation correction (producer)	[0.2 - 0.8]
λ_{s}	Coefficient of expectation correction (storage firm)	[0.2 - 0.8]
c	Physical storage cost	0.1
S_{max}	Storage capacity	[2.5 - 10]
r	Interest rate	[5% - 20%]
T	Transportation and handling costs	0.01

For simplicity, we assume that the parameters of the demand curve are the same in both seasons.

Table 2: Comparison of annual price series from nonlinear cobweb model with storage to actual series and D&L (1992) series

	Mean	Coefficient of variation	AC 1	Skewness	Kurtosis
Simulated series of nonlinear cobweb with storage (1)					
(1) Figure 1	1.8	1.00E-10	-0.01	16.2	-0.76
(2) Figure 2	4.82	0.7	0.29	1.6	0.12
(3) Figure 3	3.65	0.78	0.6	1.96	0.28
(4) Figure 4	2.03	0.40	0.53	1.82	-0.80
(5) Figure 5	3.28	0.56	1.18	3.26	0.22
Actual series					
Mean		0.39	0.8	1.06	2.38
Minimum		0.17	0.62	0.04	-0.98
Maximum		0.6	0.91	3.24	16.52
Simulated series of Deaton & Laroque's model					
(a)		0.1	0.08	0.43	-0.29
(b)		0.5	0.48	1.99	5.5
(c)		0.1	0.08	0.61	0.17
(d)		0.53	0.33	3.41	24.22
(e)		0.53	0.29	3.15	16.43

AC 1 stands for autocorrelation of first rank

Actual series are for 13 commodities as reported in D&L (1992)

Simulated series of Deaton & Laroque's model are from Table 2 in D&L (1992)

Table 3: Simulations of the nonlinear cobweb model with storage

	$\alpha=5$ $a=2.5$	$\alpha=5$ $a=1.5$	$\alpha=3$ $a=2.5$	$\alpha=3$ $a=1.5$
Cobweb model with inter and intra-annual storage				
Price season 1 (summer)	3.18	3.16	2.06	2.40
Price season 2 (winter)	2.34	1.89	1.85	2.05
Production	1.35	1.49	1.27	0.22
Stock in season 1	3.02	3.39	2.47	1.80
Stock in season 2	3.63	4.03	3.08	2.40
Coefficient of variation of annual price	0.58	0.61	0.39	0.43
Coefficient of variation of production	0.19	0.29	0.13	0.22
AC 1 of annual price	0.49	0.36	0.63	0.34
Number of simulations	32	32	32	32
Percentage of cases with chaotic dynamics	56%	94%	13%	50%
Cobweb model with intra-annual storage only				
Price season 1 (summer)	4.03	3.56	2.01	1.96
Price season 2 (winter)	3.68	3.49	1.78	1.73
Production	1.45	1.95	1.25	1.41
Stock in season 1	0.00	0.00	1.79	0.00
Stock in season 2	0.77	1.14	2.39	0.69
Coefficient of variation of annual price	1.05	1.12	0.31	0.54
Coefficient of variation of production	0.31	0.49	0.11	0.23
AC 1 of annual price	-0.41	-0.43	0.60	-0.50
Number of simulations	32	32	32	32
Percentage of cases with chaotic dynamics	50%	31%	10%	41%

For each cell, the mean of the relevant indicator is given for the indicated number of simulations, extracted from a set of 192, with three values for the slope of demand curve, and two values for the slope of supply curve, the interest rate, the elasticity of expectation, the producer risk aversion coefficient, and the storage capacity. Only chaotic results have been included in the statistics reported above (Converging or periodic series are ignored).

The price refers to the price on the central market.

Table 4 : Effects of changes in the interest rate on prices and production

Rate of interest	*5%*	*20%*
Price in Season 1 (summer)	2.81	2.89
Price in Season 2 (winter)	1.94	1.93
Producer Price	1.93	1.92
Production	1.39	1.43
Stock in Season 1 (summer)	3.68	1.68
Stock in Season 2 (winter)	4.30	2.32
Coefficient of Variation of Annual Central Price	0.50	0.62
AC 1 of Annual Central Price	0.47	0.31
Number of simulations	54	54
Percentage of cases with chaotic dynamics	74%	52%

We vary the value of the interest rate *r* . For each value of *r* , we run 54 simulations changing the values of storage capacity, expectation elasticity, the demand and supply slopes.Only chaotic results have been included in the results above, converging or periodic series are ignored.

Table 5 : Effects of changes in the storage capacity on prices and production

Storage capacity	2.5	10
Price in Season 1 (summer)	1.87	2.00
Price in Season 2 (winter)	2.98	2.71
Producer Price	1.86	1.99
Production	1.38	1.43
Stock in Season 1 (summer)	1.25	4.37
Stock in Season 2 (winter)	1.85	5.03
Coefficient of Variation of Annual Central Price	0.51	0.58
AC 1 of Annual Central Price	0.17	0.62
Number of simulations	64	64
Percentage of cases with chaotic dynamics	55%	52%

We vary the value of the storage capacity. For each value of the storage capacity, we run about 30 simulations changing the values of the interest rate, expectation elasticity, the demand and supply slopes.The table includes mean values for the simulations that gave chaotic results. Converging or periodic series are ignored.

Appendix:

Equation (6) is derived as follows:

Inventory holders are risk averse and myopic: they only take into account the current period and the next period while making a decision. In season 1 of year t, in choosing the quantity stored for season 2 of year t $s_{t,2}$, the inventory holder maximizes the expected utility from income *y* in the second season (t,2). The inventory holder's expected income for the next season is given by:

$$\hat{y}_{t,2} = \hat{p}^{c}_{t,2} s_{t,2} \quad \text{(A.1)}$$

The expected utility of the income of the risk averse inventory holder, is therefore:

$$\hat{W}_{t,2} = \hat{y}_{t,2} - \frac{1}{2} A_s Var(\hat{y}_{t,2}) \quad \text{(A.2)}$$

that is,

$$\hat{W}_{t,2} = \hat{p}_{t,2} s_{t,2} - \frac{1}{2} A_s s^2_{t,2} \, \hat{\sigma}^2_{t,2} \quad \text{(A.3)}$$

where A_s is the absolute risk aversion coefficient of the inventory holder and $\hat{\sigma}^2_{t,2}$ is the expected variance of $\hat{p}_{t,2}$ and is given by equation (5b).

At equilibrium, the marginal welfare gain of storage equals the expected marginal cost of storage. The marginal welfare gain of storage is given by:

$$\frac{d\hat{W}_{t,2}}{ds_{t,2}} = = \hat{p}_{t,2} - A_s s_{t,2} \, \hat{\sigma}^2_{t,2} \quad \text{(A.4)}$$

The expected marginal cost of storage is the future value during season 2 of year *t* of the foregone income from selling in the current season $p^c_{t,1}$ plus the cost of storage c, that is given by $p^c_{t,i}(1+r)+c$ (A.5)

where *r* is the interest rate. Equating equations (A.4) and (A.5) above leads to equation (6a):

$$\hat{p}^c_{t,2} - A_s s_{t,2}\hat{\sigma}^2_{t,2} = p^c_{t,1}(1+r)+c \qquad (6a)$$

Repeating the same process above for the storage decision made in season 2 of year t for season 1 of year t+1, we get:

$$\hat{p}^c_{t+1,1} - A_s s_{t+1,1}\hat{\sigma}^2_{t+1,1} = p^c_{t,2}(1+r)+c \qquad (6b)$$