Fordham University
Department of Economics
Discussion Paper Series

Investors Facing Risk: Prospect Theory and Non-Expected Utility in Portfolio Selection

Erick W. Rengifo
Fordham University, Department of Economics
Emanuela Trifan
Bayerngas Energy
Rossen Trendafilov
Truman State University, Department of Economics

Discussion Paper No: 2014-03

January 2014

Department of Economics
Fordham University
441 E Fordham Rd, Dealy Hall
Bronx, NY 10458
(718) 817-4048

Investors Facing Risk: Prospect Theory and Non-Expected Utility in Portfolio Selection

ERICK W. RENGIFO‡ EMANUELA TRIFAN†

ROSSEN TRENDAFILOV*

September 2013

Abstract

This paper focuses on the attitude of non-professional investors towards financial losses and their decisions on wealth allocation, and how these change subject to behavioral factors. Our contribution concerns the integration of behavioral elements into the classic portfolio optimization. Individual perceptions are modeled according to an extended prospect-theory framework: Losses loom larger than gains of the same size (loss aversion) and the past risky-portfolio performance changes the subjective valuation of risky investments. The utility of financial investments is overemphasized (myopia). The portfolio model with individual VaR delivers an optimal wealth assignment between risky and risk-free assets.

KEYWORDS: VaR, Non-Professional Investor, Prospect Theory, Non-Expected Utility

JEL Classification: G10, G11, D81, E27

The authors would like to thank Duncan James, Jerome Lahaye and Hrishikesh D. Vinod for their useful comments that helped improve the quality of the paper. We also would like to thank the participants of the economic seminars at Fordham University. The usual disclaimers apply.

‡Corresponding Author. Department of Economics and the Center for International Policy Studies (CIPS) at Fordham University New York. 441 East Fordham Road, Dealy Hall, Office E513, Bronx, NY 10458, USA. Phone: +1 (718) 817 4061, fax: +1 (718) 817 3518, e-mail: rengifomina@fordham.edu.
† Quantitative analyst at Bayerngas Energy. Albert-Roshupter-Str. 52, D-81369 Munich, Germany. Phone: +49 (0) 175 9849421, e-mail: ema.trifan@gmail.com.
* Department of Economics at Truman State University Missouri, 100 E. Normal Ave, McClain Hall, Office MC230, Kirksville, MO 53501, USA. Phone: +1 (660) 785 4577, e-mail: rossent@truman.edu.

1. INTRODUCTION

This paper addresses the behavior of non-professional investors who derive utility from both consumption and financial wealth fluctuations. Specifically, we account for consumption as additional source of utility besides financial investments. This study builds on the work of Rengifo and Trifan (2009 and 2010) [8,9]. In this paper we introduce an improvement on their model using non-expected utility theory. The data set is the same as in Rengifo and Trifan (2009) [8], which allows for comparison of the results.

Non-professional investors have to decide upon the optimal wealth allocation between consumption and financial investments in total. The latter category offers a further choice between a risky portfolio and a risk-free asset. In particular, one can think of this decision process of non-professional investors as unfolding in two main steps: First, they determine the total sum of money to be invested in financial markets. Second, in order to optimally split this money among different financial instruments, they ask for professional advice. In so doing, non-professional investors commit the technical details of the optimization of their asset portfolio to professional managers. Moreover, non-professional investors provide managers with information about the level of risk they are ready to bear. Acting on this information, managers finally derive the optimal capital allocation for their particular clients. What is important for non-professional investors in this context is simply how their wealth can be (optimally) split between risky and risk-free assets.

We adopt the formal views regarding the subjective perception of risky vs. risk-free investments – i.e. *the prospective value* – and how it enters the wealth-allocation problem of non-professional investors. In this context, loss aversion is quantified by two measures: the *loss-aversion coefficient* and the *global first-order risk aversion* gRA developed by Rengifo and Trifan (2009) [8], which extends Rengifo and Triffan (2010) [9]. Wealth allocation is expressed by the wealth percentages dedicated to consumption and to financial assets. In addition, we rely on the theoretical approach of Barberis, Huang, and colleagues (2001, 2004, 2006) [1,2,3,4], according to which investors decisions rely on the maximization of recursive non-expected utility with first-order risk aversion where, the utility function is shaped in order to account for the excessive focus (in technical terms *narrow framing* or *myopia*) on financial investments and for the influence of past portfolio performance on the current perceptions of risky investments.

We analyze the loss attitude and wealth allocation in the aggregate equilibrium with a representative investor. In this paper we concentrate our efforts in a setting with non-expected utility and compare it with the one where expected utility used in Rengifo and Trifan (2009) [8]. We derive the equilibrium equation and then infer the variables of interest from these equations. The single variable for which both settings deliver expressions in equilibrium is the prospective value. It further serves for obtaining equilibrium-equivalent measures of the loss attitude, specifically the loss-aversion coefficient and gRA. Under non-expected utility, the percentages of total wealth allocated to consumption and of post-consumption wealth invested in risky assets are, direct equilibrium estimates.

The theoretical part is implemented based on the same data set used in as in Rengifo and Trifan (2009) [8], since one of our goals is to compare our results with theirs. In particular, we consider S&P 500 and 3-months T-bill nominal returns as proxies for a well-diversified risky portfolio and the risk-free investment, respectively. In addition, we employ quarterly data of the aggregate per-capita consumption that provide for consumption values at (only) two different evaluation horizons of the risky-portfolio performance: one year and three months. This allows us to analyze the myopic aversion.

We simulate how non-professional investors behave in an environment where consumption and financial markets are characterized by general parameters, such as the risk-free returns and the dynamics of consumption and of expected returns, derived from the sets of real data at hand. We account for various investor profiles by choosing different combinations of our behavioral parameters, such as the degree of narrow framing, the consumption-related risk aversion, the weight of financial utility, the sensitivity to past losses, the way of accounting for past performance, etc. Moreover, in order to avoid the impossibility of covering current consumption needs from financial revenues over the entire investing interval, we consider that investors periodically dispose of exogenous additional incomes, the level of which can vary as well.

We show that our setting deliver different recommendations based on the two measures of loss aversion – the loss-aversion coefficient and gRA. In particular, loss aversion can manifest in multiple ways and depends on the measure used to quantify it. Myopic loss aversion can be tested only over two evaluation frequencies, but in multiple ways. We demonstrate that is supported under expected-utility maximization and only when loss attitudes are measured by the loss-aversion coefficient (what we denote to be *myopic loss aversion* in the strict sense); It also holds with respect to the perception of risky investments captured by the prospective value, but this time exclusively under non-expected utility; None of the two settings provides evidence for myopic loss aversion with respect to the money dedicated to risky assets. Moreover, the non-expected utility maximization appears to be better suited to describe individual behaviors, based on the robustness of the estimates and a more intuitive economic interpretation.

The remainder of this work is organized as follows: Section 2, briefly reviews the model in Rengifo and Trifan (2010)[9]. In so doing, first we focus on variables that describe perceptions; then we address the optimal wealth allocation with individual levels of risk VaR*. The beginning of section 3 following Rengifo and Triffan (2009) [8] sets the stage for two sources of individual utility: financial wealth fluctuations and consumption. In essence, the wealth-allocation problem in Rengifo and Trifan (2010) [9] is now augmented by a step splitting money between consumption and financial investments—that should be placed before partitioning the last sum between risk-free and risky assets. The second part of section 3 presents the important case of non-expected utility which according to Barberis, Huang, and Thaler (2006) [3] better captures the utility of decisions under risk. The forth section, presents the implementation of the theoretical model for two different sets of behavioral parameters. It further compares the results with the outcomes Rengifo and Triffan (2009) [8]. Finally, the last section of this chapter summarizes our findings and concludes.

2. THEORETICAL MODEL

This section contains the main theoretical considerations of the paper. It starts by reviewing the portfolio selection model in Campbell, Huisman, and Koedijk (2001) [6]. This model uses VaR as its measure of risk. Our setting, subsequently formulated, incorporates the individual perception of risky projects as captured in the extended prospect theory framework of Barberis, Huang, and Santos (2001) [4]. It is shown in detail the construction of a measure of individual loss aversion VaR* and its implications for the wealth allocation decisions of non-professional investors. We also add to the formal representation of investor attitudes towards financial losses by introducing the notion of *global* first-order risk aversion.

2.1. Optimal Portfolio Selection with "Exogenous" VaR

The model in Campbell, Huisman, and Koedijk (2001) [6] follows the common procedure of portfolio optimization, where market risk is assessed by means of *Value-at-Risk* (VaR). In particular, financial assets are chosen in order to maximize expected returns, subject to a twofold restriction: the budget and risk constraints. The maximum expected loss from holding the risky portfolio should not exceed what is denoted as *exogenous* VaR (VaR^{ex}). VaR^{ex} stands for the risk level that the non-professional client is disposed to bear. It is indicated to the portfolio manager in form of a single, fixed number. In this model, managers consider $\mathrm{VaR}^{\mathrm{ex}}$ as a constraint exogenous to the optimization problem.

The objective of the optimization problem in Campbell, Huisman, and Koedijk (2001) [6] is maximizing the next-period wealth W_{t+1}. The risky portfolio consists of $i = 1,\dots,n$ financial assets with single time t prices $p_{i,t}$ and portfolio weights $w_{i,t}$, such that $\sum_{i=1}^{n} w_{i,t} = 1$. $a_{i,t}$ is the number of shares of the asset i contained in the portfolio at time t. Thus:

$$W_{t+1}(w_t) = (W_t + B_t)E_t[R_{t+1}(w_t)] - B_t\, R_f \underset{w_t}{\rightarrow} max \quad (1)$$

$$W_t + B_t = \sum_{i=1}^{n} a_{i,t} p_{i,t} = a_t' p_{\,t} \quad \textit{(budget constraint)} \quad (2)$$

s.t.

$$P_t[W_{t+1}(w_t) \leq W_t - VAR^{ex}] \leq 1 - \alpha \quad (risk\ constraint)$$

where $R_{t+1}(w_t)$ stands for the *portfolio gross returns* at the next trade, $E_t[R_{t+1}(w_t)]$ for the corresponding expected returns. Henceforth, it is referred to the gross returns of the risky portfolio by "*returns*" or "*portfolio returns*".

In the above Equations (1) and (2), B_t denotes the *risk-free investment* and the fixed risk-free gross return rate is R_f. Finally, P_t denotes *the* conditional probability given the information at time t, and $1 - \alpha$ the chosen confidence level.

Campbell, Huisman, and Koedijk (2001) [6] obtain the optimal weights of the risky portfolio as:

$$w_t^{opt} \equiv arg\, max_{w_t} \frac{E_t[R_{t+1}(w_t)] - R_f}{W_t R_f - W_t q_t(w_t, \alpha)} \quad (3)$$

where $q_t(w_t, \alpha)$ represents the quintile of the distribution of portfolio gross returns $R_{t+1}(w_t)$ for the confidence level $1 - \alpha$, i.e. $P_t[R_{t+1}(w_t) \leq q_t(w_t, \alpha)] \leq 1 - \alpha$. Thus, the optimal mix of risky assets depends merely on the distribution of the portfolio gross returns and on the significance level α.

Equation (3) shows that, *the two-fund separation theorem* applies: Neither the (non-professional) investors' initial wealth nor the desired risk level VaR^{ex} affects the maximization procedure. In other words, investors first determine the optimal risky portfolio and second, they decide upon the extra amount of money to be borrowed or lent. The latter reflects by how much the portfolio VaR, defined as:

$$VaR_t = W_t\big(q_t\big(w_t^{opt}, \alpha\big) - 1\big) \tag{4}$$

varies according to the investor degree of loss aversion measured by the selected VaR^{ex} level.

The optimal investment in risk-free assets can be then written as:

$$B_t = \frac{VaR^{ex} + VaR_t}{R_f - q_t\big(w_t^{opt}, \alpha\big)} \tag{5}$$

and hence the value of the risky investment at time $t+1$ yields:

$$S_{t+1} = (W_t + B_t)R_{t+1} \tag{6}$$

Since it is considered that non-professional investors are mainly concerned with how to split their money between risky and risk-free assets, the optimal investments in risk-free and risky assets in Equations (5) and (6) represent fundamental variables in the model.

2.2. The individual loss level VaR*

Coming from the main ideas of the setting in Campbell, Huisman, and Koedijk (2001) [6], the current model goes a step further by asking how non-professional investors actually arrive at their desired level of loss aversion. As far as the optimization procedure presented above is concerned, one can think of VaR* formally replacing VaR^{ex} in the above equations, but remaining an exogenous input (or constraint). However, the value of this risk constraint forms in our approach on the basis of individual behavioral parameters and affects the final wealth allocation between risky and risk-free assets, as apparent from Equation (5). This extension of the allocation problem is the reason to denote VaR* as the *endogenous individual loss level*.

2.3. The Value Function

Investors' desires and attitudes – hence their subjective loss level VaR* – depend on their perception of the value of financial investments. The prospect theory (PT) in Kahneman and Tversky (1979) [7] and Tversky and Kahneman (1992) [10] suggests how individual perceptions of financial performance can be formalized by means of the so-called *value function* v. Accordingly, human minds take for actual carriers of value not the absolute outcomes of a project, but their changes defined as departures from an individual reference point. The deviations above (below) this reference are labeled as gains (losses). Thus, the value function is kinked at the reference point and exhibits distinct profiles in the domains of gains and losses, being steeper for losses (a property known as *loss aversion*). It also shows diminishing sensitivity in both domains, i.e. it is concave for gains but convex for losses.

As noted in Barberis, Huang, and Santos (2001) [4], individual perceptions can be additionally influenced by the past performance of risky investments. This past performance is captured by the *cushion* concept. Formally, the cushion corresponds to the difference between the current value of the risky investment St and a historical benchmark level of the risky value Z_t. When this difference is positive, investors made money from investing in risky assets in the past, otherwise they made losses.

The approach relies on the extended formulation of the value function proposed in Equations (15) and (16) by Barberis, Huang, and Santos (2001) [4]. In the following $x_t = R_t - R_{ft}$ is the *risk premium*, $S_t - Z_t$ is the *(absolute) cushion*, and $z_t = Z_t/S_t$ is *the relative cushion*. The positive (negative) past performance corresponds to a positive (negative) cushion that can be termed as $Z_t \leq S_t$ $(Z_t > St)$ or equivalently as $z_t \leq 1$ $(z_t > 1)$. The value function takes different courses depending on the past performance and can be expressed as follows:

$$v_{t+1} = \begin{cases} v_{t+1}^{prior\,gains} & , for\, z_t \leq 1 \\ v_{t+1}^{prior\,losses} & , for\, z_t > 1 \end{cases} \tag{7}$$

where:

$$v_{t+1}^{prior\,gains} = \begin{cases} S_t x_{t+1} & , for\, x_{t+1} + (1 - z_t)R_{ft} \geq 0 \\ \lambda S_t x_{t+1} + (\lambda - 1)(S_t - Z_t)R_{ft} & , for\, x_{t+1} + (1 - z_t)R_{ft} < 0 \end{cases} \tag{8}$$

and

$$v_{t+1}^{prior\,losses} = \begin{cases} S_t x_{t+1} & , for\, x_{t+1} \geq 0 \\ \lambda S_t x_{t+1} + k(Z_t - S_t)x_{t+1} & , for\, x_{t+1} < 0 \end{cases} \tag{9}$$

The parameter λ in Equations (8) and (9) is termed the *coefficient of loss aversion*. According to PT, investors are loss averse when $\lambda > 1$, while $\lambda = 1$ points to loss neutrality.

The parameter $k \geq 0$ captures the influence of previous losses on the perception of current ones: The larger the previous losses are, the more painful the next losses become. It is denoted as the *sensitivity to past losses*.

Note that the gain branches of both value functions in Equations (8) and (9) are invariable to the past performance Z_t. The loss branches are yet distinct. However, they both contain a first term $\lambda S_t(R_{t+1} - R_{ft})$ that resembles the original PT, but also a second one revealing the impact of the cushion $S_t - Z_t$. Also, the reference point shifts in dependence on the past performance.

Henceforth, the following probability notations are used:

$$\pi_t = P_t(z_t \leq 1)$$
$$w_t = P_t(x_{t+1} \geq 0 | z_t > 1) \tag{10}$$
$$\psi_t = P_t(x_{t+1} + (1 - z_t)R_{ft} \geq 0 | z_t \leq 1)$$

where π_t stands for the probability of past gains, and w_t for the probability of a positive premium given past losses. Finally, ψ_t is the probability of obtaining a risk premium $x_{t+1} + (1 - z_t)R_{ft}$, higher than the risk premium x_{t+1}, that expresses raised expectations resulting from recurrent gains.

2.4. The Prospective Value of the Risky Investment

In Equation (5), the risk-free investment depends, among others, on the risk level VaR^{ex} indicated by the non-professional client to the portfolio manager. The traditional approach does not account for the way in which non-professional investors ascertain this level. This ascertainment should take place according to individual perceptions of financial losses which can, in line with PT, substantially differ from the actual losses. In this section, we define a new measure of the individual loss level (more specifically, the individually accepted or desired loss level) that we denote as VaR*.

In so doing, we start from the literal definition of VaR*: the maximum loss that can be a-priori expected by someone investing in risky assets. We concentrate on the terms “loss”, “individual”, and “maximum” encompassed by this definition. First, VaR* quantifies losses. According to PT, what actually counts for individual (non-professional) investors is not the absolute magnitude of a loss, but rather the subjectively perceived one, as captured by the value function described above. Hence, VaR* should rely on the subjective value of losses expressed in the loss branches of the value functions in Equations (8) and (8). It thus depends on individual features, originating in the subjective view over gains and losses, and can vary over time, for instance with the past performance of risky investments. Moreover, we are looking for a maximal value. This is obtained in that, in calculating VaR*, investors ascribe a maximal occurrence probability (of 1) to the losses in the value function, so that $\pi_t(1-\psi_t)+(1-\pi_t)(1-w_t)=1$ Finally, VaR* should correspond to the concept of Value-at-Risk and hence represent a quintile, namely, according to the above considerations, a quintile of the subjective loss distribution.

Therefore, we suggest the following formal definition for the individual loss level:

$$\begin{aligned}
&VaR^{*}_{t+1}=E_t[loss-value_{t+1}]-\varphi\sqrt{VaR_t[loss-value_{t+1}]}\\
&=\lambda S_t E_t[x_{t+1}]-kE_t[x_{t+1}](S_t-Z_t)\\
&+\sqrt{\pi_t(1-\psi_t)}\left(\sqrt{\pi_t(1-\psi_t)}-\varphi\sqrt{1-\pi_t(1-\psi_t)}\right)\left((\lambda-1)R_{ft}+kE_t[x_{t+1}]\right)(S_t+Z_t)\\
&=\lambda S_t E_t[x_{t+1}]+\left(\zeta_t(\lambda-1)R_{ft}+(\zeta_t-1)kE_t[x_{t+1}]\right)(S_t-Z_t)
\end{aligned} \tag{11}$$

where “loss-value” stands for the subjective value ascribed to financial losses according to the loss branch of the value functions in Equations (8) and (9), and the subjectively perceived losses are assumed to follow a distribution (e.g. normal or Student-t) with the lower quintile φ. Moreover, $\mathrm{Et}[x_{t+1}]=\ \mathrm{Et}[R_{t+1}]-R_{ft}$ denotes the expected risk premium. The last expression in Equation (11) is obtained using the simplifying notation $\zeta_t=\sqrt{\pi_t(1-\psi_t)}\big(\sqrt{\pi_t(1-\psi_t)}-$ $\varphi\sqrt{1-\pi_t(1-\psi_t)}\big)$

We distinguish two terms of the VaR*-expression in Equation (11): The first one accounts for the expected risky return (relative to the risk-free rate) $S_t E_t[x_{t+1}]$, weighted by the loss aversion coefficient ¸. As it consequently resembles the prospective value according to the original PT, we denote this term as the PT-term. The last term is responsible for the influence of the previous performance captured by the cushion $S_t = Z_t$ in Barberis, Huang, and Santos (2001) [4]. For this reason, we denote it as the cushion term. The corresponding weight is a linear combination of the expected risky and the risk-free returns.

Once non-professional investors set their minds about the desired VaR*, they communicate it to the portfolio manager. In the view of the latter, this client indication represents an exogenous risk level that corresponds to VaR^{ex} in Equation (5) and is applied in order to determine the optimal level of borrowing or lending B_t. When VaR* is lower in absolute value than the portfolio VaR, B_t is negative, which formalizes the profile of more risk-averse investors who prefer to increase the proportion of wealth invested in risk-free assets. In contrast, for a VaR* higher than VaR in absolute value, investors augment their risky investments by borrowing extra money, i.e. they are less risk averse. Thus, analyzing the evolution of B_t can shed some light on the behavior of non-professional investors confronted with financial losses.

A further interesting topic to investigate lies in estimating the equivalent loss aversion parameter λ_t^* that can be obtained for a fixed $\overline{VaR^*}$ under the traditional approach. Common assumptions of this approach are significance levels of 1%, 5%, or 10% and no dependency on past performance $k = 0$. The formula of λ_t^* is then immediate from the definition in Equation (11) for k = 0. This yields:

$$\lambda_{t+1}^* = \frac{\overline{VaR^*} + \zeta_t R_{ft}(S_t - Z_t)}{S_t E_t[x_{t+1}] + \zeta_t R_{ft}(S_t - Z_t)} \tag{12}$$

The estimation of the individually maximum acceptable loss level VaR* represents only the first step in the analysis. VaR* dictates the optimal choice of the non-professional investors in terms of wealth percentages allocated between risky and risk-free assets.

It is also of interest the attitude of non-professional investors towards financial losses in general, as this attitude influences the level of the individual VaR*. The loss attitude results from the perception of the utility generated by financial investments. The corresponding PT-concept of expected utility is the *prospective value* V and it depends on the fixed VaR*. In this framework, the prospective value of the risky portfolio can be formulated as:

$$
\begin{aligned}
&V_{t+1} = \pi_t E_t\left[v_{t+1}^{prior\ gains}\right] + (1-\pi_t)E_t\left[v_{t+1}^{prior\ losses}\right] \\
&= \pi_t\left[\psi_t S_t E_t[x_{t+1}] + (1-\psi)\left(\lambda S_t E_t[x_{t+1}] + (\lambda-1)(S_t - Z_t)R_{ft}\right)\right] \\
&+ (1-\pi_t)[w_t S_t E_t[x_{t+1}] + (1-w_t)(\lambda S_t E_t[x_{t+1}] + k(Z_t - S_t)E_t[x_{t+1}])] \\
&= \left(\pi_t\psi_t + (1-\pi_t)w_t + \left(\pi_t(1-\psi_t) + (1-\pi_t)(1-w_t)\right)\lambda\right)S_t E_t[x_{t+1}] \\
&+ \left(\pi_t(1-\psi_t)(\lambda-1)R_{ft} - (1-\pi_t)(1-w_t)kE_t[x_{t+1}]\right)(S_t - Z_t)
\end{aligned}
\tag{13}
$$

The first term of the last expression in Equation (13), that subsequently is denoted as the *PT-effect*, captures the expected risky-investment value relative to the safe bank investment $S_t E_t[x_{t+1}]$. The corresponding probability weight is the sum of perceived gain and loss probabilities, laxly put $P_t(gain) + \lambda P_t(loss)$. It points out that, losses are larger than gains, being additionally penalized by the loss aversion coefficient λ.

The last term of the prospective value in Equation (13) covers the cushion influence and it is referred as the *cushion effect*. The weight of the cushion $S_t - Z_t$ is in this term a combination of expected losses obtained under the consideration of the performance history. Specifically, when current losses follow past gains – which occur with the joint probability $\pi_t(1-\psi_t)$ – the past performance is valued at the risk-free rate R_{ft} and is amended by how much the loss aversion coefficient λ exceeds the loss-neutral value of 1. Indeed, if risky investments were successful in the past, a current loss has value only compared to the alternative of having put the entire money in risk-free assets. When losses extend from past to present – where $(1-\pi_t)(1-w_t)$ is the joint probability of current and past losses – the valuation implies a comparison of the risk-free rate to the risky performance $E_t[x_{t+1}]$ in view of the sensitivity to past losses k.

It is of interest to examine the evolution of the prospective value not only in time but also for different portfolio evaluation frequencies. The rationale for this is that revising portfolio performance at different time intervals implies, first, drawing back on distinct return values. Second, these return changes implicitly impact, at later times, on further model parameters, such as the cushion and the probabilities of past and current gains and losses. Therefore, the prospective value in Equation (13) is affected in multiple ways.

In so doing, a further notion is applied referring to the investor attitudes towards financial risks that attempts to capture more complex dependencies than the simple coefficient of loss aversion λ. According to PT, loss aversion corresponds to risk aversion of first order in the loss domain. The first derivative of the prospective value with respect to the expected risk premium is termed as the *global first-order risk aversion* (gRA). Formally, gRA yields:

$$gRA_t = \frac{\partial V_{t+1}}{\partial E_t[x_{t+1}]} = \big(\pi_t\psi_t + (1-\pi_t)w_t + \big(\pi_t(1-\psi_t) + (1-\pi_t)(1-w_t)\big)\lambda\big)S_t$$

$$-(1-\pi_t)(1-w_t)k(S_t - Z_t) \qquad (14)$$

gRA reflects the sensitivity – in terms of first-order changes – of the prospective value to the variation of expected returns. Due to the linearity the prospective value in the expected risk premium $E_t[x_{t+1}]$, gRA is independent of this premium.

Moreover, since gRA directly reflects changes in the prospective value – which is proportional to the attractiveness of financial investments – higher gRA-values point to a more relaxed loss attitude. This can be recognized in Equation (14): The first term increases with the sum invested in risky assets S_t; The second is inversely proportional to the cushion $S_t - Z_t$. Note yet that this second term accounts for the situation when past losses are followed by current losses, which occurs with the probability $(1-\pi_t)(1-w_t)$. In such a case, cushions are most probably negative $S_t - Z_t \leq 0$. Smaller (negative) cushions will then render this second term higher. In sum, gRA grows both when investors put more money in risky assets and when they manage to reduce recurrent losses.

3. TWO-DIMENTIONAL UTILITY: CONSUMPTION VS. FINANCIAL ASSETS

Following Rengifo and Triffan (2009) [8] this section presents sets the theoretical framework describing how non-professional investors perceive financial risks and accordingly allocate their wealth between consumption and financial assets in order to maximize perceived utility. Further in line with Barberis and Huang (2004, 2006) [1,2], a recursive non-expected utility with first-order risk aversion formulation for the maximization problem is adopted. This setting accounts for narrow framing of financial projects and for the influence of past performance on the perceived value of risky investments.

Investor attitudes depend on the subjective perception of financial investments and on the possible losses associated with these investments. Perceptions are modeled according to the extended prospect-theory framework by Barberis, Huang, and Santos (2001) [4]. Accordingly, risky performance is mentally split – with respect to a subjective reference point – in gains and losses; moreover, losses loom larger than gains of the same size, and past performance influences current perceptions.

Two distinct cushion definitions are considered: the myopic cushions for which the benchmark level of past performance was taken to be identical to the last-period risky holdings $Z_t = S_{t-1}$; and the dynamic cushions which assumed the same benchmark to be a combination of past references and current risky investment values $Z_t = \eta Z_{t-1}\bar{R} + (1-\eta)S_t$ where the parameter η measured how far in the past the investor memory stretches. Hence, myopic cushions amount to $S_t - S_{t-1}$ and dynamic ones $\eta(S_t - Z_{t-1}\bar{R})$.

Central to the analysis is the derivation of the *prospective value* V from Equation (13). This variable captures the subjectively perceived utility of the risky portfolio (relatively to the risk-free rate) and hence is related to the attitude adopted towards financial losses. One goal of the present work is to determine the prospective value ascribed – in the equilibrium of the aggregate market – to financial investments by investors who derive utility from consumption and financial investments.

Drawing on the idea that individual attitudes towards financial losses can be measured by means of the loss-aversion coefficient, it is interesting to compute a loss-aversion coefficient $\bar{\lambda}$ that is equivalent to the prospective value $\bar{V}$. From Equation (13) $\bar{\lambda}$ formally yields:

$$\overline{\lambda_t} = \frac{\overline{V_{t+1}} - (\pi_t\psi_t + (1-\pi_t)w_t)S_tE_t[x_{t+1}] + \left(\pi_t(1-\psi_t)R_{ft} + (1-\pi_t)(1-w_t)kE_t[x_{t+1}]\right)(S_t - Z_t)}{\left(\pi_t(1-\psi_t) + (1-\pi_t)(1-w_t)\right)S_tE_t[x_{t+1}] + \pi_t(1-\psi_t)R_{ft}(S_t - Z_t)} \tag{15}$$

The coefficient $\bar{\lambda}$ plays a central role in our model, as it stands for an equilibrium equivalent measure of the attitude towards financial losses. Note that established research (based on PT) works often with values of 2.25 for the loss-aversion coefficient.

However, the simple loss-aversion coefficient fails to capture the influence of past performance that is yet explicitly considered in the extended PT by Barberis, Huang, and Santos (2001) [4]. Consequently, equation (14) has introduced a further measure of the loss attitude denoted as the *global first-order risk aversion*. In the applied part of the present section, the evolution of gRA is analyzed in the two-dimensional utility equilibrium. In essence, higher gRA-values point to more relaxed loss attitudes, as this measure directly reflects changes in the attractiveness of financial investments captured by the prospective value.

The non-professional investors are not merely concerned with financial investments and the utility they generate - this assumption appears to be better suited to professional investors than to non-professional ones. The activity of the former demands a strictly investment-oriented perspective, and their main task reduces to making money that is going to be reinvested in financial markets. In contrast, non-professional investors sooner regard financial investments as a source of income dedicated to covering consumption needs. In other words, consumption should be the main generator of individual utility for non-professional investors. However, financial investments might be perceived as an equally important source of utility. The main reason resides in the above mentioned narrow framing, i.e. the excessive focus on financial investments, which appears to be driven by the fear of registering losses when faced with financial risks.

Based on these considerations, the current model allows for two sources of individual utility: financial wealth fluctuations and consumption. In so doing, the model relies on Barberis, Huang, and Santos (2001) [4], Barberis and Huang (2004) [1], and Barberis and Huang (2006) [2]. The present section details the theoretical background of our contribution.

The above wealth allocation problem based on one-dimensional utility is augmented with an additional step: splitting money between consumption and financial investments. Strictly speaking, the non-professional investors decide first on how much money should be dedicated to consumption needs and to financial assets in total; Only afterwards they can partition the latter sum between risk-free and risky assets. As the performance of risky investments is mostly measured with respect to risk-free assets, it is possible formally to merge these two successive steps into a single decision. The common goal is then the maximization of total utility derived from consumption and risky – relative to risk-free – financial investments.

Following Barberis and Huang (2004) [1], an aggregate market which lacks perfect substitution is considered. Thus, one can focus on absolute pricing and avoid possible arbitrage opportunities generated by narrow framing. The total utility is formulated in order to account for the above-mentioned two-dimensional origin and yields the sum of discounted utilities of consumption $U(C)$ and of perceived values of financial investments $\tilde{V}$, that is:

$$U = U(C) + \tilde{V} = \sum_{t=0}^{\infty}\left(\rho^t U(C_t) + \rho^{t+1} b_t \tilde{V}_{t+1}\right) \tag{16}$$

where ρ is referred to as the *discounting factor* and $0 < \rho < 1$. According to Equation (16), at each time t the current consumption is discounted with ρ^t, while the prospective value – that encompasses subjective perception of the next-period performance– has to be provided with a corresponding ρ^{t+1}.

In line with Barberis, Huang, and Santos (2001) [4], b_t is an exogenous scaling factor designed to map the perceived value of gains and losses into consumption units. It follows the rule stated in their Equation (11), namely $b_t = b_0 \bar{C}_t^{-\gamma}$ where $\bar{C}_t$ represents the exogenous *aggregate per-capita consumption* at time t, and b_0 measures the *degree of narrow framing*. Finally, we denote γ is as the *consumption-related coefficient of risk aversion*.

In line with Barberis and Huang (2006) [2], it is possible to develop an equilibrium framework in the aggregate market with a representative investor. In this paper the equilibrium conditions are derived when a recursive non-expected utility function with first-order risk aversion is optimized. Throughout, the assumptions of narrow framing and dependence of current decisions on past portfolio performance are formally incorporated.

Furthermore is presented the phenomenon of *myopic loss aversion* (mLA). Introduced by Benartzi and Thaler (1995) and supported by numerous experimental tests, mLA refers to the fact that narrow framing (or myopia) strengthens the loss aversion, so that investors reduce their risky investments when risky performance is checked on more frequently. In view of the manifold possibilities to quantify the loss attitude, the notion of mLA is refined in the following sense: It is denoted as mLA in the strict sense the enhancement of loss aversion with the evaluation frequency. According to the model, the loss aversion can be quantified either by the loss-aversion coefficient or by the extended measure gRA. Thus, mLA in the strict sense holds if either the loss- aversion coefficient increases or gRA decreases with the evaluation frequency. As both loss-aversion measures are derived from individual perceptions of risky investments, one can also measure mLA in the large sense with respect to the prospective value. mLA in the large sense can be supported if the prospective value falls at higher evaluation frequencies. Finally, mLA in the monetary sense is defined as the decrease of monetary risky holdings – in percentages of the total wealth – in consequence of more frequent portfolio evaluations. In addition, one can speak about myopic aversion towards financial investments when the wealth percentages dedicated to consumption increase with the evaluation frequency. Note that our data sets constrain us to check on mLA only at two evaluation frequencies. For the development of similar idea using the expected utility approach we refer the reader to read Rengifo and Trifan (2009) [8]. In this paper the topic is fully develop based on the non-expected utility approach.

3.1. The Non-Expected Utility Approach

Although the expected-utility maximization represents the most widespread theoretical approach so far, Barberis, Huang, and Thaler (2006) [3] claim that another specification captures better the utility of decisions under risk: *the non-expected recursive utility with first-order risk aversion* (R-FORA). Yet, simple R-FORA specifications account merely for loss aversion and hence have to be extended in order to accommodate with both loss aversion and narrow framing, since these two phenomena appear to be crucial in financial markets. Henceforth, the R-FORA setting with narrow framing is referred as the non-expected utility approach. We rely on the approach proposed in Barberis and Huang (2006) [2], according to whom investors maximize a recursive utility-function U_t, that is defined as:

$$U_t = \diamond \langle C_t, \mu(U_{t+1} + b_0 E_t[v(G_{t+1})] | F_t \rangle \tag{17}$$

Where

$$\diamond \langle C, y \rangle = \left((1-\beta)C^{1-\gamma} + \beta y^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \text{ , } for\ 0 < \beta < 1, 0 < \gamma \neq 1 \tag{18a}$$

$$\mu(y) = (E[y^{1-\gamma}])^{\frac{1}{1-\gamma}} \text{, } for\ 0 < \gamma \neq 1 \tag{18b}$$

$$G_{t+1} = \theta_t (W_t - C_t)(R_{t+1} - R_{ft}) \tag{18c}$$

$$v(y) = \begin{cases} y, for\ y \geq 0 \\ \lambda x, for\ y < 0 \end{cases} \quad for\ \lambda > 1 \tag{18d}$$

Here, $\diamond \langle .,. \rangle$ is an aggregator function, $\mu(.)$ the certainty equivalent of the distribution of future utility conditional on the information F_t at time t and G_{t+1} the next-period value of the risky investment. The parameter β is henceforth referred to as the *weight of financial utility*.

According to Barberis and Huang (2004) [1], the certainty equivalent $\mu(.)$ is assumed to be homogenous of degree one and, in order to ensure tractability, the individual value function v must also be homogenous. Consequently, for the equilibrium conditions to be necessary and sufficient, v has to take the piecewise-linear form in equation (18d). In other words, a good behaved equilibrium with non-expected utility does not allow for the influence of past performance on current perceptions of financial risk as proposed in the extended PT-framework by Barberis, Huang, and Santos (2001) [4], but merely for loss aversion as in the initial PT of Kahneman and Tversky.

Therefore, the non-expected utility equilibrium reduces to imposing the condition of nil cushions $S_t = Z_T$ in all equations of the theoretical model in the previous section. This condition can be interpreted as a particular case with dynamic cushions $\eta(S_T - Z_{t-1}\bar{R})$, where $\eta = 0$ or, in other words, when investors have no memory of the past performance. It is this case that is extensively studied in the applicative. Obviously, all influence of the sensitivity to past losses k on the model estimates is discarded.

In the non-expected utility maximization, the analysis is restricted to the general equilibrium for aggregate markets with a representative investor, in line with Equations (60)-(62) and with the subsequent Example 6.1 for stock markets in Barberis and Huang (2004) [1]. The focus remains on non-professional investor decisions concerning wealth allocation among consumption, the risky portfolio with gross returns R_t, and the risk-free asset with the gross return R_{ft}.

Let α_t be the fraction of total wealth dedicated to consumption, which is formally:

$$\alpha_t = \frac{C_t}{W_t} \tag{19}$$

When a fraction θ_t of post-consumption wealth is to be invested in the risky portfolio and another fraction of total wealth α_t to be consumed, the following Euler equations yield necessary and sufficient conditions at equilibrium:

$$\beta R_{ft} E_t\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right]\left(\beta E_t\left[\frac{\bar{C}_{t+1}}{C_t}\right] R_{t+1}^{tot}\right)^{\frac{\gamma}{1-\gamma}} = 1 \tag{20a}$$

$$\frac{E_t\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}(R_{t+1}-R_{ft})\right]}{E_t\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right]} + b_0 R_{ft}\left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{1-\alpha_t}{\alpha_t}\right)^{-\frac{\gamma}{1-\gamma}} E_t\left[v\left(R_{t+1}-R_{ft}\right)\right] = 0 \tag{20b}$$

$$\frac{E_t\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}(R_{t+1}^{tot}-R_{ft})\right]}{E_t\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right]} + b_0 R_{ft}\left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{1-\alpha_t}{\alpha_t}\right)^{-\frac{\gamma}{1-\gamma}} \theta_t E_t\left[v\left(R_{t+1}-R_{ft}\right)\right] = 0 \tag{20c}$$

where $R_{t+1}^{tot} = \theta_t R_{t+1} + (1-\theta_t) R_{ft}$ is the *total gross return of financial assets*. Equation (20c) is derived from Equation (20b) by multiplying it withθ_t.

The next period financial wealth formulated as:

$$W_{t+1} = (W_t - C_t)\left(\theta_t R_{t+1} + (1-\theta_t) R_{ft}\right) \tag{21}$$

can be now rewritten as $W_{t+1} = (W_t - C_t) R_{t+1}^{tot}$ and thus we obtain:

$$R_{t+1}^{tot} = \frac{\alpha_t}{\alpha_{t+1}(1-\alpha_t)} \frac{C_{t+1}}{C_t} \tag{22}$$

Assuming time constancy for the portfolio wealth fraction θ, the consumption ratio α, and the risk-free return R_f, the total gross return results in:

$$R_{t+1}^{tot} = \frac{1}{1-\alpha}\frac{C_{t+1}}{C_t} \Longrightarrow \log(R_{t+1}^{tot}) = -\,log(1-\alpha) + c + \sigma_c \epsilon_{t+1} \tag{23}$$

Thus, the equilibrium Equations (20) yield:

$$\beta^{\frac{1}{1-\gamma}}(1-\alpha)^{-\frac{\gamma}{1-\gamma}} R_f E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right]\left(E\left[\left(\frac{\bar{C}_{t+1}}{C_t}\right)^{1-\gamma}\right]\right)^{\frac{\gamma}{1-\gamma}} = 1 \tag{24a}$$

$$\frac{E_t\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}(R_{t+1}-R_f)\right]}{E_t\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right]} + b_0 R_{ft}\left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}} E\left[\bar{v}\left(R_{t+1}-R_f\right)\right] = 0 \tag{24b}$$

$$\frac{E_t\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\left(R_{t+1}^{tot}-R_f\right)\right]}{E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right]}+\ b_0 R_f\left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}}\theta E\left[\bar{v}\left(R_{t+1}-R_f\right)\right]=0 \qquad (24c)$$

The parameter dynamics of consumption and returns are the same as the ones assumed in Barberis and Huang (2006) [2]. Thus we take:

$$log\left(\frac{C_{t+1}}{C_t}\right)=c+\sigma_c\epsilon_{t+1} \qquad (25a)$$

$$log(R_{t+1})=r+\sigma_r\eta_{t+1} \qquad (25b)$$

$$\begin{pmatrix}\epsilon_{t+1}\\ \eta_{t+1}\end{pmatrix}\sim N\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}1 & \sigma_{cr}\\ \sigma_{cr} & 1\end{pmatrix}\right), i.i.d. \, over\ time \qquad (25c)$$

Also, it is considered that $\bar{V}=E\left[\bar{v}\left(R_{t+1}-R_f\right)\right]$, where v corresponds to the value functions in Equation (7) under the condition that $S_t=Z_t$, are equivalent in expectation to the prospective value in Equation (13). Thus, the equilibrium Equations (24) can be further restated as follows:

$$\beta^{\frac{1}{1-\gamma}}(1-\alpha)^{-\frac{\gamma}{1-\gamma}}R_f\ exp\left(\frac{\gamma\sigma_c^2}{2}\right)=1 \qquad (26a)$$

$$-\,exp\left(-\gamma c+r+\frac{\gamma^2\sigma_c^2+\sigma_r^2}{2}-\gamma\sigma_{cr}\right)+R_f\ exp\left(-\gamma c+\frac{\gamma^2\sigma_c^2}{2}\right)$$

$$=b_0 R_f\left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}}exp\left(-\gamma c+\frac{\gamma^2\sigma_c^2}{2}\right)\bar{V}$$

(26b)

$$-\frac{1}{1-\alpha}exp\left((1-\gamma)c+\frac{(1-\gamma)^2\sigma_c^2}{2}\right)+R_f\ exp\left(-\gamma c+\frac{\gamma^2\sigma_c^2}{2}\right)$$

$$=b_0 R_f\left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}}exp\left(-\gamma c+\frac{\gamma^2\sigma_c^2}{2}\right)\theta\bar{V}$$

(26c)

4. APLICATION

This section presents numerical findings based on the theoretical results from the above sections. We first review the general assumptions made in order to facilitate the estimation procedure and to render it comparable to Rengifo and Trifan (2009) [8]. The exposition focuses on two main aspects: On the one hand, the evolution of the attitude towards financial losses is analyzed. This attitude is described first by the loss-aversion coefficient and second by our extended measure gRA. On the other hand, the decisions on wealth allocation among consumption, risky, and risk-free assets are examined. This is quantified by the wealth fractions dedicated to the respective sources of utility. Throughout, a note is taken if mLA continues to manifest in equilibrium.

4.1. General Assumptions

The first data set includes nominal returns of the stock index S&P 500 and of the three-month Treasury bill – as proxies for the risky and the risk-free investment, respectively – from 01/02/1962 to 03/09/2006 (11,005 daily observations). This data is divided into two parts: The observations before 03/01/1982 serve to estimate the empirical mean and the standard deviation of the portfolio returns at the date considered to be the beginning of the trade, namely on 03/01/1982; The second part, from 03/01/1982 to 03/09/2006 (6,010 observations), is the actual data used for performing simulations.

Additionally, aggregate per-capita consumption data between 01/02/1962 and 12/31/2005 sampled at quarterly intervals, provide a basis for the calculation of the log-consumption's mean and variance. Note that the consumption data set only allows assessing consumption values corresponding to portfolio evaluations horizons of one year and three months. We will, analyze how the recommendation of our model changes for these two evaluation frequencies.

After smoothing out the outlier corresponding to the October 1987 market crash, quarterly and yearly returns are constructed from the actual data set and used to derive the main variables that describe the loss attitude and the optimal wealth allocation of our non-professional investors. In so doing, it is assumed that investors start by spreading their wealth equally between consumption and financial assets. The latter fraction is further allocated in equal parts to the risky index and the risk-free T-bill. The investors are long- lived beyond the VaR-horizon and are not allowed to quit the market during the trading period. Portfolio gross returns are assumed to be normally distributed, and future portfolio returns to be estimated as the unconditional mean of past returns. In addition, the risk-free returns, the mean log-consumption, and the mean risky returns are set identical to the means of the corresponding variable, computed over the actual trade period from 03/01/1982 to 03/09/2006, specifically as $\hat{R}_f = mean\left[R_{ft}\right]$, $\hat{c} = mean[\log(C_{t+1}) - \log(C_t)]$, and $r = mean[\log(R_t)]$, respectively.

A final and more specific assumption, tackles an issue emerging from our considerations that investors are long-lived and view financial investments as single source of wealth: It is possible that financial investments do not generate sufficient revenues in order to cover consumption needs over the entire trade interval. This potential problem is circumvented by considering that, at each time t, investors dispose of additional incomes I_t. Such incomes represent, for instance, the wages earned by non-professional investors from their main employment. They are *exogenous*, that is, they stem from outside of those investments that constitute the decision making object at hand. Under this assumption, the total wealth in Equation (21) results from both financial investments and additional incomes and yields at $t + 1$:

$$W_{t+1} - I_{t+1} = (W_t - C_t)\left(\theta_t R_{t+1} + (1 - \theta_t) R_{ft}\right) \tag{27}$$

The additional income I_t may cover a part of the consumption needs of the current period and hence we define it as follows:

$$I_t = \frac{C_t}{\alpha\delta} \tag{28}$$

where α represents the percentages of total wealth dedicated to consumption and δ is an arbitrary constant. Of course, both $\alpha, \delta > 0$.

Apparently, $\delta \leq 1/\alpha$ ($\delta > 1/\alpha$) the additional income exceeds (does not entirely meet) the consumption needs of the period $I_t \geq C_t$ ($I_t < C_t$). Two particular cases are distinguished which, due to their lack of practical meaning, are of *no* interest in the present framework: First, for $\delta = 1$, the current additional income yields a fraction of the consumption needs $I_t = C_t/\alpha$ and investors should assign no money to financial assets in total $R_{t+1}^{tot} = 0$. Second, for $\delta = 1/\alpha$ the additional income covers exactly the current consumption $I_t = C_t$. Then, the total financial investment would be $R_{t+1}^{tot} = C_{t+1}/C_t$ and hence independent of α, which eliminates any connection between the investment decision and the subjective perception of financial investments in the non-expected utility equilibrium. Consequently, one should be looking for values of $\delta \in R^{+}\{0,1,1/\alpha\}$.

The choice of α-values is such that it conforms with the equilibrium estimates of the total-wealth percentages allocated to consumption in the non-expected-utility setting. In particular, the level of the additional income I_t is varied by changing the parameters β and δ. The rationale is that α depends on the financial weight parameter β, according to Equations (24). It observed that I_t increases (decreases) subject to higher values of $\beta(\delta)$ as higher additional incomes are equivalent to more relaxed requirements for financial investments. We take $\beta \in \{0.1;\ 0.5;\ 0.9;\ 0.98\}$, where the last value is the one estimated in Barberis and Huang (2004) [1], and $\delta \in \{0.1;\ 0.5;\ 0.9;\ 2;\ 10;\ 100\}$. Note that, in the expected-utility setting, β has no direct intuitive meaning and thus it is easier to interpret the changes in the additional income resulting from different (β, δ)-combinations.

Further assumptions concern the remaining model parameters, that are in the main of behavioral nature and that are varied in order to study their influence on the main equilibrium variables. In particular, different values are used for the initial loss- aversion coefficient $\lambda \in$

$\{1;\ 2.25;\ 3\}$, where only the latter two corroborate with the non-expected equilibrium according to Equation (21.18d). Barberis, Huang, and Santos (2001) [4] and Barberis and Huang (2006) [2] provide the risk-aversion degree $\gamma \in \{0.5;\ 1;\ 1.5\}$ for the expected-utility setting, where higher values point to increased aversion. In line with the condition $\gamma \neq 1$ from Equation (21.18a), we use $\gamma \in \{0.5;\ 1.5\}$. Furthermore, narrow-framing degrees considered are $b_0 \in \{0.001;\ 0.1;\ 0.5;\ 1;\ 5;\ 10;\ 100;\ 1000\}$, where the first value stands for the situation with (almost) no narrow framing since $b_0 \neq 0$ according to:

$$\rho = \frac{1}{R_f} exp\left(\gamma c - \frac{\gamma^2 \sigma_c^2}{2}\right) \tag{29a}$$

$$\bar{V} = \frac{\bar{C}_t^{\gamma}}{b_0}\left(\frac{1}{\rho} - exp\left(-\gamma c + r + \frac{\gamma^2 \sigma_c^2 + \sigma_r^2}{2} - \gamma \sigma_{cr}\right)\right) \tag{29b}$$

Following the same authors, it is account for no, moderate, and high influence of past losses on the perception of risky investments $k \in \{0;\ 3;\ 10\}$. Recall yet that k plays no role in the non-expected utility equilibrium. The non-expected utility equilibrium allows only for memory less dynamic cushions $\eta = 0$.

Since the lack of space does not allow for an extensive presentation of all obtained results, we subsequently focus on few cases that appear to be the most realistic and that entail plausible. Further interesting situations are explicitly indicated. In particular, we refer to risk-averse investors $\gamma = 0.5$ who narrowly frame financial investments $b_0 \geq 1$. We furthermore consider three qualitatively different levels of the average additional income I, i.e. low, middle, and high, that correspond to the combinations ($\beta = 0.1, \delta = 0.9$), ($\beta = 0.5, \delta = 0.5$), and ($\beta = 0.9, \delta = 0.1$), respectively. Also, we briefly address the middle-range income combination ($\beta = 0.9, \delta = 2$) at the end of the applicative sections.

4.1. Main Results

In this section, the main variables in the equilibrium with non-expected utility are estimated and analysis is performed on how they change *on average* subject to *the ceteris paribus* variation of chosen parameters.

Cushions are the result of a memory less dynamic assessment, i.e. with $\eta = 0$, so that the sensitivity to past losses k exerts no influence on the equilibrium variables. Moreover, from the test values of the consumption-related risk aversion and of the initial loss-aversion coefficient, the non-expected equilibrium allows only for $\gamma \in \{0.5;\ 1.5\}$ and $\lambda \in \{2.25;\ 3\}$, respectively. Finally, the parameter β can be directly interpreted in the context of Equation (18a) as the weight put on that utility piece which stems from financial investments.

The main equilibrium variables are now the percentage of total wealth dedicated to consumption α, the post-consumption wealth invested in risky assets θ, and the prospective value $\bar{V}$. They are derived under the assumption of periodical additional incomes of $I_t = C_t/(\alpha\delta)$. Accordingly, the total gross returns from financial investments in Equation (23) results in:

$$R_{t+1}^{tot} = \frac{1}{1-\alpha}\frac{C_{t+1} - \alpha I_{t+1}}{C_t} = \frac{1}{1-\alpha}\frac{\delta - 1}{\delta}\frac{C_{t+1}}{C_t}$$

$$\Rightarrow log(R_{t+1}^{tot}) = log(\delta - 1) - log(\delta) - log(1-\alpha) + c + \sigma_c \epsilon_{t+1}$$

(30)

and hence the equilibrium Equation (26c) changes to:

$$-\frac{\delta - 1}{\delta(1-\alpha)} exp\left((1-\gamma) + \frac{(1-\gamma)^2\sigma_c^2}{2}\right) + R_f\, exp\left(-\gamma c + \frac{\gamma^2\sigma_c^2}{2}\right)$$

$$= b_0 R_f \left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}} exp\left(-\gamma c + \frac{\gamma^2\sigma_c^2}{2}\right)\theta\bar{V}$$

(31)

For a fixed weight of financial utility β in Equation (18a), α and θ are derived by dividing Equations (26b) and (31) and plugging the result into Equation (26a). Equation (26b) is reformulated in order to obtain an expression for $\bar{V}$. The following expressions are obtained:

$$\alpha = 1 - \beta^{\frac{1}{\gamma}} R_f^{\frac{1-\gamma}{\gamma}} exp\left(\frac{(1-\gamma)\sigma_c^2}{2}\right) \tag{32a}$$

$$\theta = \frac{R_f - \frac{\delta-1}{\delta(1-\alpha)} exp\left(c + \frac{(1-2\gamma)\sigma_c^2}{2}\right)}{\left(R_f - exp\left(r + \frac{\sigma_r^2}{2} - \gamma\sigma_{cr}\right)\right.} = \frac{\delta\beta^{\frac{1}{\gamma}} R_f^{\frac{1}{\gamma}} - (\delta-1)\, exp\left(c - \frac{\gamma\sigma_c^2}{2}\right)}{\delta\beta^{\frac{1}{\gamma}} R_f^{\frac{1-\gamma}{\gamma}}\left(R_f - exp\left(r + \frac{\sigma_r^2}{2} - \gamma\sigma_{cr}\right)\right)} \tag{32b}$$

$$\begin{aligned}\bar{V} &= \frac{1}{b_0 R_f}\left(\frac{\alpha\beta}{(1-\alpha)(1-\beta)}\right)^{-\frac{\gamma}{1-\gamma}}\left(R_f - exp\left(r + \frac{\sigma_r^2}{2} - \gamma\sigma_{cr}\right)\right) \\ &= \frac{1}{b_0}\beta(1-\beta)^{\frac{\gamma}{1-\gamma}} exp\left(\frac{\gamma\sigma_c^2}{2}\right)\frac{R_f - exp\left(r + \frac{\sigma_r^2}{2} - \gamma\sigma_{cr}\right)}{\left(1 - \beta^{\frac{1}{\gamma}} R_f^{\frac{1-\gamma}{\gamma}} exp\left(\frac{(1-\gamma)\sigma_c^2}{2}\right)\right)^{\frac{\gamma}{1-\gamma}}}\end{aligned} \tag{32c}$$

The non-expected utility equilibrium provides direct estimates of the main wealth-allocation variables, namely the proportion of total wealth dedicated to consumption α and the post-consumption wealth fraction to be put in risky assets θ.

Note that the percentages θ from Equation (32b) are inversely proportional to each the additional-income parameter δ and the total-wealth percentage dedicated to consumption in equilibrium α. Thus, according to Equation (32a) and to the assumption that $I_t = C_t(\alpha\delta)$, θ increases for each higher weight β and higher δ-value.

Some further observations regarding the interpretation of $\bar{V}$ and θ have to be made: In both Equations (32b) and (32c) we observe the presence of the same term that stands for the profitability, taken with inverse sign, of risky investments, that is $R_f - \exp(r + \sigma_r^2/2 - \gamma\sigma_{cr})$. The prospective value is directly proportional to this term, according to the first-line expression in Equation (32c), while the post-consumption wealth percentages invested in risky assets evolve inversely proportional to it, as implied by Equation (32b). In the data set, this term is always negative. Highly negative values in fact arise in situations of higher profitability, when, in consequence, both $\bar{V}$ and θ should be highly positive. Thus, a meaningful interpretation can be given only to the absolute values $|\bar{V}|$ and $|\theta|$. Note also that values of $|\theta| > 1$ point to the fact that investors borrow extra-money at the risk-free rate and invest it in the risky portfolio.

One more note on how to interpret the results regards the following: It is expected that the estimated loss-aversion coefficients in equilibrium take negative values $\hat{\lambda} < 0$. This negative sign is imposed, so to speak "by theoretical construction", namely by the sign of $\bar{V}$ and the condition that $\eta = 0$. In particular, the nil cushions $S_t - Z_t = 0$ implied by the memory less process with $\eta = 0$, transform Equation (15) as follows:

$$\bar{\lambda}_{t+1} = \frac{\bar{V} - (\pi_t \psi_t + (1-\pi_t) w_t) S_t E_t[x_{t+1}]}{(\pi_t (1-\psi_t) + (1-\pi_t)(1-w_t)) S_t E_t[x_{t+1}]} \tag{33}$$

The prospective value $\bar{V}$ always dominates the second term in the numerator of this expression. Therefore, the negative – and hence meaningful – equilibrium estimates $\hat{V} < 0$ necessarily imply negative values of $\hat{\lambda}$ from the above expression, although the negative sign is only artificial. Again, an economic interpretation can be given only to the absolute values $|\hat{\lambda}|$. This is always true with $\eta = 0$.

In the analysis of the obtained estimates and their variation with the chosen behavioral parameters, the concentration is on the case with investors who are both risk- and loss-averse, i.e. with $\gamma = 0.5$ and an initial $\lambda = 2.25$, narrowly frame financial investments $b_0 \geq 1$, and dispose of investment-exogenous incomes I of low, middle, or high magnitudes given by the combinations $(\beta = 0.1, \delta = 0.9)$, $(\beta = 0.5, \delta = 0.5)$, and $(\beta = 0.9, \delta = 0.1)$, respectively.

Table 1 (2) presents the equilibrium estimates of the variables in Equations (32) for yearly (quarterly) evaluations of risky performance.

The prospective value $|\hat{V}|$ decreases subject to the intensity of narrow framing b_0, so that the perception of financial investments depreciates when more attention is paid to these investments. However, financial investments are now subjectively considered to be less attractive when the risky performance is evaluated more often, i.e. $|\hat{V}|$ is lower for quarterly portfolio evaluations. This comes in line with mLA in the large sense.

(Insert Table 1 about here)

(Insert Table 2 about here)

Next is presented the examination of the attitude towards financial losses resorting to the two specific measures: the loss-aversion coefficient and gRA. Tables 1 and 2 suggest that the equilibrium-equivalent loss-aversion coefficient $|\hat{\lambda}|$ is almost invariant with respect to the degree of narrow framing b_0, but only as long as there is indeed narrow framing $b_0 > 0.001$. The same invariance holds approximatively, with respect to the average additional income I, too. Finally, $|\hat{\lambda}|$ grows just slightly for higher $\gamma = 1.5$ and thus can be considered to be robust to the consumption-related risk aversion as well.

In essence, $|\hat{\lambda}|$ lies above the value of 1 across all considered configurations of parameters, as long as there is narrow framing $b_0 > 0.001$. For the cases considered in Tables 1 and 2, $|\hat{\lambda}| \approx 2.$ (1.25) for yearly (quarterly) evaluations, which speaks for loss aversion. The smaller values obtained for quarterly portfolio revisions contradict mLA in the strict sense.

The analysis concerning the loss attitude is refined by focusing on the extended measure gRA. It can be derived from the main equilibrium estimate $|\hat{V}|$ in Equation (32c), according to Equation (17). The corresponding gRA-values for yearly (quarterly) data and our usual cases are included in Table 3 (Table 4).

Relative to the simple loss-aversion coefficient, the extended measure gRA appears again to capture more consistently the loss attitude subject to individual behavioral profiles: Its values in the non-expected equilibrium setting are always positive and change with the behavioral parameters for both evaluation frequencies, as it is to be intuitively expected. In particular, gRA falls with the degree of narrow framing b_0, other things being equal, showing that a narrower focus on financial investments is coupled with a higher reluctance towards potential losses from these investments. Moreover, gRA grows with the magnitude of the average additional income I. In particular, it is insensitive to the free-choice parameter δ, but grows with the weight β of the financial utility. This comes in line with the idea that more relaxed attitudes towards risky investments are to be expected when investors perceive the importance of these investments to be higher. Finally, gRA is higher for quarterly performance evaluations, which is again at odds with mLA in the strict sense.

(Insert Table 3 about here)

(Insert Table 4 about here)

The equilibrium fractions of total wealth to be consumed $\hat{\alpha}$ and of post-consumption wealth to be invested in risky assets $\hat{\theta}$ are particularized in Tables 1 and 2 for the usual cases. They are both independent of the narrow-framing coefficient b_0 .

Moreover, Equation (32a) indicates that α does not vary with δ. Therefore, the weight of financial utility β is the only parameter that dictates the changes of α with the magnitude of the additional income I. In particular, the estimated wealth fractions dedicated to consumption $\hat{\alpha}$ are considerably lower when investors ascribe a higher importance β to financial investments as a source of utility. Specifically, when β grows from 0.1 to 0.9, these wealth fractions drop from over 98.9% to around 13.1% (17.6%) for yearly (quarterly) checks on the risky performance. Note also that $\hat{\alpha}$ is slightly higher for increased evaluation frequencies, which underpins the idea of myopic aversion with respect to financial investments in general. Finally, extremely risk-averse investors with $\gamma = 1.5$ allocate less money to consumption. This counterintuitive result leads to the conclusion that too high a consumption-related risk aversion might be incompatible with the present framework.

Concerning the post-consumption wealth percentages to be put in risky assets, Tables 1 and 2 suggest that investors who have more money at their disposal, i.e. higher additional incomes I, allocate smaller fractions of their wealth after consumption $\hat{\theta}$ to risky assets. Moreover, recall that $|\hat{\theta}| > 1$ stands for the case when investors enhance their investments in risky assets by borrowing additional sums of money at the risk-free rate. This appears to be the case for all combinations $(\beta \geq 0.5, \delta = 0.9)$ and both evaluation frequencies. In particular, $|\hat{\theta}|$ lies above 65.3% (65.6%) for yearly (quarterly) portfolio evaluations. Thus, $|\hat{\theta}|$ is somewhat higher, on average, for more frequent portfolio evaluations, which are the only relevant ones as far as the aversion to financial risks is concerned. The differences are yet very small and do not reflect how the total wealth is split between risky and risk-free assets.

Note also that $|\hat{\theta}|$ takes extremely high – and hence implausible – values for the combinations $(\beta = 0.1, \delta \leq 0.5)$.The reason is that $\beta = 0.1$ stands for the case when investors consider consumption as the main source of utility and consequently allocate the main part of their wealth α to it. Then, these tremendously high percentages of remaining wealth entail reasonable values for the percentages of total wealth dedicated to risky assets $(1 - \hat{\alpha})|\hat{\theta}|$. The same should be kept in mind when we observe that $|\hat{\theta}|$ falls when investors are extremely risk-averse $\gamma = 1.5$.

In order to analyze mLA in the monetary sense, the fractions $(1 - \hat{\alpha})|\hat{\theta}|$ of total wealth dedicated to risky assets in equilibrium is estimated. The results for our usual case with $\lambda = 2.25$, $\gamma = 0.5$ and for yearly (quarterly) portfolio evaluations are to be found in Tables 5 and 6.

(Insert Table5 about here)

(Insert Table 6 about here)

Across all considered configurations of parameters, multiplying $(1-\hat{\alpha})$ by $|\hat{\theta}|$ amounts to around 6.96-563.5% (7-569.9%) of total wealth dedicated to risky assets when their performance is evaluated yearly (quarterly). Thus, mLA in the monetary sense does not hold in the non-expected-utility setting: When investors revise risky performance more often, they dedicate similar to slightly higher portions of their total wealth to risky assets.

The estimated percentages $(1-\hat{\alpha})|\hat{\theta}|$ grow dramatically for higher additional incomes I, which shows that having more money at their disposal renders investors much more open to risky investments. This variation can be split, ceteris paribus, into an increase in β and a decrease in δ. The intuition for the change with β is straightforward: The chances that risky assets are perceived as more attractive should be higher when investors manifest a more pronounced inclination to financial investments in general – that is when these investments are considered as a sufficiently important source of utility relative to consumption. Moreover, when investors dispose of more money in consequence of lower δ-values, other things being equal, their attitude towards financial assets in total does not change, as α is independent of δ. However, when investors are confronted with the more refined choice between risky and risk-free assets, they decide to put more money in the former category. Thus, their reluctance towards financial risk appears to fall.

Surprisingly, $(1-\hat{\alpha})|\hat{\theta}|$ are higher for $\gamma = 1.5$, although extremely risk-averse investors should be disposed to allocate less money to risky investments. This supports our earlier claim that $\gamma = 1.5$ does not fit in the equilibrium framework.

Furthermore, note that the estimates of all variables are almost identical to those presented above for higher initial $\lambda = 3$.

(Insert Table7 about here)

(Insert Table 8 about here)

(Insert Table 9 about here)

(Insert Table 10 about here)

(Insert Table 11 about here)

The results for $(\beta = 0.9, \delta = 2)$ are included in Tables 7 –11. Although the additional income corresponding to this (β, δ)-combination is of middle level, most estimates resemble rather those for high additional incomes given by $(\beta = 0.9, \delta = 0.1)$. This supports the notion that the free-choice parameter δ appears to play a secondary role compared to the weight of financial utility β with respect to investors' decisions. Noticeable discrepancies emerge only for the wealth variables related to the investment in risky assets: For $\delta = 2$, $|\hat{\theta}|$ lies substantially below the corresponding values for $\delta \geq 5$ and even below 1. This attests the fact that investors borrow money in order to put it in risky assets. Moreover $(1 - \hat{\alpha})|\hat{\theta}|$ are also much lower than for our usual $\delta \geq 0.5$ and also decline for quarterly evaluations of the risky performance, which underpins mLA in the monetary sense. Thus, investors who ascribe higher weights $\beta = 0.9$ to the financial utility and dispose of scarce additional incomes $\delta = 2$ appear to be identically reluctant towards financial investments in total (as $\hat{\alpha}$ remains at the same level), but less open to risky investments (since $(1 - \hat{\alpha})|\hat{\theta}|$ is lower) with respect to their peers who have more money from exogenous sources $\delta = 0.1$.

4.2. Expected vs non-expected utility

A last question emerging in the present context is which of the two settings with expected and non-expected utility describes better the behavior of non-professional investors who derive utility from both consumption and financial investments, narrowly frame the latter, and reluctantly perceive financial losses. On the one hand, the expected-utility setting offers the advantage of being formally less complex and more intuitive. On the other hand, the non-expected utility approach provides immediate estimates of more variables of interest, especially of those related to the optimal wealth allocation.

A rigorous comparison of these two settings is yet not straightforward. In spite of the “preventive measures” adopted in order to ensure such a comparison (namely taking similar parameter values), they rely on different equilibrium conditions, employ distinct estimation procedures, and hence deliver different results.

This section attempts to put together the pieces of evidence gathered so far and to enrich them with further comparative results. In this section we present a brief confrontation, in a qualitative sense, of the common and specific results under expected (Rengifo and Trifan 2009 [8]) and non-expected utility. In particular, we rely on the general conclusions and recommendations of the two settings underlined in the above applicative sections. A comparison in a quantitative sense is outside the reach of this paper and we left it for future research

4.2.1 A qualitative comparison

First, our qualitative findings with respect to the subjective value of financial investments for individual investors are, in main, consistent between the settings: A more intense narrow framing of financial assets b_0 yields higher prospective values in equilibrium $|\hat{V}|$. However, mLA in the large sense holds only under non-expected utility, in that the perception of financial investments depreciates at higher performance- evaluation frequencies, which is yet not the case under expected utility.

Second, recall that we measure the loss attitude by means of two variables: the loss-aversion coefficient and the global first-order risk aversion. Within each setting, these two measures are consistent with each other: The estimated loss-aversion coefficient – that is $\hat{\lambda}$ with expected utility and $|\hat{\lambda}|$ with non-expected utility – and gRA vary in opposite directions subject to behavioral parameters (such as the narrow-framing degree b_0, and the sensitivity to past losses

k) and to the additional income I. This is to be expected, since the former coefficient is proportional to the loss reluctance and the latter to the loss acceptance. However, this does not necessarily hold with respect to the evaluation frequency: When the loss attitude is measured by the loss-aversion coefficient, mLA in the strict sense holds with expected utility (and hence the loss aversion increases with the portfolio evaluation frequency), but not with non-expected utility. When, in contrast, gRA quantifies the loss attitude, both settings reject mLA in the strict sense. We also note some problems encountered with respect to the loss-aversion coefficient, for instance its inconsistent variation with k under the maximization of expected utility; Also, this coefficient is very low for yearly evaluations in the expected-utility setting and indicates a loss-loving attitude. In light of its more clear and intuitive variation patterns, gRA appears to be somewhat better suited as a measure of loss attitudes in both settings.

Third, the wealth allocation is quantified by means of the wealth fractions dedicated to consumption and to risky financial assets. Under expected utility, we can merely approximate these variables, while under non-expected utility they result as equilibrium values. In spite of this fundamental discrepancy in the methodology, the estimates provided by the two settings behave similarly: The wealth allocation is invariant with respect to the degree of narrow framing b_0 in both settings. As shown by the wealth fractions dedicated to consumption, $\bar{C}/\bar{\hat{V}}$ with expected utility and $\hat{\alpha}$ with non-expected utility, investors are myopically averse towards financial investments in general, since they allocate more money to consumption – and proportionally less to financial assets in total – when the risky performance is evaluated more often. This aversion decreases when higher additional incomes I are available. This behavior is more pronounced for expected-utility maximization. Moreover, mLA in the monetary sense does not hold in any of the two settings, since constant to higher total-wealth fractions – that is $(1 - \bar{C}/\hat{V})\hat{\theta}$ with expected utility and $(1 - \hat{\alpha})|\hat{\theta}|$ with non-expected utility – are dedicated to risky assets. The only exception is observed for $\delta = 2$ under non-expected utility. The part of total wealth to be put in risky assets grows with the additional income I. However, both maximizers of expected and of non-expected utility appear to behave myopically averse towards financial investments in general, since they dedicate larger fractions of their total wealth $\bar{C}/\hat{V}$ and $\hat{\alpha}$ respectively to consumption and hence proportionally less to financial investments.

Finally, both settings speak rather against the compatibility of too high consumption-related risk aversion coefficients γ with the equilibrium framework. In particular, γ =1.5 mostly delivers implausible estimates of certain variables. The dynamic cushion appears to be less well suited to the estimations under expected utility. Also, from the two parameters that determine the change in the additional income I, β appears to be more important than δ in eliciting investor reactions in both settings. Recall yet that β can be directly interpreted as the weight of financial utility only with non-expected utility. Note also that too high values of each β, such as $\beta = 0.98$, and δ, such as $\delta > 2$, deliver implausible results in both settings.

Note that, in general, the estimates under non-expected utility maximization are robust to changes in the behavioral profile. This is indeed the expected result, as we analyze here the aggregate market with a single representative investor (and hence consider behavioral profiles "on average"). Therefore, we incline to sustain the claim of Barberis, Huang, and Thaler (2006) that non-expected utility better describes decision making under risk.

5. SUMMARY AND CONCLUSIONS

This paper presents a portfolio model with a two-dimensional utility framework. We are interested in how non-professional investors, who now derive utility from both consumption and narrowly-framed financial investments, behave when faced with financial risk. We also study how these investors change their perception of losses and how they consequently split their money between consumption, and (risky vs. risk-free) financial assets.

Following Barberis, Huang, and Thaler (2006) [5] and Barberis and Huang (2004, 2006) [1,2], we consider an aggregate market with a representative investor who maximizes subjective utility. The equilibrium is derived considering the maximization of non-expected utility. We explicitly account for the narrow framing of financial investments, as well as for the impact of past performance on current perceptions. Note that this setting requires specific conditions in order to attain the aggregate equilibrium. For instance, the non-expected utility equilibrium does not allow for the influence of past performance. It also restricts the set of feasible values of several behavioral parameters, such as the risk- and loss-aversion coefficients.

However, this setting delivers direct equilibrium estimates of the prospective value, i.e. of the subjective utility of financial investments. From this variable, we obtain equilibrium-equivalent measures of the loss attitude, such as the loss-aversion coefficient and the global first-order risk aversion. Byproducts of the estimation procedure in the non-expected-utility setting are wealth-allocation variables, such as the percentages of total wealth dedicated to consumption and of post-consumption wealth to be invested in risky assets.

The theoretical results are subsequently tested and extended in an applied context. We use the S&P 500 and the 3-months T-bill nominal returns, as proxies for a well-diversified risky portfolio and the risk-free investment, respectively, as well as quarterly aggregate per-capita consumption data between 1982-2006. In order to avoid the impossibility of covering current consumption needs from financial revenues throughout the entire investing period, we also consider that investors dispose of exogenous additional incomes at each decision time. These incomes are shaped in order to ensure the equivalency of the two settings with expected and non-expected utility. General market parameters (such as the risk-free returns and the dynamics of consumption and of expected returns) are estimated on the basis of the above real data. As such an estimation is not possible for the behavioral parameters (such as the degree of narrow framing, the risk aversion to consumption, the weight of financial utility, the sensitivity to past losses, the way of accounting for past performance, etc.), we work with wide value-sets of these behavioral parameters in order to detect plausible (combinations of) values. We investigate how the main variables that express loss attitudes and wealth allocation, change subject to different behavioral profiles of non-professional investors, at different levels of the additional income, and for two distinct horizons of risky performance evaluation, specifically of one year and three months.

The two settings with expected and non-expected utility can be straightforwardly compared only in a qualitative sense that is with respect to their general recommendations and to the variation patterns of the main variables. A quantitative comparison of the variable values is possible only for the cases based on common identical assumptions, from which the most important regards the memory less cushions. We leave this last comparison for future research.

Since the estimates under non-expected utility are more informative, more robust, and change more intuitively with the behavioral investor profile, we consider this setting to be better suited to describe behavior and decision making of non-professional investors.

REFERENCES

[1]Barberis, N., and M. Huang. 2004. "Preferences with Frames: A New Utility Specification that Allows for the Framing of Risks." *Working paper, University of Chicago, Stanford University*.

[2]Barberis, N., and M. Huang. 2006. "The Loss Aversion / Narrow Framing Approach to the Equity Premium Puzzle." *NBER Working Paper No. 12378*.

[3]Barberis, N., M. Huang, and R. Thaler. 2006. "Individual Preferences, Monetary Gambles and the Equity Premium." *American Economic Review* 96:4, 1-53.

[4]Barberis, N., M. Huang, and T. Santos. 2001. "Prospect Theory and Asset." *Quarterly Journal of Economics*, 116, 1-53.

[5]Benartzi, S., and R. H. Thaler. 1995. "Myopic Loss Aversion and the Equity Premium Puzzle." *Quarterly Journal of Economics*, 110:1, 73-92.

[6]Campbell, R., R. Huisman, and K. Koedijk. 2001. "Optimal portfolio selection in a Value-at-Risk framework." *Journal of Banking and Finance*, 25, 1789-1804.

[7]Kahneman, D., and A. Tversky.1979. "Prospect theory: An analysis of decision." *Econometrica*, 47:2, 263-291.

[8]Rengifo, E. W., and E. Trifan. 2009. "How Investors Face Financial Risk Loss and Wealth Allocation with Two-Dimentional Utility: A VaR Application." Chap. 23 in *The VaR Implementation Handbook*, edited by G.N. Gregotiou, 485-512. McGraw Hill.

[9]Rengifo, E. W., and E. Trifan. 2010. "How Investors Face Financial Risk: Loss Aversion and Wealth Allocation." *CENTRUM Cathedra*. 3:1, 41-59

[10]Tversky, A., and D. Kahneman. 1992. "Advances in Prospect Theory: Cumulative Representation of Uncertainty." *Journal of Risk and Uncertainty*, 4:5, 297-323.

Table1 The main variable estimates in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional incomes I and narrow-framing degrees b_0.

		$b_0 = 1$	$b_0 = 5$	$b_0 = 10$	$b_0 = 100$	$b_0 = 1000$
	$\lvert\hat{V}\rvert$	0.16819	0.033638	0.016819	0.0016819	0.00016819
$Low\ I$	$\lvert\hat{\lambda}\rvert$	2.0141	2.0145	2.0146	2.0146	2.0146
$(\beta = 0.1, \delta = 0.9)$	$\hat{\alpha}$	0.98927	0.98927	0.98927	0.98927	0.98927
	$\lvert\hat{\theta}\rvert$	6.4843	6.4843	6.4843	6.4843	6.4843
	$\lvert\hat{V}\rvert$	0.63158	0.12632	0.063158	0.0063158	0.00063158
$Middle\ I$	$\lvert\hat{\lambda}\rvert$	2.0143	2.0145	2.0146	2.0146	2.0146
$(\beta = 0.5, \delta = 0.5)$	$\hat{\alpha}$	0.73178	0.73178	0.73178	0.73178	0.73178
	$\lvert\hat{\theta}\rvert$	2.7058	2.7058	2.7058	2.7058	2.7058
	$\lvert\hat{V}\rvert$	1.2705	0.25411	0.12705	0.012705	0.0012705
$High\ I$	$\lvert\hat{\lambda}\rvert$	2.0146	2.0146	2.0146	2.0146	2.0146
$(\beta = 0.9, \delta = 0.1)$	$\hat{\alpha}$	0.13095	0.13095	0.13095	0.13095	0.13095
	$\lvert\hat{\theta}\rvert$	6.4843	6.4843	6.4843	6.4843	6.4843

Table 2 The main variable estimates in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional incomes I and narrow-framing degrees b_0.

		$b_0 = 1$	$b_0 = 5$	$b_0 = 10$	$b_0 = 100$	$b_0 = 1000$
$Low\ I$ $(\beta = 0.1, \delta = 0.9)$	$\lvert\hat{V}\rvert$	0.15926	0.031851	0.015926	0.0015926	0.00015926
	$\lvert\hat{\lambda}\rvert$	1.2486	1.2547	1.2555	1.2561	1.2562
	$\hat{\alpha}$	0.98982	0.98982	0.98982	0.98982	0.98982
	$\lvert\hat{\theta}\rvert$	6.9149	6.9149	6.9149	6.9149	6.9149
$Middle\ I$ $(\beta = 0.5, \delta = 0.5)$	$\lvert\hat{V}\rvert$	0.58727	0.11745	0.058727	0.0058727	0.00058727
	$\lvert\hat{\lambda}\rvert$	1.2532	1.2556	1.2559	1.2562	1.2562
	$\hat{\alpha}$	0.74562	0.74562	0.74562	0.74562	0.74562
	$\lvert\hat{\theta}\rvert$	2.8611	2.8611	2.8611	2.8611	2.8611
$High\ I$ $(\beta = 0.9, \delta = 0.1)$	$\lvert\hat{V}\rvert$	0.89663	0.17933	0.089663	0.0089663	0.00089663
	$\lvert\hat{\lambda}\rvert$	1.2561	1.2562	1.2562	1.2562	1.2562
	$\hat{\alpha}$	0.17581	0.17581	0.17581	0.17581	0.17581
	$\lvert\hat{\theta}\rvert$	6.9149	6.9149	6.9149	6.9149	6.9149

Table 3 The estimated global first--order risk aversion (gRA) in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional incomes I and narrow-framing degrees b_0.

	$b_0 = 1$	18.654
$Low\ I$	$b_0 = 5$	3.7308
$(\beta = 0.1, \delta = 0.9)$	$b_0 = 10$	1.8654
	$b_0 = 100$	0.18654
	$b_0 = 1000$	0.018654
	$b_0 = 1$	70.05
$Middle\ I$	$b_0 = 5$	14.01
$(\beta = 0.5, \delta = 0.5)$	$b_0 = 10$	7.005
	$b_0 = 100$	0.7005
	$b_0 = 1000$	0.07005
	$b_0 = 1$	140.92
$High\ I$	$b_0 = 5$	28.184
$(\beta = 0.9, \delta = 0.1)$	$b_0 = 10$	14.092
	$b_0 = 100$	1.4092
	$b_0 = 1000$	0.14092

Table 4 The estimated global first--order risk aversion (gRA) in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional incomes I and narrow-framing degrees b_0.

	$b_0 = 1$	54.682
$Low\ I$	$b_0 = 5$	10.936
$(\beta = 0.1, \delta = 0.9)$	$b_0 = 10$	5.4682
	$b_0 = 100$	0.54682
	$b_0 = 1000$	0.054682
	$b_0 = 1$	201.64
$Middle\ I$	$b_0 = 5$	40.329
$(\beta = 0.5, \delta = 0.5)$	$b_0 = 10$	20.164
	$b_0 = 100$	2.0164
	$b_0 = 1000$	0.20164
	$b_0 = 1$	307.87
$High\ I$	$b_0 = 5$	61.573
$(\beta = 0.9, \delta = 0.1)$	$b_0 = 10$	30.787
	$b_0 = 100$	3.0787
	$b_0 = 1000$	0.30787

Table 5 The estimated total-wealth fractions dedicated to risky assets $(1-\hat{\alpha})|\hat{\theta}|$ in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional incomes I and narrow-framing degrees b_0.

$Low\ I$	$(\beta = 0.1, \delta = 0.9)$	0.069569
$Middle\ I$	$(\beta = 0.5, \delta = 0.5)$	0.72575
$High\ I$	$(\beta = 0.9, \delta = 0.1)$	5.6351

Table 6 The estimated total-wealth fractions dedicated to risky assets $(1-\hat{\alpha})|\hat{\theta}|$ in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional incomes I and narrow-framing degrees b_0.

$Low\ I$	$(\beta = 0.1, \delta = 0.9)$	0.07036
$Middle\ I$	$(\beta = 0.5, \delta = 0.5)$	0.72782
$High\ I$	$(\beta = 0.9, \delta = 0.1)$	5.6991

Table 7 The main variable estimates in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional income levels I given by $(\beta = 0.9, \delta = 2)$ and narrow-framing degrees b_0.

	$b_0 = 1$	$b_0 = 5$	$b_0 = 10$	$b_0 = 100$	$b_0 = 1000$
$\lvert\hat{V}\rvert$	1.2705	0.25411	0.12705	0.012705	0.0012705
$\lvert\hat{\lambda}\rvert$	2.0139	2.0143	2.0145	2.0146	2.0146
$\hat{\alpha}$	0.13095	0.13095	0.13095	0.13095	0.13095
$\lvert\hat{\theta}\rvert$	0.25236	0.25236	0.25236	0.25236	0.25236

Table 8 The main variable estimates in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional income levels I given by $(\beta = 0.9, \delta = 2)$ and narrow-framing degrees b_0.

	$b_0 = 1$	$b_0 = 5$	$b_0 = 10$	$b_0 = 100$	$b_0 = 1000$
$\lvert\hat{V}\rvert$	0.89663	0.17933	0.089663	0.0089663	0.00089663
$\lvert\hat{\lambda}\rvert$	1.2451	1.254	1.2551	1.2561	1.2562
$\hat{\alpha}$	0.17581	0.17581	0.17581	0.17581	0.17581
$\lvert\hat{\theta}\rvert$	0.22905	0.22905	0.22905	0.22905	0.22905

Table 9 The estimated global first--order risk aversion (gRA) in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional income levels I given by $(\beta = 0.9, \delta = 2)$ and narrow-framing degrees b_0.

$b_0 = 1$	140.92
$b_0 = 5$	28.184
$b_0 = 10$	14.092
$b_0 = 100$	1.4092
$b_0 = 1000$	0.14092

Table 10 The estimated global first--order risk aversion (gRA) in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, various additional income levels I given by $(\beta = 0.9, \delta = 2)$ and narrow-framing degrees b_0.

$b_0 = 1$	307.87
$b_0 = 5$	61.573
$b_0 = 10$	30.787
$b_0 = 100$	3.0787
$b_0 = 1000$	0.30787

Table 11 The estimated total-wealth fractions dedicated to risky assets $(1-\hat{\alpha})|\hat{\theta}|$ in the non-expected utility equilibrium for yearly portfolio evaluations, initial loss aversion $\lambda = 2.25$, risk-aversion $\gamma = 0.5$, memoryless dynamic cushions with $\eta = 0$, an additional levels I given by $(\beta = 0.9, \delta = 2)$.

Evaluation Frequency	
Yearly	Quarterly
0.21931	0.18878