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# The Individually Acceptable Loss (IAL)\*

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## Abstract

This paper proposes a new, *individual* measure of market risk, denoted as the individually acceptable loss (IAL). This measure can be used by portfolio managers in order to better meet the individual profiles of their non-professional clients, including psychological traits. It can be easily assessed from general subjective and objective parameters. We formally define the IAL of loss averse investors, who narrowly frame financial investments, and are sensitive to the past performance of their risky portfolio. This individual risk measure is applied to the classic portfolio optimization framework in order to derive the optimal wealth allocation among different financial assets. Our empirical results suggest that previous optimization relying on a portfolio-exogenous VaR-formulation, underestimates the aversion of individual investors towards financial losses.

*Keywords:* market risk, prospect theory, loss aversion, capital allocation, Value-at-Risk

*JEL Classification:* C32, C35, G10

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# 1 Introduction

According to the top-down strategy, portfolio optimization can be described by means of a threefold decision procedure: A first step, referred as the *capital allocation decision*, deals with the choice between risky and risk-free assets. A second decision, denoted as the *asset allocation decision*, focuses on the selection of different classes of risky assets that can become part of the risky portfolio. Third, the so-called *security allocation decision* refers to the specific securities to be held within each particular risky asset class chosen before. In practice, the last two decisions are usually made by professional portfolio managers with no intervention of their non-professional clients. However for the first decision, the participation of these non-professionals becomes necessary, as it allows the professional portfolio managers to determine the optimal portfolio allocation that best fits the individual risk-profiles of their (non-professional) clients.

This paper focuses on the capital allocation decision. In particular, we aim at understanding the possible determinants of the subjective risk-profiles of non-professional investors and their effect on the decisions regarding the optimal capital allocation between risky and risk-free assets. Note that we do *not* attempt to discuss “quality” (that is the validity or the possible failures) or to compare the performance of any theoretical models of portfolio optimization, as they are shaped to address merely the second and third of the decisions mentioned above.

Traditionally, market risk is measured by the *variance* (or equivalently, by the standard deviation) of portfolio returns. Being a symmetric measure, the variance equally accounts for high gains and high losses. However, as shown by numerous empirical studies and formalized in the Kahneman and Tversky’s prospect theory (abbr. PT), losses appear to loom larger than gains of the same size in individual perception, a phenomenon denoted as loss aversion. Therefore, more sophisticated measures have been developed for the purpose of better capturing the actual perception of market risks. The *Value-at-Risk* (abbr. VaR) defines the maximum (or worst) expected loss from a financial investment over a given time horizon and at a given confidence level  $1 - \alpha$ . Closely related, the *expected shortfall* (abbr. ES) represents the limit in probability of the expected losses in  $\alpha\%$  worst cases. For continuous distributions, ES is equivalent to the so-called *conditional VaR* (abbr. CVaR). It is hence apparent that VaR, ES, and CVaR measure the downside risk, as they merely refer to the left tail of the distribution corresponding to losses.

However, none of these measures considers the individual features of investors. For non-institutional clients who deal with capital allocation decisions, this subjective (mainly psychological) profile becomes very important, as it dictates the final capital allocation. In this context, our paper introduces an individual measure of financial losses, that is denoted as the *individually acceptable loss* (abbr. IAL). It serves for quantifying market

risks from the subjective perspective of non-professional investors, which is necessary to determine the optimal capital allocation. This measure is based on the so-called desired VaR\* proposed in Rengifo and Trifan (2007).

As every subjective quantity, IAL is derived on the basis of individual perceptions. In particular, these perceptions account, according to PT, for discrepancies between gains and losses, as well as for the investment performance history, as suggested in Barberis, Huang, and Santos (2001). Loosely speaking, IAL is the non-professional counterpart of professional risk measures that quantify downside market risk, such as VaR. As portfolio managers need to comply with their client wishes, it is reasonable to think that IAL can be actively used in portfolio management in order to adapt the optimal capital allocation to the subjective perceptions of the market evolution. In essence, portfolio managers can derive the individual IAL of their non-professional clients based on the subjective investor profile and on objective information regarding the past performance of the risky portfolio and market characteristics. The individual profile includes very general features, such as how investors weight out between gains and losses (in technical terms the loss aversion coefficient), how long in the past does their perceptions reach (which serves for instance to determine how they build cushions and how hardly they penalize past losses), and which are their education and experience (that could help in establishing the type of return-expectation model used for deriving expected return premia). Clearly, this profile can be easily assessed by the simple direct contact with the client. After determining the individual IAL, it can be employed in the optimization framework as a capital allocation risk constraint. For the present work, it is hence interesting to investigate how the optimal portfolio allocation changes when IAL is used as measure of risk, instead of the classical measures enumerated above.

Our paper resolves for defining IAL and presents an application of this individual loss measure aimed at determining the optimal capital composition, i.e. the wealth proportion invested in the risky portfolio and in risk-free assets.

Formally, IAL is set to be the maximum expected loss perceived by individual investors. It encompasses two main terms, denoted as the *PT-term* and the *cushion-term*. In essence, the former comes in line with the original PT and accounts for the expectation of future portfolio returns weighted by the loss aversion coefficient specific to the individual investor. The latter shows the influence of previous performance on current individual perceptions, as stressed in Barberis, Huang, and Santos (2001). Past performance is expressed through the monetary cushions accumulated from past trades, that are weighted in the cushion-term by an expression that depends, among others, on the probability of experiencing past losses and on the penalty imposed on them.

After computing the IAL, we show how it can be applied by for deriving the optimal

wealth allocation among different financial assets. To this end, we consider the framework developed in Campbell, Huisman, and Koedijk (2001) as working example. In this setting, the optimal mix of risky assets is found under the consideration of both a budget and a risk constraint. The byproduct of this optimization is the amount of money to be additionally borrowed (lent), in other words invested in risk-free assets, in order to increase (decrease) the total risky investment. In the Campbell, Huisman, and Koedijk (2001)'s model, the risk constraint of the optimization problem is given by the so-called desired risk level and denoted as  $\text{VaR}^*$ . This desired level is fixed by the non-professional client and communicated to the portfolio manager. Managers equate the client indication to the common theoretical VaR-concept, hence they interpret it in terms of confidence levels and investment horizons. Campbell, Huisman, and Koedijk (2001) consider non-professional investors with two degrees of risk aversion that correspond, in the manager view, to significance levels ranging of 1% and 10%. The risky portfolio, where the portfolio VaR is assessed using a 5%-significance level, is taken as benchmark. We show how our definition of IAL can be employed in this model (in the place of the exogenous  $\text{VaR}^*$ ), in order to individually adapt the portfolio risk constraint. The capital allocation delivered by the optimization procedure is consequently shaped to the subjective risk-profile of the client. In the empirical part of our paper, we calculate the significance levels, the wealth percentages invested in risky assets, and the loss aversion coefficients that are necessary, according to the definition of the risk constraint in Campbell, Huisman, and Koedijk (2001), for reaching the IAL derived in line with our model and on the basis of real market data.

The remainder of the paper is organized as follows. Section 2 presents the main theoretical considerations. In Section 2.1, we refresh the notion of value function as theoretical pendant of the subjective perception of financial risk. To this end, we rely on the original Kahneman and Tversky (1979, 1992)'s prospect theory and on the extended perspective in Barberis, Huang, and Santos (2001). Section 2.2 formally introduces the notion of IAL and proposes two different ways of quantifying it. Finally, Section 2.3 details optimal portfolio selection model with exogenous  $\text{VaR}^*$  as risk constraint in Campbell, Huisman, and Koedijk (2001). This model underlies our empirical application in Section 3. In particular, Section 3.2.1 derives theoretically-equivalent significance levels of portfolio risk corresponding to the IAL inferred on the basis of our data. In the same vein, Section 3.2.2 computes equivalent values of the loss aversion coefficient and of the wealth percentages dedicated to risky assets, that result from the average IAL computed from our data and according to our model equations. The overall results are summarized in the final Section 4. Further results are included in the Appendix.

## 2 The theoretical setting

This section details the construction of our subjective measure of financial losses that is the individually acceptable loss IAL. Subsequently, it shows how this measure can be applied as risk constraint in a portfolio optimization setting. In so doing, we rely on Section 2 in Rengifo and Trifan (2007).

### 2.1 The value function

Human decisions are not based on pure real facts but on images of reality that form in our minds through perception. Thus, individual measures such as IAL depend on the investor perception of the value of financial projects. The Kahneman and Tversky's prospect theory (abbr. PT) elaborates on individual perceptions of financial performance. According to Kahneman and Tversky (1979) and Tversky and Kahneman (1992), the subjective value of one unit of risk asset is formalized by means of the so-called *value function*. This function captures several basic features of perception. First, human minds take for actual carriers of value not the absolute outcomes of a project, but their changes defined as departures from an individual reference point. The deviations above (below) this reference are labelled as gains (losses). Thus, the value function is kinked at the reference point. Second, people appear to be more reluctant to accept losses than open to assume gains of the same size, a property known as *loss aversion*. Hence, the value function exhibits distinct evolution in the domains of gains and losses, i.e. it is steeper for losses. Moreover, it shows diminishing sensitivity in both domains, i.e. it is concave for gains but convex for losses.

Barberis, Huang, and Santos (2001) extend this original PT-view by additionally accounting for the potential impact of past performance on current individual perceptions of risky investments. They suggest that the value function reflects not only the discrepancies between gains and losses, but also the influence of so-called monetary *cushions* accumulated from past trades. Formally, the cushions are defined as the difference between the current value of the risky investment  $S_t$  and a benchmark level from the past  $Z_t$  (e.g. the purchasing price of the respective asset). When this difference is positive, investors made money from past risky investment, otherwise they accumulated losses.

Our approach relies on this latter extended formulation of the value function, specifically on Equations (15) and (16) in Barberis, Huang, and Santos (2001). Accordingly, the reference point changes with the past performance, that is from  $z_t R_{ft}$  for  $z_t \leq 1$  to  $R_{ft}$  for  $z_t > 1$ , where  $z_t = Z_t/S_t$ . For facilitating the subsequent understanding and derivation of IAL, we formally rearrange these definitions. The goal is to obtain identical reference points and similar courses in the loss domain for both considered cases with positive and

negative cushions. This would be similar to the original PT-formulation, where gains are defined as the difference between the value function argument (here  $R_{t+1}$ ) and the reference point. To this end, we fix the reference value in both cases (i.e. with prior gains  $z_t \leq 1$  and prior losses  $z_t > 1$ ) to  $R_{ft}$  and rewrite Equations (15) and (16) in Barberis, Huang, and Santos (2001) as:

$$v_{t+1} = \begin{cases} S_t(R_{t+1} - R_{ft}) & , \text{ for } R_{t+1} \geq R_{ft} \\ \lambda S_t(R_{t+1} - R_{ft}) + (\lambda - 1)(S_t - Z_t)R_{ft} & , \text{ for } R_{t+1} < R_{ft} \end{cases} , \text{ for } z_t \leq 1 (\Leftrightarrow Z_t \leq S_t), \quad (2.1)$$

and

$$v_{t+1} = \begin{cases} S_t(R_{t+1} - R_{ft}) & , \text{ for } R_{t+1} \geq R_{ft} \\ \lambda S_t(R_{t+1} - R_{ft}) + k(Z_t - S_t)(R_{t+1} - R_{ft}) & , \text{ for } R_{t+1} < R_{ft} \end{cases} , \text{ for } z_t > 1 (\Leftrightarrow Z_t > S_t). \quad (2.2)$$

Here,  $\lambda$  is denoted as the coefficient of loss aversion. The parameter  $k > 0$  captures the influence of previous losses on the perception of current ones (specifically, the larger the previous loss is, the more painful the next losses become). We note that while the gain branches of both value functions are invariable to the past performance  $z_t$ , the loss branches contain a first term  $\lambda S_t(R_{t+1} - R_{ft})$  that resembles the original PT, but also a second one revealing the impact of the cushion  $S_t - Z_t$ .

## 2.2 The IAL

Our formal definition of IAL follows Section 2.3 in Rengifo and Trifan (2007) and is based on its literal definition. Accordingly, the *individually acceptable loss* IAL represents the maximum loss-level acceptable (in terms of expectations) by each investor. Thus, we focus on the notions of “maximum”, “loss”, and “individual”. First, IAL quantifies losses. According to the PT, what actually counts for individual investors is not the absolute magnitude of a loss, but rather the subjectively perceived one, as captured by the value function. Hence, IAL should rely on the subjective value (or utility) of losses expressed in the loss branches of the value functions (2.1) and (2.2). It depends on individual investor characteristics, that originate in the subjective view over gains and losses, and can vary over time. Second, IAL should represent a (subjective) expectation because the next period returns  $R_{t+1}$ , on which the evaluation of risky investments is based, are still unknown at the decision time  $t$ . Third, we are looking for a maximal value so that in calculating IAL, investors must ascribe a maximal occurrence probability  $P_t(E_t[R_{t+1}] < R_{ft}) = 1$  to the losses in the value function.

Therefore, we propose that IAL accounts for the maximum expectation of sustainable

losses as resulting from individual valuations of the risky investment. However, investors may be sophisticated enough in order to consider that not only the mean, but also the variation of prospective losses influences the maximum acceptable loss level. Thus, we subsequently extend the IAL-definition by adjusting for the loss variance.

Henceforth, we consider that value functions are weighted by the pure probabilities of occurrence, instead of the non-linear probability functions proposed in the cumulative PT of Tversky and Kahneman (1992). We denote by  $\pi_t$  the probability of experiencing past gains  $P_t(z_t \leq 1)$ , and by  $x_{t+1}$  the equity return premium  $R_{t+1} - R_{ft}$ . Thus, the expected equity premium becomes  $E_t[x_{t+1}] = E_t[R_{t+1}] - R_{ft}$ . According to Equations (2.1) and (2.2), we then have:

$$\begin{aligned} E_t[\text{loss-utility}_{t+1}] &= \pi_t(\lambda S_t E_t[x_{t+1}] + (\lambda - 1)(S_t - Z_t)R_{ft}) \\ &\quad + (1 - \pi_t)(\lambda S_t E_t[x_{t+1}] + k(Z_t - S_t)E_t[x_{t+1}]) \\ &= \lambda S_t E_t[x_{t+1}] + (\pi_t(\lambda - 1)R_{ft} - (1 - \pi_t)kE_t[x_{t+1}])(S_t - Z_t) \end{aligned} \quad (2.3a)$$

$$\begin{aligned} Var_t[\text{loss-utility}_{t+1}] &= E_t[\text{loss-utility}_{t+1}^2] - E_t^2[\text{loss-utility}_{t+1}] \\ &= \pi_t(\lambda S_t E_t[x_{t+1}] + (\lambda - 1)(S_t - Z_t)R_{ft})^2 \\ &\quad + (1 - \pi_t)(\lambda S_t E_t[x_{t+1}] + k(Z_t - S_t)E_t[x_{t+1}])^2 - E_t^2[\text{loss-utility}_{t+1}] \\ &= (\lambda S_t E_t[x_{t+1}])^2 + (\pi_t(\lambda - 1)^2 R_{ft}^2 - (1 - \pi_t)k^2 E_t[x_{t+1}]^2)(S_t - Z_t)^2 \\ &\quad + 2(\pi_t(\lambda - 1)R_{ft} - (1 - \pi_t)kE_t[x_{t+1}])\lambda S_t E_t[x_{t+1}](S_t - Z_t) - (\lambda S_t E_t[x_{t+1}])^2 \\ &\quad - (\pi_t(\lambda - 1)R_{ft} - (1 - \pi_t)kE_t[x_{t+1}])^2(S_t - Z_t)^2 \\ &\quad - 2(\pi_t(\lambda - 1)R_{ft} - (1 - \pi_t)kE_t[x_{t+1}])\lambda S_t E_t[x_{t+1}](S_t - Z_t) \\ &= \pi_t(1 - \pi_t)((\lambda - 1)R_{ft} + kE_t[x_{t+1}])^2(S_t - Z_t)^2. \end{aligned} \quad (2.3b)$$

The first term of the expected losses in Equation (2.3a) is similar to the loss-formulation in the PT. Yet the remaining terms point out the influence of the cushion accumulated over past trades. By contrast, the variance of losses in Equation (2.3b) is exclusively dictated by the cushion-part, as individually perceived by investors. It depends on the probability of having made gains or losses in the past, on the variance of expected returns with respect to the reference risk-free rate, and on the squared cushion.

As mentioned above, in a first approximation we keep with the literal definition of IAL as an *expectation* and design IAL as the maximum expected loss:

$$\begin{aligned} IAL_{t+1}^1 &= E_t[\text{loss-utility}_{t+1}] \\ &= \lambda S_t E_t[x_{t+1}] + (\pi_t(\lambda - 1)R_{ft} - (1 - \pi_t)kE_t[x_{t+1}])(S_t - Z_t). \end{aligned} \quad (2.4)$$

We refer to Equation (2.4) as the *simple* IAL or the  $IAL^1$ .



However, investors may consider the loss-variance as equally important for the determination of the maximal sustainable loss. Then, assuming that IAL follows a certain distribution (i.e. normal or Student-t) with the value  $\varphi$ , we introduce the second definition that we refer as the *variance-adjusted* IAL or plainly the IAL:<sup>1</sup>

$$\begin{aligned} \text{IAL}_{t+1} &= E_t[\text{loss-utility}_{t+1}] - \varphi \sqrt{\text{Var}_t[\text{loss-utility}_{t+1}]} \\ &= \lambda S_t E_t[x_{t+1}] \\ &\quad + ((\pi_t - \varphi \sqrt{\pi_t(1 - \pi_t)})(\lambda - 1)R_{ft} - (1 - \pi_t + \varphi \sqrt{\pi_t(1 - \pi_t)})kE_t[x_{t+1}])(S_t - Z_t), \end{aligned} \tag{2.5}$$

where the last equality was obtained from Equations (2.3). Again, the variance-adjusted expression of IAL in Equation (2.5) encompasses the twofold loss effect, stemming from the loss aversion coefficient of the original PT and from the cushion introduced in Barberis, Huang, and Santos (2001). Subsequently, we mostly rely on this second, more sophisticated definition. Section 3.1 compares the impact of both the simple IAL<sup>1</sup> and the variance-adjusted IAL on the wealth allocation between risky and risk-free assets.

We denote the first term of IAL in both Equations (2.4) and (2.5) as the *PT-term*. It points out the dependency of the individually acceptable loss on the loss aversion coefficient specific to each individual, and the expected returns of the risky investment.<sup>2</sup> The second term is different in Equations (2.4) and (2.5), yet refers to the impact of monetary cushions accumulated from past trades  $S_t - Z_t$  and is thus designated as the *cushion-term*. The weighting coefficient of the cushion is an expression that depends on the probability of experiencing past losses and the penalty imposed on them.

As apparent in both definitions of IAL, this measure can be inferred on the basis of two types of data, that we can denote as objective and subjective. The former category describe the evolution of the risky and risk-free investments, and comprise the current value of the risky investment  $S_t$ , the previous values of the same investment that underlie the derivation of  $Z_t$ , the risky return  $R_{t+1}$ , and the risk-free rate  $R_{ft}$ . Clearly, these parameters are directly observable to the portfolio manager, wherefrom the denomination of “objective”. The latter parameter category relies on psychological features of the individual investors and on their subjective view over the risky investment, and encompass the loss aversion coefficient  $\lambda$ , the penalty imposed on past losses  $k$ , the formation rule of expected returns  $E_t[R_{t+1}]$ , the probability of current gains  $\pi_t$ , and for Equation (2.5) the assumed gross return distribution  $\varphi$ . This information forms the subjective profile of the non-professional client can be assessed by portfolio managers by personal contact or even

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<sup>1</sup>Equation (2.5) results from the assumption that:  
 $(\text{IAL}_{t+1} - E_t[\text{loss-utility}_{t+1}]) / (\sqrt{\text{Var}_t[\text{loss-utility}_{t+1}]}) = \varphi \sim N(0, 1)$  or  $t(5)$ .

<sup>2</sup>Obviously, the expected returns are obtained multiplying the current risky investment  $S_t$  by the expected equity premium  $E_t[x_{t+1}]$ .

direct questioning of the client. Thus, we consider that in practice, the computation of IAL should not be a very difficult task.

It is interesting to observe that for a fixed IAL-level, that we denote by  $\overline{\text{IAL}}$ , we can infer the equivalent loss aversion coefficient  $\lambda_t^*$ . Hence, fixing the desired IAL, such that it corresponds for example to the classic portfolio VaR for commonly used significance levels such as 1%, 5% or 10%, entails certain equivalent loss aversion values that can be compared to the prescriptions of the original PT. The result for the equivalent  $\lambda_t^*$  is immediate from Equation (2.5):

$$\lambda_{t+1}^* = \frac{\overline{\text{IAL}} + [(\pi_t - v\sqrt{\pi_t(1-\pi_t)})R_{ft} + (1 - \pi_t + v\sqrt{\pi_t(1-\pi_t)})kE_t[x_{t+1}]](S_t - Z_t)}{S_t E_t[x_{t+1}] + (\pi_t - v\sqrt{\pi_t(1-\pi_t)})R_{ft}(S_t - Z_t)}. \quad (2.6)$$

Recall that as  $\lambda_{t+1}^*$  depends on the *fixed*  $\overline{\text{IAL}}$ , there should be no further causal relationship between past and future losses. Thus, we can set  $k = 0$  and Equation (2.6) simplifies to:

$$\lambda_{t+1}^* = \frac{\overline{\text{IAL}} + [(\pi_t - v\sqrt{\pi_t(1-\pi_t)})R_{ft}](S_t - Z_t)}{S_t E_t[x_{t+1}] + (\pi_t - v\sqrt{\pi_t(1-\pi_t)})R_{ft}(S_t - Z_t)}. \quad (2.7)$$

### 2.3 An example: Optimal portfolio selection with IAL

In essence, every portfolio optimization under risk constraints follows the same main steps, independently of the measured employed for quantifying market risks. In particular, the chosen risk measure (e.g. the variance or the VaR) is first minimized for various expected portfolio returns. This subsequently allows for the derivation of the mean-variance or mean-VaR feasible set. Finally, the respective efficient frontier can be inferred. However in most of the cases, the key ingredient for determining the optimal capital allocation is inadequately treated. This key ingredient is the individual (non-professional) risk-profile, and managers mostly consider it as exogenously given and attempt to represent it in the theoretical terms of a risk constraint. In Section 2.2, we proposed a method to quantify individual risk-profiles based on the Kahneman and Tversky (1979, 1992)'s prospect theory and on its extended version developed in Barberis, Huang, and Santos (2001). This section outlines a possible application into the portfolio optimization setting with VaR as risk measure in Campbell, Huisman, and Koedijk (2001).

The definition of IAL as the maximum loss-level that is individually acceptable (in terms of expectations) turns this quantity into a general measure of loss aversion. Recalling that, in general, investors are loss averse, and that loss aversion represents risk aversion of first order, we can even consider IAL as an individual measure of risk aversion of non-professional investors. As mentioned in the introduction, IAL quantifies downside market risks from the subjective view of non-professional investors, and represents the individual counterpart of VaR. Thus, replacing other measures of market risk, commonly

used in portfolio optimization (such as the standard deviation or the portfolio VaR), by IAL allows to the portfolio managers to shape their recommendations according to the subjective profile of their clients.

Let us now take an example of portfolio selection where the subjective IAL can be applied. Specifically, we consider the model with exogenous desired risk introduced in Campbell, Huisman, and Koedijk (2001). Accordingly, financial assets are allocated by maximizing the expected return subject to the usual budget constraint, as well as to an additional risk constraint. In the manager view, the risk is measured by the so-called desired Value-at-Risk (abbr. VaR\*) which corresponds to the risk level desired by (or sooner acceptable for) the non-professional client. The optimal portfolio is derived such that the maximum expected loss does not exceed this risk level indicated by the client. In other words, the client specification of the acceptable risk is considered by the portfolio manager as exogenous to the optimization procedure and enters it in form of a constraint. In Campbell, Huisman, and Koedijk (2001), managers are not interested in how their clients set the desired risk-level. To them, this level corresponds to a certain VaR-value computed according to the definition of VaR, namely for a chosen investment horizon and at a given confidence level (from here the denomination of VaR\*). Obviously, from the viewpoint of the client, the desired risk level corresponds to our measure IAL. Our goal is to observe how the portfolio allocation changes with respect to the results in Campbell, Huisman, and Koedijk (2001), when the fixed, VaR-equivalent formula used in this framework, is replaced by IAL-values obtained on the basis of our definitions in Section 2.2 and of real market data as in Section 3.

Note that Campbell, Huisman, and Koedijk (2001) allow to investors to additionally borrow (lend) money at the market interest rate in order to increase (decrease) the value of their risky portfolios. The wealth at time  $t$  is denoted by  $W_t$ , and  $B_t$  represents the amount of money to borrow ( $B_t > 0$ ) or lend ( $B_t < 0$ ) at the fixed risk-free gross return rate  $R_f$ . Recall that the individually desired risk level is designed by the manager as VaR\*, as it corresponds to the theoretical VaR-concept. However, VaR\* equals the value given by the non-professional client to our subjective measure IAL, and is communicated in form of a single number to the manager. Thus, we henceforth use the notation IAL.

Let the risky portfolio consist of  $i = 1, \dots, n$  financial assets with single time  $t$  prices  $p_{i,t}$  and define the set of portfolio weights at time  $t$  as  $[w_t \in R^n : \sum_{i=1}^n w_{i,t} = 1]$ . Moreover,  $x_{i,t} = w_{i,t}(W_t + B_t)/p_{i,t}$  represents the number of shares of the asset  $i$  contained in the portfolio at time  $t$ . Obviously, the portfolio gross return at next trade  $R_{t+1}$  depends on the portfolio composition at the current date  $w_t$ . With the budget constraint:

$$W_t + B_t = \sum_{i=1}^n x_{i,t} p_{i,t} = x_t' p_t, \quad (2.8)$$

the value of the portfolio at  $t + 1$  results in:

$$W_{t+1}(w_t) = (W_t + B_t)R_{t+1}(w_t) - B_tR_f. \quad (2.9)$$

As the investor desired risk-level (VaR\* for the manager, IAL for the client) corresponds in the view of the portfolio manager to a VaR-specification, it stands then for the maximum expected loss over a given investment horizon and for a given confidence level  $1-\alpha$ .<sup>3</sup> Thus, we can write the risk constraint that managers have to take into account in addition to the budget constraint (2.8) when searching for optimal portfolio weights as:

$$P_t[W_{t+1}(w_t) \leq W_t - \text{IAL}] \leq 1 - \alpha, \quad (2.10)$$

where  $P_t$  is the conditional probability on the available information at time  $t$ .

The portfolio optimization problem can be now expressed in terms of the maximization of expected portfolio returns  $E_t[W_{t+1}(w_t)]$ , subject to both the budget restriction and the IAL-constraint:

$$w_t^* \equiv \arg \max_{w_t} \{(W_t + B_t)E_t[R_{t+1}(w_t)] - B_tR_f\}, \text{ s.t. (2.8) and (2.10)}. \quad (2.11)$$

Here,  $E_t[R_{t+1}(w_t)]$  represents the expected return of the portfolio given the information at time  $t$ .

After few facile manipulations, the optimal portfolio composition yields:

$$w_t^* \equiv \arg \max_{w_t} \frac{E_t[R_{t+1}(w_t)] - R_f}{W_tR_f - W_tq_t(w_t, \alpha)}. \quad (2.12)$$

Equation (2.12) shows that, similarly to the traditional mean-variance framework, the two-fund separation theorem applies. Specifically, neither the non-professional investor's initial wealth, nor the desired risk-level (IAL) affect the maximization procedure. In other words, we can assume a two-step process. First, the professional portfolio manager determines the optimal composition of the risky portfolio (that is the optimal capital allocation among different risky assets). Second, the non-professional client communicates the desired risk-level (that is the IAL, that is yet viewed as VaR\* by the manager), in order to determine amount money to be borrowed or lent (that is invested in risk-free assets). The latter reflects by how much the portfolio-VaR varies according to the investor degree of risk aversion measured by the selected IAL-level.

As already mentioned, we focus on the decisions of non-professional investors. Therefore, we are *not* interested in the first step of the process described above, that has been exhaustively treated by the numerous portfolio optimization models in the literature. It

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<sup>3</sup>Note that losses are considered in absolute value, thus IAL (VaR\*) is positive.

is thus no our intention to compare the performance of such models.

The optimal sum to be borrowed or lent results further in:

$$B_t = \frac{\text{IAL} + \text{VaR}_t}{R_f - q_t(w_t^*, \alpha)} \quad (2.13)$$

where  $\text{VaR}_t = W_t[q_t(w_t^*, \alpha) - 1]$  stands for the portfolio VaR.<sup>4</sup> Thus, the desired IAL is imposed by the client *prior* to the portfolio formation and enters the portfolio optimization problem in form of a constraint. By contrast, the portfolio VaR is an *output* of this optimization and measures the actual maximum loss that can be incurred at time  $t$  at the confidence level  $1 - \alpha$  for the obtained optimal portfolio  $w^*$ .

We can now specify the time  $t + 1$ -value of the risky investment that underlies the definition of IAL from Equations (2.4) and (2.5), that is:

$$S_{t+1} = (W_t + B_t)R_{t+1}. \quad (2.14)$$

Thus, the definition of IAL serves to determine the optimal level of borrowing or lending ( $B_t$ ) from Equation (2.13). When IAL lies “to the left” of the portfolio VaR (specifically, when it is lower in absolute value than VaR),  $B_t$  is negative, hence investors become more risk averse and save money. By contrast, for an IAL higher than VaR in absolute value, investors augment their risky investment by borrowing extra money.

### 3 An empirical application

This chapter presents empirical findings complying with the theoretical results derived in Section 2. In particular, we apply the IAL defined in Equations (2.4) and (2.5) as risk constraint, now subjectively assessed, in the portfolio optimization framework of Campbell, Huisman, and Koedijk (2001) detailed in Section 2.3. In so doing, we rely on the results in Sections 3.1 and 3.4 Rengifo and Trifan (2007).

Our empirical analysis is based on daily data between 01/02/1962 to 03/09/2006 (11,005 observations) for the SP500 index (considered as proxy for an well diversified risky portfolio) and the 10-year nominal returns bond (viewed as the risk-free investment alternative).<sup>5</sup> From this data set, we construct daily, weekly, monthly, and up to eleven months (with a one-month increment), as well as yearly, two-yearly, and up to eight-yearly (with a one-year increment) returns. Due to the financial reform in 1979 that significantly changed the trading conditions, we can consider that the early 80s mark the beginning of

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<sup>4</sup>The expression  $q_t(w_t^*, \alpha) - 1$  should be viewed as the quantile of the net returns  $R_t - 1$  and corresponds to the quantile  $q_t(w_t^*, \alpha)$  of the gross returns  $R_t$ . Equation (2.13) can be restated in terms of net returns as:  $B_t = (\text{IAL} + \text{VaR}) / [(R_f - 1) - (q_t(w_t^*, \alpha) - 1)]$ .

<sup>5</sup>Descriptive statistics can be found in Tables 6 and 7 in the Appendix.

a new era of financial markets. Hence our sample is divided into two parts, from which only the second one (from 03/01/1982<sup>6</sup> to 03/09/2006, specifically 6,010 observations) is relevant for inferring current market evolutions. We denote it as the “active” data set and it underlies the subsequent empirical investigations. The first part of the sample (from 01/02/1962 to 03/01/1982) serves to assess the empirical mean and the standard deviation of the portfolio returns at “date zero” of the trade (03/01/1982). Yet, the active data set contains an outlier corresponding to the October 1987 market crash. As the real market data serves in our work merely as support for simulating trading behaviors, that we view as more general, this outlier may distort the results. Consequently, we smoothed it out by replacing it with the mean of the ten before and after data points.<sup>7</sup>

We consider that non-professional investors perceive risky investments according to the value functions in Equations (2.1) and (2.2), and calculate the maximum loss level according to Equation (2.5). The active data set allows us to run the model on the basis of Sections 2.3 and 2.1 and to derive the desired IAL-level used to determine the wealth proportion invested in the risky portfolio (which is here in the SP500 index). The remaining money is assumed to be automatically put in the risk-free 10-year bond. Moreover, our investors start trading with an even initial wealth allocation between the risky portfolio and the bond.<sup>8</sup> We also assume that the number of investors is constant, specifically that no investors can enter or exit the market during the trading interval (that corresponds to the second part of the data).<sup>9</sup>

### 3.1 The investment in risky assets

Before entering the details of the portfolio optimization with IAL as risk measure according to Section 2.3, we investigate how risky investments develop subject to different portfolio evaluation frequencies. This is important, as our IAL in Equation (2.5) depends on the current value of these investments  $S_t$ . Finally, we discuss the impact of applying the simpler definition IAL<sup>1</sup> (that merely accounts for maximum expected losses) on the wealth percentages invested in the risky portfolio.

As noted in Rengifo and Trifan (2007), the current values of risky investments  $S_t$  depends on the frequency at which risky performance is evaluated. Specifically, they prove that for more frequent evaluations, investors dedicate on average lower percentages

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<sup>6</sup>As it took several years until the financial reform became operative.

<sup>7</sup>We consider that this method is appropriate for preserving some of the particularities of less probable market events such as crashes, while at the same time allowing for circumvention of excessive impacts due to extreme outliers.

<sup>8</sup>A similar assumption is made in Thaler, Tversky, Kahneman, and Schwartz (1997).

<sup>9</sup>This assumption implies that the evaluation period is shorter than the lifetime of our loss averse agents or, equivalently, that investors are long-lived beyond the VaR horizon. Identical assumptions are made in Basak and Shapiro (2001), Berkelaar, Kouwenberg, and Post (2004), Berkelaar and Kouwenberg (2006).

of their wealth  $S_t/W_t$  to risky assets. This supports the idea of *myopic loss aversion* suggested in Benartzi and Thaler (1995).<sup>10</sup>

The subsequent Table 1 replicates the results of Table 1 in Rengifo and Trifan (2007), for their usual case with cumulative cushions  $Z_t = \sum_{i=0}^t S_i$  that amass from the date zero of the trade. Here,  $S_t$  is derived according to Equation (2.14) and two distributions of the portfolio gross returns are considered, i.e. (standard) normal and Student-t (with five degrees of freedom). Moreover, investors are assumed to ascertain expected returns  $E_t[R_{t+1}]$  as unconditional mean of past returns.<sup>11</sup> In particular, the portfolio VaR (see the comments below Equation (2.13)) is computed for the given distribution of the portfolio gross returns and for a significance level of 5%. Following Tversky and Kahneman (1992) and Barberis, Huang, and Santos (2001), we set the initial loss aversion coefficient to  $\lambda = 2.25$  and the penalty imposed on past losses to  $k = 3$ . The probability of past gains  $\pi_t$  is estimated as well as the empirical frequency of the cases where  $z_t \leq 1$ . The individually accepted loss level  $IAL_{t+1}$  is derived according to Equation (2.5) and subsequently plugged into Equation (2.13) in order to determine the optimal level  $B_t$  of borrowing or lending.

Evaluation frequency	Portfolio returns	
	Normal	Student-t
1 year	49.31	44.83
6 months	14.18	12.14
4 months	19.38	14.96
3 months	18.50	13.73
1 month	2.05	1.88
1 week	0.44	0.39
1 day	0.15	0.15

Table 1: Average wealth percentages invested in SP500.

According to Table 1, when investors are loss averse and use the  $IAL_{t+1}$  from Equation (2.5) as measure of the maximal acceptable risk, higher portfolio evaluation frequencies entail lower investments in the risky portfolio. This result is consistent with previous findings, such as Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001), namely that loss-averse investors who perform high frequency evaluations and narrow-frame financial projects (by overly focusing on long series of past performances) become extremely risk averse. In essence, investors who use cumulative cushions and daily evaluate the performance of their risky portfolio end up by putting almost all their money

<sup>10</sup>Specifically, Benartzi and Thaler (1995) show that the magnitude of the equity premium observed in practice can be fully explained by considering loss-averse investors who evaluate the performance of their portfolios once a year, and employ a linear value function with conventional PT parameter values.

<sup>11</sup>Rengifo and Trifan (2007) obtain similar results for the case where expected returns are derived as zero mean or the AR(1) process.

in the risk-free asset. Their reluctance towards risky assets is more pronounced when the gross portfolio returns are considered to be Student-t distributed, compared to the normal distribution. Interestingly, the yearly results under the normal distribution almost perfectly match the so called TIAA-CREF typical allocation (with slightly less than 50% as stock investment) mentioned in Benartzi and Thaler (1995).

We close this section by referring the simple IAL<sup>1</sup> in Equation (2.4), that exclusively accounts for maximum expected losses. Investors who merely account for the average expected losses in formulating individual risk constraints continue to reduce their risky investments for higher evaluation frequencies. In other words, they continue to manifest myopic loss aversion. In effect, this phenomenon becomes somewhat differently manifest, as shown in Table 2, that relies on identical parameter values and considerations as the above Table 1. Accordingly, the IAL<sup>1</sup>-investors start with lower risky allocations for yearly evaluations than their more sophisticated IAL-peers. They reduce yet their risky investments subject to higher evaluation frequencies more slowly (faster) up to (above) five months, ending up by investing nothing in risky assets. We can conclude that in essence, the standard evaluation frequency of one year renders the variance-adjustment in the IAL-formula unimportant with respect to the wealth percentages dedicated to risky assets.

<b>Evaluation frequency</b>	<b>Portfolio returns</b>	
	Normal	Student-t
1 year	47.04	43.41
6 months	26.84	26.30
4 months	8.27	7.80
3 months	7.86	6.82
1 month	0.00	0.00
1 week	0.00	0.01
1 day	0.00	0.00

Table 2: Average wealth percentages invested in SP500 using the simple IAL<sup>1</sup>.

### 3.2 A comparison with the portfolio optimization framework

This section proposes to translate the results obtained in our framework (where investors subjectively derive the maximum acceptable level of losses) in terms of the portfolio optimization “language” spoken by professional managers. To this end, we calculate the significance levels, the wealth percentages invested in risky assets, and the average coefficients of loss aversion that correspond to the IAL derived according to our model equations and on the basis of our data set, as if these IAL-values result from the VaR-concept applied by the portfolio manager in Campbell, Huisman, and Koedijk (2001).



### 3.2.1 IAL-equivalent significance levels

One further question of interest arises from the use of IAL as measure of risk in the portfolio optimization model in Section 2.3. In the view of portfolio managers, who equate the risk level indicated by the client by a common VaR-specification, IAL represents the lower quantile of portfolio returns at a given (i.e. fixed) significance level  $\alpha$  (or confidence level  $1 - \alpha$ ), where usually  $\alpha \in [1, 10]\%$ . (Recall the denomination of  $\text{VaR}^*$  used in Campbell, Huisman, and Koedijk (2001).) The individually optimal  $\text{IAL}_{t+1}$  is yet set by clients on the basis of subjective considerations, as captured by our Equation (2.5). The portfolio manager compares then the IAL-indication of the client to the portfolio VaR, in order to determine how wealth is to be optimally split between the risky portfolio and the risk-free bond. The sum to be additionally invested in risk-free assets is formalized in Equation (2.13).

We denote by  $\alpha_t^*$  the significance level that corresponds to the  $\text{IAL}_{t+1}$  computed in our model. Thus, if the portfolio VaR at time  $t$  corresponds to an  $\alpha > \alpha_t^*$  (or equivalently, to a confidence level  $1 - \alpha < 1 - \alpha_t^*$ ), then the sign of Equation (2.13) is negative. In other words, too much risk would arise by putting the entire wealth in the risky portfolio, so that, in order to accommodate the desired (lower) risk level, a percentage of the investor wealth should be lent, i.e. invested in the risk-free asset ( $B_t < 0$ ). On the contrary, if  $\alpha < \alpha_t^*$ , then the portfolio risk meets the individual risk requirements (being lower than the subjective risk threshold) and investors borrow extra money ( $B_t > 0$ ) in order to increase their SP500-holdings.

In this section, we determine the significance levels corresponding to the values of  $\text{VaR}_{t+1}^*$  derived from Equation (2.5) for normally and Student-t distributed gross returns and cumulative cushions. Their average over time  $\alpha^*$  is given in Table 3.

Evaluation frequency	Portfolio returns	
	Normal	Student-t
1 year	0.00	0.00
6 months	0.00	0.00
4 months	0.00	0.00
3 months	0.00	0.00
1 month	0.00	0.00
1 week	0.00	0.00
1 day	0.00	0.00

Table 3: Average significance levels  $\alpha^*$  equivalent to the estimated  $\text{IAL}_{t+1}$ .

As stated above, classical portfolio selection models based on VaR assume that investors chose significance levels  $\alpha$  in the interval  $[1, 10]\%$ . Our findings in Table 3 show that for any evaluation frequency higher than one year, this assumption does *not* comply

with the real market data when non-professional investors assess their maximum acceptable loss level according to Equation (2.5). Specifically, the equivalent significance level  $\alpha^*$  lies below the theoretically acceptable interval (being practically zero). Thus, even the lowest significance level of 1% proposed in standard portfolio optimization models is not able to capture the risk aversion of non-professional investors acting according to our setting. In other words, investors may be substantially more risk averse than considered in theory and in usual practice.

### 3.2.2 Portfolio-equivalent indices of loss aversion

The previous section suggested that non-professional investors, who differently perceive gains and losses and are influenced by their personal investment history, are more risk averse than commonly described by in terms of portfolio significance levels  $\alpha \in [1, 10]\%$ . In the same context, we now address the impact of an exogenous desired risk level as originally employed in Campbell, Huisman, and Koedijk (2001), on the wealth percentages invested in risky assets computed according to Equations (2.8), (2.13) and (2.14) in our model, as well as on the values of the loss aversion coefficient  $\lambda_{t+1}^*$  from Equation (2.7). To this end, we go back to the conventional significance levels of 1% and 10% and estimate an homologous IAL that would correspond to the portfolio VaR (see formula below Equation (2.13)), at one of these two significance levels. This equivalent IAL serves to compute  $\lambda_{t+1}^*$  according to Equation (2.7).

Tables 4 and 5 present the equivalent wealth percentages (specifically, the average of  $S_t/W_t$ ) that would be invested in the risky portfolio at 1%- and 10%-significance, but are obtained imposing the IAL-values that result from our model. The values in these tables are to be read relative to a benchmark, which is the portfolio VaR below Equation (2.13) estimated for 5%-significance (in other words, this benchmark corresponds to 100% risky portfolio). The same tables also show the average equivalent coefficient of loss aversion  $\lambda^*$  and consider the cases with normally or Student-t distributed portfolio returns and cumulative cushions.

Accordingly, the equivalent recommendations from our model at 1%- (10%-) significance lie well below (above) the benchmark VaR at 5%. This points out a higher (lower) risk aversion in our endogenous IAL-framework, after restating it in terms of the exogenous-VaR model, relative to the portfolio risk measured by VaR. Comparing Tables 4 and 5, we can observe that the lower the significance level is (or the higher the confidence level), the more risk averse the non-professional investors become, as the wealth proportion invested in the risky portfolio is smaller than 100%. However, even the lowest percentages in Table 4 are still much higher than those in Table 1, where IAL is treated as endogenous (estimated on the basis of the individual investor profile). In-

Evaluation frequency	Wealth %		$\lambda^*$	
	Portfolio returns		Portfolio returns	
	Normal	Student-t	Normal	Student-t
1 year	69.10	47.65	0.91	0.83
6 months	65.54	42.53	0.97	0.97
4 months	64.48	41.04	0.97	0.97
3 months	63.43	39.59	0.97	0.97
1 month	61.64	37.12	0.98	0.99
1 week	60.14	35.10	1.07	0.99
1 day	59.35	34.05	1.00	1.00

Table 4: Wealth percentages invested in SP500 and the average  $\lambda^*$ , for  $\alpha = 0.01$ .

Evaluation frequency	Wealth %		$\lambda^*$	
	Portfolio returns		Portfolio returns	
	Normal	Student-t	Normal	Student-t
1 year	116.47	120.91	0.68	1.12
6 months	118.37	122.95	1.09	1.09
4 months	118.94	123.55	1.12	1.09
3 months	119.49	124.13	1.15	1.08
1 month	120.45	125.11	1.09	1.07
1 week	121.25	125.92	1.02	1.02
1 day	121.67	126.34	1.01	1.02

Table 5: Wealth percentages invested in SP500 and the average  $\lambda^*$ , for  $\alpha = 0.10$ .

terestingly, the results for  $\alpha = 1\%$  are consistent with our previous findings supporting the myopic loss aversion, as the wealth percentage invested in risky assets decreases for higher evaluation frequencies. By contrast, when  $\alpha$  increases to 10%, this phenomenon is reversed, and investors appear to allocate more money to the risky portfolio for more frequent evaluations. As the myopic loss aversion is a widely documented phenomenon, we can conclude that the traditional portfolio optimization framework (with VaR as risk constraint) fails once more to capture the real investor behavior in a consistent way.

Similar conclusions can be drawn for the loss aversion coefficient  $\bar{\lambda}^*$  derived for conventional significance levels assumed in previous research. Its values are much lower than 2.25, the empirical level estimated in the original PT and largely used in previous empirical research.<sup>12</sup> For the majority of the considered combinations of  $\alpha$ -values and evaluation frequencies, we obtain  $\bar{\lambda}^* \simeq 1$ , a level that indicates identical perception over gains and losses according to the value function from Equations (2.1) and (2.2) (and recalling that  $k = 0$ ). Actually, this “neutral” level of 1 is exceeded merely for high evaluation frequencies, i.e. over one week (six months) for  $\alpha = 1\%$  (10%). This reinforces our earlier claim that even assuming low significance levels, such as  $\alpha = 1\%$ , entails an underestimation of the loss attitude of real investors captured by the specific coefficient  $\lambda$ .

## 4 Summary and conclusions

This paper proposes a new measure of (downside) market risk that is shaped in line with the individual profile of non-professional investors (including psychological traits). We denote this measure as the individually acceptable loss (IAL) and suggest that it can be used by professional portfolio managers in order to better meet the individual wishes of their clients. Moreover, this measure can be easily assessed from general parameters, that are on the one hand subjective and reveal the client preferences and attitudes towards financial losses, and on the other hand describe the general market and investment evolutions. Acting on the basis of our measure, portfolio managers can also provide a solution to their clients’ concerns referring to the optimal wealth allocation among different financial assets, given the subjective risk attitude of these clients.

In line with Kahneman and Tversky (1979, 1992) and Barberis, Huang, and Santos (2001), we design the non-professional clients as being loss averse, sensitive to the past performance of the risky portfolio, and narrowly frame financial investments. Considering that these investors derive utility merely from financial investments, we theoretically model their perceptions regarding the utility of risky assets and define their subjective maximum acceptable level of financial losses, which is our measure IAL. This IAL-level

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<sup>12</sup>Such as Barberis, Huang, and Santos (2001), Benartzi and Thaler (1995).

is subsequently communicated to the professional portfolio manager in charge of finding the optimal capital composition. We present evidence for how the optimal mix of risky and risk-free assets can be found when this subjectively assessed IAL enters the optimization procedure in form of a risk constraint. To this end, we rely on the portfolio allocation model developed in Campbell, Huisman, and Koedijk (2001) as an application example. Analyzing how IAL performs in other portfolio optimization frameworks that apply different risk measures remains an interesting issue of future research.

We check and amend our theoretical findings with empirical results obtained on the basis of real market data. In particular, we consider the price series of the SP500 and the US 10-year bond as proxies for the risky portfolio and the risk-free asset, respectively. In line with Rengifo and Trifan (2007), we first observe that the investor behavior depends on the frequency at which the performance of the risky portfolio is revised. Specifically, investors allocate smaller fractions of their wealth to risky investments, as they revise portfolios more often. This result comes in line with the notion of myopic loss aversion introduced in Benartzi and Thaler (1995). In the sequel, we attempt to provide an equivalency between the setting in Campbell, Huisman, and Koedijk (2001) and our own one. In the former model, portfolio managers equate the risk levels indicated by their clients to a VaR-specification. In so doing, they mostly apply common significance levels, such as 1%, 5%, or 10%. By contrast, our setting allows for the derivation of IAL on the basis of the subjective client profile. Yet the desired risk level, either computed by managers from the standard VaR-definition (VaR\*) or derived by clients according to their subjective profiles (IAL), enters the optimization procedure in the form of a risk constraint. For the IAL-values calculated in the subjective framework of our model and on the basis of real data, we thus deduce equivalent significance levels that would correspond to the VaR\*-specification. Similarly, we assess equivalent wealth percentages invested in risky assets and equivalent loss aversion coefficients, that correspond to our IAL, and compare them with the values proposed in the previous literature. Our results show that both procedures imply an underestimation of the non-professional investors' reluctance towards financial losses in the portfolio optimization framework with VaR as risk constraint.

## A Appendix

<b>SP500</b>	<b>Evaluation frequency</b>	
	Quarterly	Yearly
Mean	0.017	0.066
Median	0.018	0.071
Std.Dev.	0.079	0.136
Kurtosis	2.661	-0.9659
Skewness	-0.671	-0.205
Max.	0.290	0.345
Min.	-0.302	-0.207
Obs.	175	43

Table 6: Log-difference of the SP500 index for quarterly and yearly portfolio evaluations.

<b>10-year</b>	<b>Evaluation frequency</b>	
	Quarterly	Yearly
Mean	0.017	0.073
Median	0.017	0.070
Std.Dev.	0.006	0.026
Kurtosis	0.623	0.974
Skewness	0.951	1.042
Max.	0.036	0.142
Min.	0.009	0.037
Obs.	175	43

Table 7: 10-year bond return for quarterly and yearly portfolio evaluations.

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