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**April 2024 Buy-Sell Guide for
Dow Jones 30 Stocks and
Modified Omega Criterion**

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Abstract

We study recent monthly data to help long-term investors buy or sell from the 30 Dow Jones Industrial Average (DJIA) Index components. The recommendations are based on six stock-picking algorithms involving comparisons of probability distributions. We use data for 30 stocks using the recent 472 months (39+ years) of monthly returns ending in March 2024. Our buy-sell recommendations also use newer “pandemic proof” out-of-sample portfolio performance comparisons from the R package ‘generalCorr.’ We include modified omega (gain-to-pain ratio) computation to compare stock performance.

1 Introduction

Wall Street investment outfits use stock prices attached to stock symbols. We use fairly long price data for 39 years and four months, ending in March 2024. If \$1 is invested in buying a stock priced at P_t at time t , if the price (adjusted for dividends) at time $t + 1$ is higher, the net return $r_t = [(P_{t+1} - P_t)/P_t]$ will exceed the initial investment of \$1. Since the net return is negative when losses are incurred, one defines gross return as $(1 + r_t) = 1 + (P_{t+1} - P_t)/P_t = P_{t+1}/P_t$. The gross return is always positive since prices are positive.

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Continuously compounded return is the exponential return, $\exp(r_t)$, which is always positive. The series expansion of $\exp(r_t)$ is $(1+r_t+r_t^2/2!+\dots)$. Assuming higher order terms in the expansion can be ignored, ($|r_t| < 1$), one can write $\exp(r_t) = 1 + r_t$. It is customary to equate the exponential return to the gross return and write $r_t = \log(P_{t+1}/P_t) = \log P_{t+1} - \log P_t$. Many published papers often use the *first difference* of logs of prices evaluated at time $t+1$ as a return from investment in the Dow Jones Industrial Average's 30 (DJ30) stocks. Since the return data do not always satisfy ($|r_t| < 1$), we use $r_t = [(P_t - P_{t-1})/P_{t-1}]$ for our monthly returns.

We begin with Tables 1 and 2 for company names, ticker symbols, relative weights in DJIA, and a (case-sensitive) single character name for identification of the company in a scatterplot of mean return on the vertical axis and standard deviation of returns measuring the volatility (risk) for that stock on the horizontal axis. The scatterplot of Figure 1 is inspired by Markowitz's efficient frontier model without the risk-free rate straight line.

The basic idea behind Figure 1 is that we imagine grouping stocks into a certain number (=7) of unequal-width ranges of standard deviations class intervals. Our 30 stocks are assigned to these seven intervals. Now, the stock yielding the highest average return for each level of risk (measured by the midpoints of the sd class interval) dominates all those below it in Figure 1. The dominating stocks from DJIA are graphically identified as (j, v, h, f, C, a, z). The corresponding longer company names of dominating stocks, according to Tables 1 and 2, are Johnson and Johnson, Visa, Home Depot, Microsoft, Salesforce, Apple, and American Express.

1.1 Descriptive statistics for the DJIA stock returns.

This section reports some basic information about our data using standard descriptive stats. We report 'min' for the smallest return, Q1 for the first quartile, where 25% of data are below Q1, and 75% above Q1. 'Median' and 'Mean' are self-explanatory. Q3 is for the third quartile (75% below and 25% above), and 'max' denotes the largest return.

In Finance, two additional descriptive stats are often used. The Sharpe ratio is the ratio of mean to standard deviation (sd) of returns. It is named after a Nobel-winning economist, Sharpe, and represents risk-adjusted average return. The second descriptive statistic in Finance is the 'expected gain to expected pain ratio.' It is called 'omega' in Keating and Shadwick (2002).

Figure 1: Mean-standard deviation Efficiency Frontier for Dow Jones 30 stocks

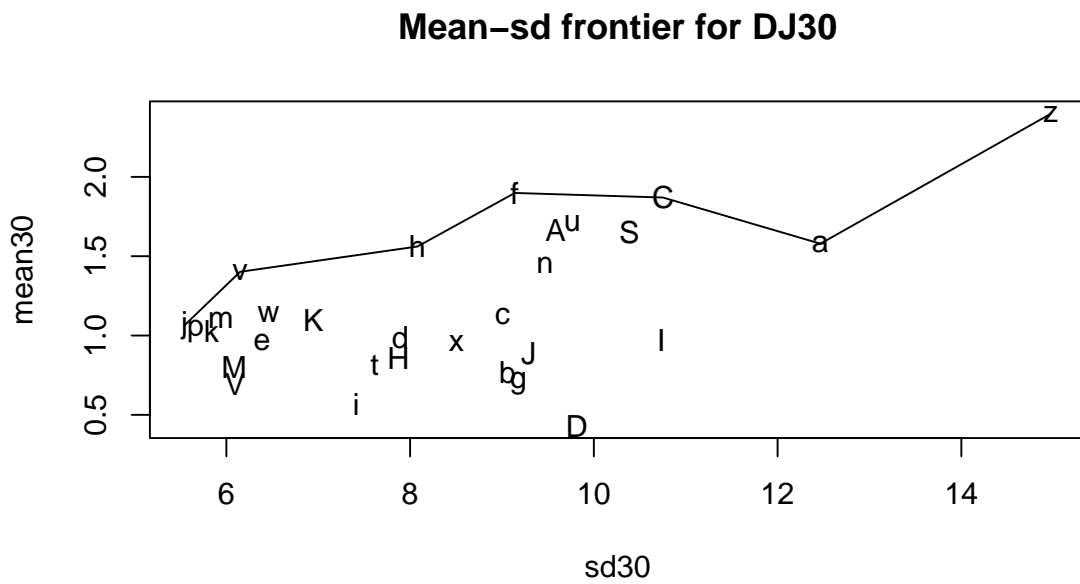


Table 1: Company names, ticker symbols, weight in DJIA, and abbreviations with only one character. Part 1

Seq.	Company	Symbol	Weight	char1
1	Apple Inc	AAPL	2.93	a
2	Amgen Inc	AMGN	4.65	A
3	Amazon.com Inc	AMZN	2.99	z
4	American Express Co	AXP	4.09	x
5	Boeing Co	BA	2.88	b
6	Caterpillar Inc	CAT	5.83	c
7	Salesforce Inc	CRM	4.71	C
8	Cisco Systems Inc	CSCO	0.83	S
9	Chevron Corp	CVX	2.85	e
10	Walt Disney Co	DIS	1.94	d
11	Dow Inc	DOW	0.97	D
12	Goldman Sachs Group Inc	GS	7.24	g
13	Home Depot Inc	HD	5.72	h
14	Honeywell International Inc	HON	3.33	H
15	Intl Business Machines Corp	IBM	2.91	i

1.1.1 Sharpe ratios for risk-adjusted stock returns

Recall that the risk (horizontal axis) versus return (vertical axis) scatterplot of Figure 1 suggests that Johnson and Johnson, Visa, Home Depot, Microsoft, Salesforce, Apple, and American Express graphically dominate others. The respective Sharpe ratios of these stocks are (0.22, 0.26, 0.23, 0.25, 0.22, 0.19, 0.15). Note that the Sharpe ratio is a direct measure of risk-adjusted return, bypassing the grouping of stock returns into standard deviation (sd) intervals.

The lowercase versions of ticker symbols for the top seven Sharpe ratios in increasing order of magnitude are amzn=0.22, jnj=0.22, crm=0.22, unh=0.23, hd=0.23, msft=0.25, and v=0.26, respectively. A stock-picking algorithm based on the Sharpe ratio is supported by Markowitz’s theory. It is one of the six algorithms discussed later in Section 3.

1.1.2 Alternate computation of “omega” for stock returns

This subsection describes the well-known “omega” measure for comparing the performance of many stocks. Since a larger omega means a larger

Table 2: Company names, ticker symbols, weight in DJIA, and abbreviations with only one character. Part 2

Seq.	Company	Symbol	Weight	char1
16	Intel Corp	INTC	0.61	I
17	Johnson & Johnson	JNJ	2.53	j
18	JP Morgan Chase & Co	JPM	3.33	J
19	Coca Cola Co	KO	1.06	k
20	McDonald's Corp	MCD	4.75	m
21	3m Co	MMM	1.58	M
22	Merck & Co. Inc.	MRK	2.25	K
23	Microsoft Corp	MSFT	6.88	f
24	Nike Inc Cl B	NKE	1.62	n
25	Procter & Gamble Co	PG	2.80	p
26	Travelers Cos Inc	TRV	3.69	t
27	Unitedhealth Group Inc	UNH	8.52	u
28	Visa Inc Class A Shares	V	4.75	v
29	Verizon Communications Inc	VZ	0.68	V
30	Walmart Inc	WMT	1.04	w

preponderance of positive returns, a stock having a larger omega is more desirable.

The idea of measuring a gain-to-loss ratio is first mentioned in Bernardo and Ledoit (2000), who define r as a risk-adjusted excess return over a target return. Their risk adjustment requires assumptions about the utility function of investors. This paper avoids any assumption regarding investor utility functions. We do retain their distinction between $r^+ = \max(0, r)$, its positive part, and $r^- = \max(r, 0)$, its negative part. Their gain-loss ratio is Ω_{bl} , where the subscript ‘bl’ identifies authors.

$$\Omega_{bl} = E(r^+)/E(r^-). \quad (1)$$

Keating and Shadwick (2002) (“KS02”) name a “cumulative probability weighted” gain-loss ratio ‘omega,’ without assuming anything about the utility function of investors. The gains and losses in KS02 are compared to a target return. If all stocks in a data set have a common target return, we can subtract it from all returns and work with returns in “excess of target.” Hence, there is no loss of generality in letting the target be zero. Therefore, our target return is mostly zero in the sequel.

Let $f(r)$ denote the probability distribution function of returns, and $F(r)$ denote the (cumulative) distribution function of returns. KS02 define

$$\Omega_{ks} = E(gain)/E(loss), \quad (2)$$

where the expectation operator from probability theory in $E(gain)$ is represented by choosing weights $(1 - F(r))$ by assuming continuous distributions. Similarly, KS02 represent $E(loss)$ by choosing the weights $F(r)$. The subscript 'ks' in Ω_{ks} identifies the authors of KS02.

With discrete data, we replace $F(r)$ with the empirical cumulative distribution function (ECDF), a step function of returns. Figure 2 illustrates the ECDF for an imaginary stock A with $n = 4$, returns $r_i = (-3, -1, 2, 5)$, for $i = 1, 2, \dots, n$. A vertical axis at $r = 0$, shown in Figure 2, separates the ECDF for stock A into the loss side on the left (for $r^- = \max(r, 0)$) and the gain side on the right (for $r^+ = \max(0, r)$).

How do we represent the E operator weights in the discrete case? Usually, mathematical expectation $E(x) = \sum x_i p_i$ (return x_i with probability p_i) is the average return. KS02 formulate the mathematical expectation of aggregate loss based on cumulative sums of negative returns r_i times corresponding cumulative probabilities from $F(r)$ as weights. Their gain side weights based on $(1 - F(r))$ are from the areas above the ECDF steps.

The ECDF on the loss side for the toy stock A has two negative ranges, $(-3, -1)$ and $(-1, 0)$, with areas under the pillars for the two ranges of $(1/n)$ and $(2/n)$, respectively. The respective gain side weights $(2/n)$ and $(1/n)$ are based on $(1 - F(r))$. These weights represent the areas above the pillars for the two ECDF ranges $(0, 2)$ and $(2, 5)$ on the right-hand side of the zero axis in Figure 2.

The KS02 weighting scheme of Ω_{ks} is similar to that of partial moments of degree 1. Hence, Ω_{ks} of (2) is the ratio of the upper partial moment (UPM) of degree 1 to the analogous lower partial moment (LPM), Viole and Nawrocki (2016). The R package called NNS, Viole (2021), has convenient functions called UPM and LPM to compute them, and hence

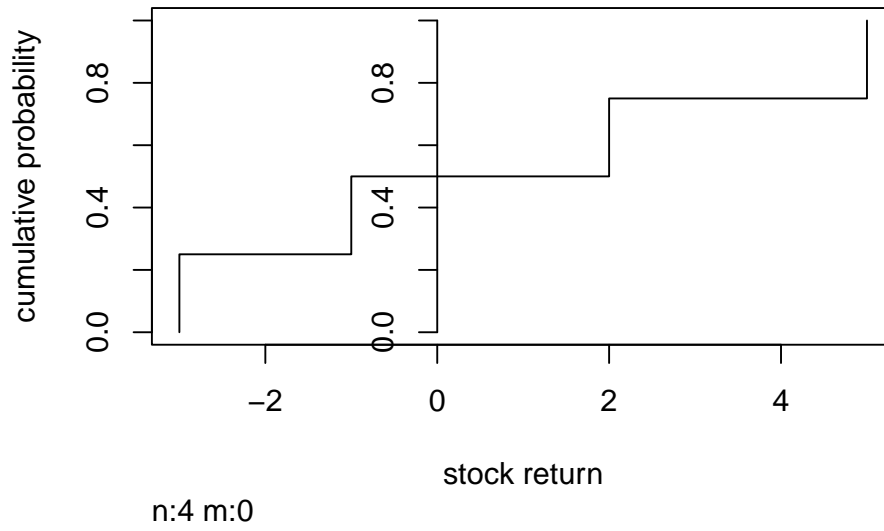
$$\Omega_{ks} = UPM(1, 0, r)/LPM(1, 0, r), \quad (3)$$

where r is a vector of returns, and where we have included the arguments (degree=1, target=0) of the R functions in the package.

This paper suggests replacing the cumbersome weighting scheme based on ECDF pillar areas used by KS02. We suggest intuitively sensible ratio of

Figure 2:

ECDF for $r=(-3,-1,2,5)$



aggregate gain to aggregate loss without using $F(r)$ and $(1 - F(r))$ weights. We simply modify equation (1) as:

$$\Omega_{sum} = \frac{\sum_i(r_i^+)}{\sum_i|(r_i^-)|}, \quad (4)$$

where the subscript ‘sum’ refers to summations in the formula. The numerator and denominator are both positive.

Alternative formulation is

$$\Omega_{avg} = \frac{\sum_i(r_i^+)/n^+}{\sum_i|(r_i^-)|/n^-}, \quad (5)$$

where n^+ denotes the number of positive returns in the data, and n^- denotes the number of negative returns in the data, and where the subscript ‘avg’ refers to the averages (expected values) in the formula. Comparing the formula Ω_{avg} with Ω_{sum} suggests that Ω_{avg} is larger when n^- is large and that Ω_{avg} is smaller when n^+ is large. We use examples to show that such behavior of Ω_{avg} cannot be justified by the underlying logic of investors’ gain-to-loss ratios in Finance.

If the stock returns were arising from an independent and identically distributed (IID) process, the probability of observing each return r_i is $1/n$. Since the market returns are almost never IID, the probability of sum of k returns ($\sum^k r_i$) equals the sum (k/n) of their individual probabilities and adjustments for all joint probabilities observing all subsets of the k returns (r_i, r_{i+1}, \dots) at the same time. Such joint probabilities are almost never known to the researcher since a stock’s return is intimately related to its past. Replacing expected values by simple averages pretending that returns are IID is incorrect. The Ω_{avg} defined in equation (5) implies an incorrect use of the probability theory concept of the “expected value.”

Now we consider a practical reason for avoiding the Ω_{avg} of (5). Recall the imaginary stock A returns be $r_i^A = (-3, -1, 2, 5)$ used before. Verify that $\Omega_{sum}^A = (2+5)/(3+1) = 1.75$. Now imagine stock B having $r_i^B = (-4, 1, 2, 4)$ having the same $\Omega_{sum}^B = 1.75$ though the $\Omega_{avg}^A \neq \Omega_{avg}^B$. The aggregate gain (=7) and aggregate loss (=4) and gain to loss ratio (7/4=1.75) to the investor is exactly the same for stocks A and B. The aggregate gain of 7 for stock B is spread over $n_B^+ = 3$ periods, while the aggregate loss 4 is spread over only one period $n_B^- = 1$. The aggregate gain and loss for stock A is spread over two periods $n_A^+ = 2 = n_A^-$. The practically irrelevant (to the investor) sizes of

(n^+, n^-) should not be allowed to contaminate the computation of the gain to loss ratio. Hence, we suggest rejecting Ω_{avg} in favor of Ω_{sum} .

Table 3: Table of basic descriptive stats. ‘Sharpe’ is the ratio of mean to standard deviation (sd). Ω_s is Ω_{sum} , the sum of all positive returns divided by the sum of all negative returns. Count of non-missing or available sample size in the last column, ‘Av.N.’ Part 1

ticker	min	Q1	Med	Mean	Q3	Max	sd	Sha- rpe	Ω_s	Av. N
aapl	-57.74	-4.75	2.50	2.35	9.66	45.38	12.21	0.19	1.7	471
amgn	-41.53	-3.62	1.59	2.15	6.31	45.88	9.95	0.22	1.8	471
amzn	-41.16	-4.75	2.47	3.61	9.76	126.38	16.61	0.22	2.0	322
axp	-32.09	-2.37	1.37	1.32	5.87	85.03	8.73	0.15	1.6	471
ba	-45.47	-3.89	1.45	1.18	7.02	45.93	8.85	0.13	1.4	471
cat	-35.91	-4.22	1.85	1.53	7.17	40.14	8.97	0.17	1.6	471
crm	-36.03	-4.39	1.83	2.48	9.00	40.26	11.07	0.22	1.8	237
csc	-36.73	-3.91	1.72	2.21	8.28	38.92	10.50	0.21	1.8	409
cvx	-21.46	-2.55	1.15	1.17	4.82	26.97	6.47	0.18	1.6	471
dis	-28.64	-3.32	1.10	1.30	5.74	31.26	7.92	0.16	1.6	471
dow	-26.45	-3.44	1.71	0.90	6.55	25.48	9.56	0.09	1.3	60
gs	-27.73	-5.21	1.32	1.13	6.59	31.38	9.24	0.12	1.4	298
hd	-28.57	-3.43	1.71	1.90	7.11	30.33	8.15	0.23	1.8	471
hon	-38.19	-2.40	1.31	1.17	5.05	51.05	7.76	0.15	1.5	471
ibm	-24.86	-3.61	0.75	0.84	5.01	35.38	7.45	0.11	1.4	471

Table 4: Table of basic descriptive stats. Sharpe is the ratio of mean to standard deviation (sd). Ω_s is Ω_{sum} , the sum of all positive returns divided by the sum of all negative returns. Count of non-missing or available sample size in the last column, ‘Av.N.’ Part 2

ticker	min	Q1	Med -ian	Mean	Q3	Max	sd	Sha- rpe	Ω_s	Av. N
intc	-44.47	-4.39	1.25	1.55	7.05	48.81	10.63	0.15	1.5	471
jnj	-16.34	-2.24	1.25	1.23	4.45	19.29	5.59	0.22	1.8	471
jpm	-32.68	-3.86	1.22	1.32	6.41	33.75	9.18	0.14	1.5	471
ko	-19.33	-2.08	1.24	1.21	4.62	22.64	5.84	0.21	1.7	471
mcd	-25.67	-2.14	1.37	1.28	5.05	18.26	5.94	0.22	1.8	471
mmm	-27.83	-2.47	1.21	0.99	4.39	25.80	6.07	0.16	1.5	471
mrk	-26.62	-3.03	1.12	1.34	5.90	23.29	6.96	0.19	1.6	471
msft	-34.35	-3.58	2.23	2.34	6.77	51.55	9.46	0.25	2.0	456
nke	-37.50	-3.33	1.84	1.90	7.06	39.34	9.42	0.20	1.7	471
pg	-35.42	-1.78	1.16	1.19	4.93	24.69	5.58	0.21	1.8	471
trv	-53.47	-3.16	1.42	1.10	5.06	52.51	7.36	0.15	1.5	471
unh	-36.51	-3.23	2.50	2.20	7.39	40.70	9.58	0.23	1.9	471
v	-19.69	-2.35	2.33	1.60	5.26	16.83	6.16	0.26	1.9	192
vz	-20.48	-2.83	0.52	0.88	4.88	37.61	6.18	0.14	1.5	471
wmt	-27.06	-2.43	1.22	1.35	5.53	26.59	6.47	0.21	1.7	471

2 Unbiased Out-of-sample Calculations

Most authors define their out-of-sample from the last few periods of the data. Since any such out-of-sample time series is sensitive to the peculiar characteristics of the last few periods, calculations using them can be biased. For example, if the out-of-sample (**oos**) series coincides with the 2020 pandemic, the calculations will have a pessimistic bias.

Vinod (2023) suggests removing the bias by “pandemic proofing” the calculations on (default= 5%) randomly chosen ‘oos’ data. Each j -th choice yields a ranking of stocks. Repeating the ranking N (=50, say) times, we compute their mean μ_i and standard deviation σ_i for the i -th stock-picking algorithm. Vinod (2023) computes a zero-cost arbitrage, where the following trades are executed. One short-sells (selling without first possessing) certain dollars worth of the worst stock in DJIA and buys the appropriate fraction of the best stock as determined by each method.

Our unbiased ‘oos’ strategy here does not seek a zero-cost arbitrage. Instead, we just compute distinct stock rankings by each method in-sample and randomly choose 40% observations for each j -th ‘oos.’

3 Ranking 30 stocks by six algorithms

We report the ranking of 30 DJIA stocks with reference to six stock-picking algorithms. We split the ranking report into three tables. Each table has ten stocks at a time in alphabetical order of their ticker symbols. See Tables 5 to 7, where we report two versions of the ranks produced by each algorithm for (a) **in-sample** ranks and (b) unbiased **out-of-sample** ranks.

1. **Sharpe-in/out:** Section 1.1.1 mentions the Sharpe ratio as a stock-picking algorithm. Sharpe (1966) devised the ratio of mean return to standard deviation of returns to represent risk-adjusted return. See also Vinod and Morey (2000) and Vinod and Morey (2001). Some adjustments in comparing negative returns, practical limitations, and extensions to allow for “downside” standard deviation as a better measure of risk are discussed in Vinod and Reagle (2005).
2. **Omega-in/out:**
Our computation is described in section 1.1.2, and equation (4) for Ω_{sum} .
3. **Decile-in/out:** It is generally agreed that the stock whose probability distribution of returns is more to the right-hand side is more desirable. One way to do this is to compare their deciles. The R package ‘generalCorr,’ Vinod (2021), offers a convenient function called `decileVote(.)`.
4. **Descr-in/out:** We compare the traditional descriptive stats of each stock’s data. Most stats are in the “the larger, the better” category and get (+1) as weight. The standard deviation represents risk and gets (-1) weight. This algorithm uses a weighted summary of these stats for stock-picking.
5. **Momen-in/out:** Moment values: The first four moments of a probability distribution provide information about centering,

variability, skewness, and kurtosis. Our weights incorporate the prior knowledge that low variability and low kurtosis are desirable, while larger mean and skewness are desirable. A weighted summary is implemented in the R package ‘generalCorr,’ function called `momentVote(.)`.

6. **Exact-in/out:** This algorithm refers to the exact stochastic dominance mentioned in section 3.1. The stochastic dominance (SD) of the first four orders is summarized in the R package ‘generalCorr.’ See the R function called `exactSd()`. The theoretical details are available in Vinod (2024), where iterated integrals of cumulative distribution functions are used. See the next subsection, 3.1, for a summary of general ideas behind the theory of stochastic dominance.

3.1 Ranking stocks by exact stochastic dominance

We apply some of the tools described in Vinod (2024) to the portfolio selection problem for the DJ30 data set used here. We use exact computation of stochastic dominance using an imaginary stock (`x.ref`) as worse than the worst performing stock in DJ30.

We shall see that stochastic dominance needs a reference stock. Accordingly, we plan to compute the return for each of the 30 DJIA stocks with reference to the return in excess of the money-losing imaginary 31-st stock (`x.ref`). The lowest return over all included DJ30 data set over all 30 stocks is -86.14151 , where the negative sign suggests a loss. Let us choose (`x.ref`) return -87.38904 , which is a little smaller than -86.14151 , implying consistently the largest losses throughout the data period. Thus, all 30 stocks in our data always dominate (`x.ref`) throughout the period with varying dominance amounts.

The exact stochastic dominance computation invented by Vinod (2024) measures the dominance of each one of the thirty DJ30 stocks over the (`x.ref`) imaginary stock. The thirty dominating amounts are comparable to each other and allow the ranking of the thirty stocks. The computation of dominating amounts depends on the order k of stochastic dominance (SD k).

The first-order computation of the dominating amount depends on the exact area between two empirical cumulative distribution functions (ECDFs). It is customary to use iterated integrals for higher-order computation of dominating areas since Levy (1973). The R package ‘generalCorr’ computes

dominating areas for SD1 to SD4 and summarizes their rankings.

Tables 5 to 7 report ranks of stock tickers named in the column heading. The stocks ranked 1 to 8 (say) are worth buying according to the algorithm implied by the row name. On the other hand, stocks ranked 23 to 30 are worth selling. Interestingly, stocks recommended for buying using in-sample data do not generally agree with the unbiased out-of-sample averages, even for the same algorithm.

Table 5: All criteria summary ranks of DJ30 stocks for in-sample and average over randomized out-of-sample returns part 1

	aapl	amgn	amzn	axp	ba	cat	crm	cSCO	cvx	dis
Sharpe-in	16.0	9.0	7.0	21.0	27.0	18.0	5.0	11.0	17.0	19.0
Sharpe-out	30.0	19.0	29.0	21.0	17.0	23.0	15.0	8.0	12.0	26.0
Omega-in	15.0	5.0	2.0	18.0	27.0	19.0	7.0	10.0	17.0	20.0
Omega-out	30.0	17.0	29.0	21.0	19.0	22.0	16.0	8.0	12.0	26.0
Decile-in	9.5	14.0	5.5	8.0	20.0	12.0	9.5	5.5	27.0	26.0
Decile-out	21.5	8.0	14.0	9.5	18.0	7.0	29.0	6.0	26.0	28.0
Descr-in	14.9	16.3	12.7	12.4	17.1	14.0	14.1	14.9	17.3	17.9
Descr-out	20.7	15.1	16.3	16.4	12.7	15.9	14.7	15.0	16.3	19.3
Momen-in	7.0	4.0	1.0	10.0	24.0	12.0	3.0	6.0	16.0	15.0
Momen-out	23.0	6.0	15.5	19.0	7.0	20.0	3.5	1.0	18.0	28.0
Exact-in	8.0	10.0	5.0	17.0	24.0	14.0	4.0	6.0	25.0	19.0
Exact-out	26.0	11.0	5.0	19.0	13.0	18.0	2.0	6.0	21.0	27.0
AvgRank	19.0	7.0	8.0	15.0	22.0	16.0	6.0	3.0	20.0	29.0

Table 8 reports abridged (case-sensitive single character) names of the top eight stocks for buying (ranked 1 to 8) and the bottom eight for selling (ranked 23 to 30) by each of our six algorithms. Algorithm names are listed in Section 3. The algorithm names also appear as row names in Tables 5 to 7. To save table space, we need to abridge the row names to only three characters. Column names are lowercase versions of the stock ticker symbols. The last row is named AvgRank refers to the average rank from all six criteria listed above.

Table 6: All criteria summary ranks of DJ30 stocks for in-sample and average over randomized out-of-sample returns part 2

	dow	gs	hd	hon	ibm	intc	jnj	jpm	ko	mcd
Sharpe-in	30.0	28.0	3.0	22.0	29.0	24.0	6.0	25.0	13.0	8.0
Sharpe-out	1.0	22.0	6.0	7.0	28.0	25.0	5.0	11.0	4.0	18.0
Omega-in	30.0	28.0	6.0	22.0	29.0	25.0	9.0	24.0	12.0	11.0
Omega-out	1.0	23.0	6.0	7.0	28.0	25.0	5.0	9.0	4.0	20.0
Decile-in	20.0	28.0	7.0	15.5	30.0	22.5	20.0	24.0	15.5	11.0
Decile-out	1.0	14.0	4.0	11.5	30.0	19.5	11.5	3.0	16.0	27.0
Descr-in	17.9	19.0	13.0	16.3	19.6	18.0	14.6	18.1	14.7	14.0
Descr-out	7.9	15.3	13.7	15.7	18.9	16.4	13.6	12.7	14.9	17.7
Momen-in	30.0	25.0	9.0	19.5	28.0	11.0	19.5	17.0	26.0	21.0
Momen-out	9.0	17.0	8.0	10.0	26.0	12.5	21.0	5.0	14.0	29.0
Exact-in	1.0	3.0	12.0	26.0	30.0	13.0	21.0	18.0	22.0	20.0
Exact-out	1.0	4.0	10.0	12.0	30.0	16.0	17.0	9.0	15.0	28.0
AvgRank	9.0	23.0	4.0	14.0	30.0	24.0	11.0	13.0	12.0	21.0

Table 7: All criteria summary rank of DJ30 stocks part 3

	mmm	mrk	msft	nke	pg	trv	unh	v	vz	wmt
Sharpe-in	20.0	15.0	2.0	14.0	10.0	23.0	4.0	1.0	26.0	12.0
Sharpe-out	9.0	14.0	16.0	27.0	3.0	20.0	13.0	2.0	10.0	24.0
Omega-in	21.0	16.0	1.0	14.0	8.0	23.0	4.0	3.0	26.0	13.0
Omega-out	10.0	18.0	14.0	27.0	3.0	15.0	13.0	2.0	11.0	24.0
Decile-in	25.0	18.0	4.0	2.0	17.0	22.5	1.0	3.0	29.0	13.0
Decile-out	17.0	23.0	24.0	19.5	9.5	14.0	2.0	5.0	21.5	25.0
Descr-in	18.6	16.3	11.7	13.7	16.6	16.7	11.6	11.1	17.4	14.6
Descr-out	15.3	17.9	15.9	19.7	12.1	18.7	13.0	10.7	13.4	19.1
Momen-in	29.0	18.0	2.0	8.0	22.5	22.5	5.0	13.0	27.0	14.0
Momen-out	22.0	25.0	2.0	27.0	15.5	24.0	3.5	11.0	12.5	30.0
Exact-in	28.0	16.0	7.0	11.0	23.0	27.0	9.0	2.0	29.0	15.0
Exact-out	22.0	24.0	7.0	25.0	14.0	23.0	8.0	3.0	20.0	29.0
AvgRank	26.0	18.0	5.0	17.0	10.0	28.0	2.0	1.0	27.0	25.0

Table 8: Single character names of top eight stocks for buying and bottom eight for selling by each of our six algorithms. Algorithm names from earlier tables are abridged to only three characters (suffix i= in sample, o= out-of-sample). Column names are ranks by the criterion named along the row.

	1	2	3	4	5	6	7	8	23	24	25	26	27	28	29	30
Shi	v	p	S	w	k	n	K	a	h	D	u	C	j	z	m	A
Sho	D	V	J	e	u	K	C	f	p	a	k	j	h	H	S	M
Omi	f	S	m	k	w	n	a	K	v	D	u	A	h	C	p	j
Omo	D	M	V	e	u	f	t	C	p	a	k	j	h	H	S	J
Dci	u	m	c	w	A	H	k	p	i	f	z	S	h	x	a	C
Dco	D	H	j	z	g	t	k	M	i	h	v	S	c	A	x	p
Dsi	v	u	f	x	z	h	n	c	V	d	D	I	J	M	g	i
Dso	v	p	b	J	u	V	j	h	K	t	i	w	d	n	a	D
Moi	z	x	I	c	v	w	d	e	C	D	A	u	S	a	n	h
Moo	S	H	v	I	V	k	z	p	C	u	w	J	A	b	h	D
Exi	D	A	n	h	I	c	w	K	g	i	C	z	S	f	a	u
Exo	D	h	A	H	b	p	k	I	v	i	g	z	S	f	u	J
avg	v	p	j	k	J	H	x	c	S	i	h	f	C	A	z	D

Table 9: Unabridged ticker symbols of the top two stocks for buying and the bottom two for selling by each of our six algorithms. Row names are algorithm names, as in earlier tables. Column names are ranks.

Criterion	1	2	29	30
Sharpe-in	v	pg	mcd	amgn
Sharpe-out	dow	vz	csc	mmm
Omega-in	msft	csc	pg	jnj
Omega-out	dow	mmm	csc	jpm
Decile-in	unh	mcd	aapl	crm
Decile-out	dow	hon	axp	pg
Descr-in	v	unh	gs	ibm
Descr-out	v	pg	aapl	dow
Momen-in	amzn	axp	nke	hd
Momen-out	csc	hon	hd	dow
Exact-in	dow	amgn	aapl	unh
Exact-out	dow	hd	unh	jpm
AvgRank	v	pg	amzn	dow

4 Final Remarks

This paper describes six stock-picking algorithms for long-term investment in the DJ30 stocks. Our implementations of omega (gain-to-pain ratio) and exact stochastic dominance appear to be new. We use monthly return data for the recent 39+ years to find that each algorithm leads to a distinct ranking. We report the ranks by each criterion, implying that rank 1 is the top stock worthy of buying and rank 30 is the bottom stock worthy of selling. As we change the time periods (e.g., quarters, months, weeks, hours, etc.) included in the selected DJ30 data sets, the entire analysis will change, and our data-driven buy-sell recommendations are also expected to change. For example, Table 8 lists the top eight one-character abbreviations of ticker symbols to buy or sell. Table 9 lists the top two ticker symbols for stocks to buy and sell.

Our research shows that the ultimate choice of stock tickers to buy or sell in suitable quantities within one's own budget is possible for anyone. Long-term investors need price data for long time intervals. It helps to compare many stock-picking algorithms along the lines shown here using a clear statement of the algorithms. We find that even for the same algorithm, the in-sample and unbiased out-of-sample ranks rarely coincide.

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