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Hyperbolic Utility Consumption-CAPM with Time Variation

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Dedication

To My Late Parents:

Soon-Hwa Hong and Yon-Gu Shim

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Appendix

1. Manipulation of ρ

$$\rho = \frac{1}{1+e^{\delta}} = \frac{1}{1+e^{\ln\frac{D_t}{P_t}}} = \frac{1}{1+\frac{D_t}{P_t}} = \frac{P_t}{P_t+D_t}$$

ρ is defined as $\rho = P^*/(P^* + D^*)$, (27)

which is equivalent in SS, $\rho = P_t/(P_t + D_t)$ (28)

Multiplying and dividing by P_{t-1}, $\rho = (P_t/P_{t-1})[P_{t-1}/(P_t + D_t)]$

Then, $\rho = (Pt/Pt\text{-}1)(Pt\text{-}1/\{Pt+Dt\}) = e^{\log(Pt/Pt-1)(Pt-1/\{Pt+Dt\})} = e^{\log(Pt/Pt-1) + \log(Pt-1/\{Pt+Dt\})}$

$= e^{g + \log(1/\{1+R\})} = e^{g+(-1)\log(1+R)} = e^{g-r}$ (Since Pt/Pt-1 = Dt/Dt-1 in SS)

Dividends and prices growing at the same time implies that $\log(P_t/P_{t-1}) = g$ (29)

Using (31), (24), and (23) $\rho = \exp(g - r)$ (30)

2. Manipulation of k

$$k = \ln(P^*+D^*) - \rho\ln P^* - (1-\rho)\ln D^*$$
$$= \ln(P^* \times \frac{P^*+D^*}{P^*}) - \rho\ln P^* - (1-\rho)\ln D^*$$
$$= \ln(P^* \times \rho^{-1}) - \rho\ln P^* - (1-\rho)\ln D^*$$
$$= (\ln P^* - \ln\rho) - \rho\ln P^* - (1-\rho)\ln D^*$$
$$= -(1-\rho)(\ln D^* - \ln P^*) - \ln\rho$$
$$= -(1-\rho)\delta - \ln\rho$$

$k = -(1 - \rho)\delta - \log\rho$ (31)

Dividing each term on RHS of (5) by P_t,

$$\rho = \frac{1}{[1 + D_t/P_t]}$$

In SS, D_t/P_t and $\log(D_t/P_t)$ are constants. The latter is in fact δ, so that (52) becomes

$$\rho = \frac{1}{[1 + \exp(\delta)]}$$ [1]

Inverted to give δ in terms of ρ, $\delta = \log[(1/\rho) - 1]$

Therefore, $k = -(1 - \rho)\log[(1/\rho) - 1] - \log\rho$ (32)

[1] Also, refer to (22).

3. a. Dividend-Price Model in Steady State

Using (27) to substitute for k in (34), [2]

$$\delta = \frac{(r-g) - [-(1-\rho)\log(1/\rho - 1) - \log\rho]}{(1-\rho)} \quad (33)$$

Since from (24), $\rho = \exp(g - r)$,

$$\delta = \frac{(r-g) + [1-\exp(g-r)]\log[1/\exp(g-r) - 1] + (g-r)}{[1-\exp(g-r)]}$$

$$\delta = \log[1/\exp(g-r) - 1] \quad (34)$$

Dividend-price ratio is then, $Dt/Pt = 1/\exp(g-r) - 1$
$= 1/(1+G)/(1+R) - 1$
$= (R-G)/(1+G)$

So, $D_{t+1}/P_t = R - G$ (35)

b. Dividend-Price Model in Dynamic State

Also, $\delta_t = \sum_{j=0}^{\infty} \rho^j E_t(r_{t+j} - \Delta d_{t+j}) - k/(1-\rho)$ (11)[3]

where $h_t = \log(1 + R_t) = \log[(P_t + D_t)/P_{t-1}]$ (36)

$\Delta d_t = \log(1 + G_t) = \log(D_t/D_{t-1}) = g$ (37)

$\delta_t \equiv d_t - p_t$ (38)

In steady state, P_t and D_t grow at the same constant rate, so from (31), (32), and (33), r_t, Δd_t (or g), and δ_t are all constant, and denoted r, g, and δ respectively.

Therefore, (31) becomes $\delta = \sum_{j=0}^{\infty} \rho^j (r-g) - [k/(1-\rho)]$

or $\delta = [(r-g) - k]/(1-\rho)$ (39)

since $0<\rho<1$ and by infinite geometric series $\sum_{j=0}^{\infty} \rho^j (r-g) = (r-g)/(1-\rho)$.

Rewriting k in terms of ρ, and ρ in terms of r and g in (44) will produce

$D_{t+1}/P_t = R - G.$ (40)

Rewriting (39) for r would give $\begin{matrix} r = \delta - \rho\delta + g + k = \xi \\ h \approx \xi = \delta - \rho\delta + g + k \end{matrix}$.

4. Gordon Growth Model Revisited

---

[2] Also refer to (4) and (7)

[3] Also, refer to (28) ~ (30)

$$P_t = E_t \frac{\sum_{i=1}^{\infty} D_{t+i}}{\prod_{j=1}^{i}(1+R_{t+j})}, \text{ where } R_t \equiv [(P_t - P_{t-1}) + D_t] / P_{t-1}$$

$$= E_t \sum_{i=1}^{\infty} \prod_{j=1}^{\infty} \frac{1+G_{t+i}}{1+R_{t+j}} D_t = \sum_{i=1}^{\infty} [(1+G)/(1+R)]^i D_t, \text{ if G and R are constant.}$$

if div growth and real returns are constant, and equal to G and R respectively.

Since $\sum_{i=1}^{\infty} [(1+G)/(1+R)]^i = (1+G)/(1+R) \sum_{i=0}^{\infty} [(1+G)/(1+R)]^i$

$= (1+G)/(1+R) \left[\dfrac{1}{1 - \dfrac{(1+G)}{(1+R)}}\right]$ if $0<G\&R<1$, by infinite geometric series

$= [(1+G)/(1+R)]\,[(1+R)/(R-G)]$

$= (1+G)/(R-G)$

Then, $P_t = [(1+G)/(R-G)]D_t$ or $D_{t+1}/P_t = R-G$

5. Linearization of HARA U-fn

$$E(g_t) = E(\beta R_t c_t^{-\gamma}) = 1$$

$$\log E(g_t) = \log(1) = 0$$

$$E\log(g_t) = E(u_t)$$

$$Je = E\log(g_t) - \log E(g_t) = E\log(g_t) = E\log R_t + E\log Z_t = 0$$

$$E\log R_t = -E\log Z_t = -E(\log\beta - \gamma\log c_t) = -\log\beta + \gamma\log c_t + u_t$$

$$where \quad \log R_t = -\log\beta + \gamma\log\left[\frac{AC_t + B}{AC_{t-1} + B}\right] + u_t$$

$$Let \quad \phi = \frac{B}{A} \quad in\ HARA \quad \chi_t = \frac{AC_t + B}{AC_{t-1} + B}$$

$$Then, \quad \chi_t = \left[A\left(\frac{C_t}{C_{t-1}}\right) + \frac{B}{C_t}\right] A^{-1}\left[1 + \phi\left(\frac{1}{C_{t-1}}\right)\right]^{-1} = \left[\frac{C_t}{C_{t-1}} + \frac{\delta}{C_{t-1}}\right] \times \left[1 + \phi\left(\frac{1}{C_{t-1}}\right)\right]^{-1}$$

$$Taking \log s, \quad \log\chi_t = \log\left[\frac{C_t}{C_{t-1}} + \left(\frac{C_t}{C_t}\right) \times \left(\frac{\phi}{C_{t-1}}\right)\right] - \log\left[1 + \phi\left(\frac{1}{C_{t-1}}\right)\right] \quad \cdots$$

6. The Logic of Algebraic Signs

The model also makes intuitive sense in terms of algebraic signs because if $D_t = C_t$ in equilibrium according to Lucas Tree Model, then, if D_t goes down, C_t goes down as well. This will lead to an increase in R_{t+1}, because D_t or C_t will be reinvested. Therefore, a decrease in δ_t will cause h_{t+1} to increase and Δd_{t+1} as well. Δd_{t+1} will also increase because if D_t falls, D_{t+1} will be relatively higher regardless of the level of pay-out. However, in an incomplete substitution between D_t and C_t, D_t and C_t are in a complementary relation, so a drop in C_t would mean an increase in D_t and an increase in R_{t+1}. Then, the following should be the correct signs: $h_{t+1} = \delta_t - \rho\delta_{t+1} + \Delta d_{t+1} - \Delta c_{t+1}$. δ_{t+1} has negative sign because if D_{t+1} is low, h_{t+2} will be high, but for h_{t+2} to be high h_{t+1} will have to be relatively low.

7. Sample Programs used in Regression

a. SAS

```
filename chev 'a:\companies\chevron(chv)\chev.prn';
data chev;
infile chev;
input  quarter rit    delta1  delta   chgdivt           rmt       invcons;
proc print data=chev;
proc reg;
model rit = delta1     delta      chgdivt     rmt       invcons;
run;
```

b. Gauss

```
new;
xx={r,c};
xx;
n=rows(xx);
" no. of observations=n= " n;
x=ones(119,1)~xx[.,2:6];
" matrix of regressors ";
x;
bigx=x;
k=cols(x);
"k=cols(x)= no of regressors including one for intercept" k;
y=xx[.,1];
```

```
" vector of dep variable data ";
y;
xtx=x'x;
"xtx";
xtx;
invxtx=invpd(xtx);
"invxtx";
invxtx;
xty=x'y;
"xty";
xty;
b=invxtx* xty;
"b";
b;
resid=y-x*b;
"resid";
resid;
"residual sum of squares =rss=resid'resid";
rss=resid'resid;
rss;
df=n-k;
"df=degrees of freedom= n-k" df;
resms=rss/df;
"est of sigma-sq= residual mean square = rss/df " resms;
" standard error of regression " sqrt(resms);
ybar=meanc(y);
"ybar=meanc(y)" ybar;
TotSS= (y-ybar*ones(n,1))'(y-ybar*ones(n,1));
" total sum of sq= (y-ybar*ones(n,1))'(y-ybar*ones(n,1))" totss;
" R-sq  by first method = 1-(rss/totss)";
rsq=1-(rss/totss);rsq;
"predicted values";
pred=x*b;pred;
ypbar=meanc(pred);
"ypbar=meanc(pred)" ypbar;
RegSS= (pred-ypbar*ones(n,1))'(pred-ypbar*ones(n,1));
"Regr sum of sq" regss;
```

```
" R-sq=  RegrSS/totalSS by 2nd method " regss/totss;
AdjRsq= 1-(1-rsq)*(n-1)/(n-k);
" Adjusted R-sq= 1-(1-rsq)*(n-1)/(n-k)" Adjrsq;
" F value = [(regss/(k-1)]/[rss/(n-k)]";
Fval = (regss/(k-1))/(rss/(n-k));fval;
"  ";
"Akaike Information criterion AIC=ln(residual-mean-sq) + 2*k/n";
aic=ln(resms) + (2*k)/n;aic;
"Amemiya prediction criterion AmPC=(residual-mean-sq)(1 + k/n)";
ampc=(resms)*(1 + (k/n));ampc;
"  ";
   { vnam,mean,var,std,min,max,valid,mis } = DSTAT(0,xx);
" ";
__con=1;
_olsres=1;
 {vnam,mom,bols,stb,vc,stderrb,sigmares,corrxy,rsqols,residols,dwols} =
                                    OLS(0,y,bigx);
" ";
varcovb=resms*invxtx;
"varcovb=resms*invxtx";
varcovb;
" ";

"Now standard errors of regr coeff's got directly from cov mtx of b";
se1=sqrt(varcovb[1,1]);
"se1=sqrt(varcovb[1,1])" se1;
se2=sqrt(varcovb[2,2]);
"se2=sqrt(varcovb[2,2])" se2;
se3=sqrt(varcovb[3,3]);
"se3=sqrt(varcovb[3,3])" se3;
se4=sqrt(varcovb[4,4]);
"se4=sqrt(varcovb[4,4])" se4;
se5=sqrt(varcovb[5,5]);
"se5=sqrt(varcovb[5,5])" se5;
se6=sqrt(varcovb[6,6]);
"se6=sqrt(varcovb[6,6])" se6;
```

```
"Now t-values as ratios of regr coeff to se's ";
t1=b[1]/se1;t1;
t2=b[2]/se2;t2;
t3=b[3]/se3;t3;
t4=b[4]/se4;t4;
t5=b[5]/se5;t5;
t6=b[6]/se6;t6;
end;
```

c. ARIMA by SAS

```
filename chev 'd:\jeffx\chevron(chv)\chev.prn';
data chev;
infile chev;
input quarter rit    delta1  delta   chgdivt          rmt       invcons;
proc arima data=chev;
i var=rit nlag=28;
run;
e p=2 q=4;
run;
/*lead=12 is also arbitrarily chosen.*/
proc autoreg data=chev;
model rit=/nlag=1 godfrey;
run;
```

d. Eigenvalue/Eigenvector Decomposition by Gauss

```
new;
x={r*c};
{va,ve}=eighv(x);
va;
ve;
```

e. Ridge Regression by Gauss

```
n=119; m=5; load x[n,m]=c:\gauss\lib\chevx.txt;
x;
n=119; m=1; load y[n,m]=c:\gauss\lib\chevy.txt;
y;
{a,b}=hkbridge(x,y);
```

```
a;
b;
proc (2) = hkbridge(x,y);
            local t, k, xs, xc, ssq, std, corr, yc, bc,
     sighat2, khat, bridge, q, covridge,
     w, stderr, crit, kold, iter;
        t = rows(x);                /* Standardize X and y */
        k = cols(x);
        xs = x[.,.];
        xc = xs - meanc(xs)';
        ssq = diag(xc'xc);
        std = sqrt(ssq);
        xc = xc ./ std';       /*Element by Element Operation*/
        corr = xc'xc;
        yc = y - meanc(y);
        bc = yc/xc;                     /*    OLS     */
        sighat2 = (yc - xc*bc)'(yc - xc*bc)/(t-k-1);
        khat = k*sighat2 ./ (bc'bc);
        bridge = invpd(corr + khat .* eye(k)) * xc'yc;
        q = invpd(corr + khat .* eye(k));
        covridge = sighat2 .* q * corr * q;
        w = diagrv(eye(k),1 ./ std);               /*    W-inv    */
        bridge = w*bridge;            /*Unstandardize */
        covridge = w*covridge*w;
        stderr = sqrt(diag(covridge));
        "HKB-noniterative Ridge Estimator";            /*    Print     */
        ?;
        " khat       : " khat;
        " Est        : " bridge';
        " Std. Err.  : " stderr';
        crit = 1;                           /* define constants */
        kold = khat;
        iter = 1;
        do until (crit le 1e-6) or (iter ge 20);    /* begin loop       */
        khat = (k) * sighat2 ./ (bridge'bridge);
        crit = abs(khat - kold);
        bridge = invpd(corr + khat .* eye(k)) * xc'yc;
```

```
kold = khat;
iter = iter + 1;
endo;
q = invpd(corr + khat .* eye(k));
covridge = sighat2 .* q * corr * q;
w = diagrv(eye(k),1 ./ std);            /* Unstandardize  */
bridge = w*bridge;
covridge = w*covridge*w;
stderr = sqrt(diag(covridge));
?;
"HKB-iterative Ridge Estimator";         /*   Print      */
?;
" khat       : " khat;
" Est        : " bridge';
" Std. Err.  : " stderr';
retp(bridge,covridge);
endp;
```

8. Charts

I. Statement of the Problem

In modern finance it seems as if there is a prevailing tendency to disregard the utility function when constructing an asset-pricing model. Whether the utility function is relevant or not in the model has become the dividing line between finance and economics. Even in economics, it has also been shown that the standard capital asset pricing model (CAPM hereafter) approximates asset pricing sufficiently, when the marginal utility of consumption is highly correlated with the return on the stock market[1]. Some theories, such as the Lucasian tree model, contend that consumption is eventually replaced by dividends in equilibrium. Although attempts have been made in the past to bridge this gap between economics and finance, I believe the relevance of the utility function must be dealt with first, prior to pursuing further with any type of real asset pricing model.

This paper is motivated by the idea that hyperbolic absolute risk aversion (HARA hereafter) based time-varying asset pricing model might be an alternative solution to close this gap between consumption CAPM (CCAPM hereafter) and the actual financial market, that several studies done in the past attempted unsuccessfully. First, Hall & Flavin, in their studies on consumption sensitivity puzzle, contended that U.S. consumption is too sensitive to changes in income. However, it was not the real puzzle at all since the actual consumption does not track the income process as in their studies. Assuming that consumption is sensitive to income as in their studies, CCAPM would seem to work well. However, they failed to recognize the random walk possibility in modeling the process of permanent income.

Second, Mehra & Prescott found that U.S. consumption is too smooth to explain the observed actual risk premium. In their paper about asset pricing puzzle, they argued that higher risk aversion parameter must be used to replicate the volatility of the stock market. All these studies served only as a detriment to the credibility of CCAPM, but they did not explain why consumption tends to be so smooth. It is, of course, a widely accepted norm in economics that consumption is not sensitive to changes in transient income.

The purpose of this dissertation, therefore, is to develop a new consumption-based real asset pricing model – one that is straightforward and rigorous in modeling technique, yet simple and easy to implement. And the model has to be able to replicate the actual asset returns more closely within the reasonable range of risk aversion. Modeling it as

---

[1] Blanchard, O.J. & Fischer, S., *Lectures on Macroeconomics*, MIT Press, 1996, pp507~510

"consumption-based" and "real" inevitably involves utility function. And involving utility function in the model generally entails the assumption of constant relative risk aversion (CRRA hereafter) in the representative individual's risk aversion mechanism. I, however, would propose to assume HARA, because i) it is a more comprehensive specification of the risk aversion mechanism.; ii) **there has not been sufficient number of known studies using HARA utility function in the asset pricing model**. Therefore, it would be a worthy effort to examine its possibilities The need for assuming HARA will be more evident as we proceed further.

In this respect, the HARA assumption seems to gain its ground as part of the core crux of the model. If the model proves to be effective and good, it will solidify the model's foundation on the utility function through the legitimate risk aversion mechanism. Diagnosis of the risk aversion parameter specified by the assumption will test this, because the utility function in the model is justified if the model proves to have high predicting power.

Once the model derivation is complete, I will proceed with testing of the model. The data to be used will consist of the time-series of 10~15 stocks selected according to the **Dividend Yield Strategy**. [2] The strategy is simple: once each year, adjust your portfolio so you own only the 10 highest yielding stocks in the Dow Jones Industrial Average. These 10 reportedly do better than the market during the down market and at least as well as the market during the up market. Therefore, these stocks can be said to rely heavily on the strategy that maximizes dividend-yield ratios. Using the dividend price ratio as a regressor thus gains a strong empirical rationale as well, apart from the solid theoretical basis to be seen later.

This will also enable us to construct a simple portfolio consisting of one share each of these stocks, and compare the portfolio's performance vis-à-vis the performance of the market portfolio/index to verify the validity of the strategy. These data are readily available for download from various websites such as Yahoo Finance.

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[2] This strategy is popularly dubbed "**Dogs of the Dow**" in the language of the financial market. There is a number of on-line resources for the Dogs of the Dow rationale. For more details refer to http://stocks.about.com/money/stocks/library/.

Again, once the model is tested, its forecasting or explanatory power can be checked with out-of-sample testing. Since the model is of a structural partial equilibrium type, the regressor data needed for *ex-ante* forecasting will have to be contemporaneous with the regressand. Therefore, it will be inevitable for an *ex-ante* forecasting to obtain futures data, which is still feasible, but not necessarily an essential task, when we can achieve the same objective with existing actual data through out-of-sample forecasting. If this model obtains significant results, it will support the validity of HARA-based asset pricing, and increase efficiency and simplicity of the time-varying asset pricing model. [3]

The next point to check is whether the new model has any advantage over the existing models. Certainly, there should be a gain either in the simplicity and ease in implementation of the model, or in the forecasting/explanatory power of the model. For instance, it can be compared with a pure data generating process (DGP hereafter) such as autoregressive integrated moving average (ARIMA hereafter) technique. We will find out if this hybrid structural model would perform better than pure time series forecasting models such as ARIMA in the short term forecasting – *i.e.* monthly or quarterly returns over short-run forecasting horizon under a year.

As mentioned previously, the new model is developed to provide a simpler and easier alternative to the existing models and this will be examined in the upcoming chapters. The out-of-sample forecasting power will also be checked. The data range to be used is mid to long term (10~30 years), so the comparison with another model will also have to be for the same time range and the same forecasting horizon. [4]

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[3] This will be tested by RMSE of out-of-sample forecasts of the competing models.

[4] For a general overview of the organization of this paper, a schematics chart is provided in Fig 1.

II. Literature Review

Consumption CAPM is derived from the Euler Equation or the first order condition of the household utility maximization problem. Merton (1973)[5] showed under what conditions the standard CAPM formulas could be derived in continuous time from intertemporal optimization of consumers over portfolio choices and consumption.[6]

Consumption CAPM is derived as follows. From the first order condition of the house hold problem we can define

return on risky asset as $U'(C_t) = \beta E_t U'(C_{t+1}) R_{it+1}$ (i)

and return on riskless asset as $U'(C_t) = \beta E_t U'(C_{t+1}) R_{ft}$ (ii)

Then, the risk premium is $0 = E_t\{U'(C_{t+1})[R_{it+1} - R_{ft}]\}$ by subtracting (ii) from (i)

$$= E_t U'(C_{t+1}) E_t [R_{it} - R_{ft}] + \text{cov}\{U'(C_{t+1})[R_{it+1} - R_{ft}]\}$$

$$= E_t U'(C_{t+1}) E_t [R_{it+1} - R_{ft}] + \text{cov}\{U'(C_{t+1}) R_{it+1}\}$$ [7]

$$E_t [R_{it+1} - R_{ft}] = -\frac{\text{cov}[U'(C_{t+1}) R_{it+1}]}{E_t U'(C_{t+1})}. \quad \text{(iii)}$$

This implies that if R_{mt+1} increases, C_{t+1} increases as well, which means that $U'(C_{t+1})$ would go down as C_{t+1} increases. Then the $\text{cov}[U'(C_{t+1}) R_{it+1}] < 0$, which ensures that $E_t [R_{it+1} - R_{ft}] > 0$.

Now to see the parallelism to the standard CAPM, suppose a composite asset or the market, m, whose return is perfectly negatively correlated with $U'(C_{t+1})$ [*i.e.*, $U'(C_{t+1}) = -\gamma R_{mt+1}$ for some γ]. It follows for all risky assets,

$$\text{cov}[U'(C_{t+1}) R_{it+1}] = -\gamma \, \text{cov}(R_{mt+1} R_{it+1}), \quad \text{(iv)}$$

where for asset m $E_t[R_{mt+1}] = R_{ft} - \dfrac{\text{cov}[U'(C_{t+1}) R_{mt+1}]}{E_t[U'(C_{t+1})]} = R_{ft} + \dfrac{\gamma \, \text{var}(R_{mt+1})}{E_t[U'(C_{t+1})]}$. (v)

By substituting (iv) and (v) into (iii) we obtain

$$E_t[R_{it+1}] - R_{ft} = \frac{\text{cov}[R_{it+1} R_{mt+1}]}{\text{var}[R_{mt+1}]} [E_t[R_{mt+1}] - R_{ft}] \quad \text{(vi)}$$

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[5] Merton, R.C., An Intertemporal Capital Asset Pricing Model, *Econometrica* 41, 1973, pp. 867~887

[6] Blanchard, O.J. & Fischer, S., *Lectures on Macroeconomics*, MIT Press, 1996, pp. 507~510

or by defining β_{im} as $\frac{\text{cov}[R_{it+1}R_{mt+1}]}{\text{var}[R_{mt+1}]}$

$$E_t[R_{it+1}] - R_{ft} = \beta_{im}\left[E_t[R_{mt+1}] - R_{ft}\right]. \qquad \text{(vii)}$$

This would be the consumption CAPM mapped into the standard CAPM format. The coefficient β_{im} can be interpreted as a regression coefficient of R_{it+1} on R_{mt+1}.

If a stock covaries negatively with the market, it provides a hedge, and therefore, consumers will be happy to hold it at an expected return that is even lower than the riskless return.[8] The equilibrium relation in (vii) between the expected market return, the riskless return and the return on any asset does not involve the specification of preferences and risk aversion. Computing the beta of a security can be done using a simple regression. This would be an advantage of CCAPM mapped into standard CAPM format.

In general, derivation of security market line is based on a two-period model in which utility is either defined directly over the mean and variance of portfolio returns or assumed to be quadratic. However, this type of utility assumption is all too implicit and does not provide clear explanation as to how it can be assumed away. In essence, **the standard CAPM is considered to be a good approximation to asset pricing when the marginal utility of consumption is highly correlated with the return on the stock market**, or more generally the portfolio of tradable assets. **The presence of a large non-tradable asset such as human wealth is likely to decrease the correlation. In this case the consumption CAPM may do better.** [9]

In practice, however, the consumption CAPM appears to describe asset returns less accurately than the standard CAPM.[10] Therefore, there is a need for proposing to model the consumption CAPM based on HARA. One reason consumption CAPM does not work as well as our expectation may be found from the studies on asset pricing puzzle. It started

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[7] Since cov[RV_1, RV_2+Constant] = cov[RV_1, RV_2].

[8] Blanchard & Fischer, *Op. cit.*

[9] Blanchard & Fischer, *Op. cit.*

[10] Mankiw & Shapiro 1986

initially with the findings by Hall & Flavin.[11] They regressed C_{t+1} on C_t, Y_t, Y_{t+1}, Y_{t+2}, $\ldots Y_{t+i}$, where the variables are all in their usual notation, and found that $\frac{\sigma_c}{\sigma_y} \approx 1$, where σ_c *and* σ_y mean standard deviations of consumption and income respectively, and concluded that the consumption is sensitive to shock in income. However, they did not consider that permanent income could also be modeled after random walk in which case $\frac{\sigma_c}{\sigma_y} < 1$. Actual U.S. consumption data showed $\frac{\sigma_c}{\sigma_y} \approx 0.45$, which is not consistent with permanent income hypothesis. Then, the stock market volatility is not explained by this smoothness of consumption.

Now the real puzzle was that the stock market volatility cannot be explained by this smooth consumption. Hence, the excess smoothness argument. Again, this was an exhibit against the efficacy of consumption CAPM. Mehra & Prescott maintained that we need higher risk aversion to replicate the relatively high market return, which has been historically around 6.98% whereas the average real return on relatively risk free short-term securities has been about 0.8%.[12] The highest risk premium obtainable by the model was only about 0.35% far short of approximately 6% of the actual market. They experimented with varying values of risk aversion parameters from 2 to10. Now, **this gives rise to another reason for assumption of a better risk aversion mechanism as the existing CCAPM assumes CRRA, and using an overblown parameter greater than one certainly seems unnatural for any risk aversion mechanism.**

As for the modeling technique Campbell & Shiller's "Time-Varying Expected Returns" suggested a possibly good candidate that fit the category of my search. Campbell expounded the present-value relations in asset pricing to a great detail in the chapter 7 of

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[11] Flavin, M.A., The Excess Smoothness of Consumption: Identification and Estimation, *Review of Economic Studies* 60, 1993, pp. 651~666; Hall, R. & Mishkin, F.S., The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households, *Econometrica* 50, 1982, pp. 446~481

[12] Mehra, R. & Prescott, E.C., The Equity Premium – A Puzzle, *Journal of Monetary Economics* 15, 1985, pp. 145~161

Campbell, Lo & McKinley (1997)[13] (CLM hereafter). He examined the relation between prices, dividends and returns, and presented a technique to approximate the present-value relation with time-varying expected returns. This technique will be expounded further in the following chapter on theoretical foundations.

CLM, then, applied the present-value relations to examine the U.S. stock price behavior. Their empirical works included prediction of stock returns over long horizon using forecasting variables other than the past returns where dividend-price ratios and interest rate variables were used. Then, CLM related long-horizon return behavior to price behavior and stock price volatility in particular. They also applied this relation to VAR to show how time-series models could be used to calculate the long-horizon implications of short-horizon asset market behavior.

Until Campbell & Shiller's results were reported, it used to be thought that expected asset returns were approximately constant and movements in prices could be attributed to news about future cash payments to investors. Therefore, Campbell & Shiller's main contribution was that they brought due recognition to the importance of time-variation in expected returns and cast light upon implications for both academics and investment professionals.

Another technique was suggested from H. Vinod's HARA-based CCAPM estimation.[14] HARA is a relatively new concept that has been explored mostly in finance and econometrics. Vinod introduced loglinearization technique of CCAPM using small sigma asymptotics (SSA hereafter). He then estimated HARA parameter based on Godambe-Durbin estimating function. The estimation function (EF hereafter) is a relatively new concept adopted in econometrics. For more on EF, it is suggested to refer to Mittelhammer, Judge & Miller chapters 11 through 13.[15]

Godambe's robust pivot functions (GPF hereafter) are functions of data and parameters with a focus on the properties of functions rather than their roots. The roots of the

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[13] Campbell, Lo & McKinley, *The Econometrics of Financial Market*, Princeton, 1997, pp. 253~287.

[14] Vinod, H.D., "Concave Consumption, Euler Equation and Inference Using Estimating Functions", *Proceedings of Business and Economic Statistics section of American Statistical Association*, Alexandria, Virginia, 1997, pp. 118-123

[15] Mittelhammer, R., Judge, G. & Miller, D., *Econometric Foundation*, Cambridge University Press, 2000

equation GPF=0 are θ, where $\theta = (\tilde{\beta}, \kappa, \phi)$ and $y = (R_t, C_t)$ in the context of this dissertation.[16] He sought robust confidence interval for κ and ϕ by constructing double bootstrap confidence intervals for these parameters with U.S. data from 1960:1 to 1987:4 on consumption, stock prices and real dividend as in Vinod 1997. He reviewed old estimation of β and γ of the CRRA and found that generalized method of moments (GMM hereafter) point estimate of β did not make economic sense for the U.S. data.

The SSA-EF estimates are simpler and more meaningful for CRRA. Vinod showed that this result would also hold for HARA using AR(4) instrument and two pivots: Royall's pivot and GPF. Royall's pivot estimation rejected ϕ=0 and did not support κ>1. However, after correcting for the median bias and Cauchy distribution problems by double bootstrap GPF yielded economically meaningful results. Double bootstrap also rejected the H_0: ϕ=0 and B=0, and showed κ>1 indicating that HARA models are statistically significantly different from the traditional CRRA.

Incidentally, as for empirically evident AR terms, autoregressive distributed lag (ADL hereafter) modeling was also considered. The question of lag length can be resolved by *t*-values of the AR terms, AIC (Akaike information criterion), or by BIC (Schwartz criterion), but theses methods involve *ad-hoc* elements. So, I also considered Koyck modeling technique, which is a kind of partial adjustment model. The Koyck technique is very convenient and powerful for simplifying the lag determination as follows.[17]

$$E_t[R_{it+1}] = \alpha + \beta_0 R_{mt} + \beta_0 \lambda R_{mt-1} + \beta_0 \lambda^2 R_{mt-2} + \cdots + E_t[u_{t+1}] \qquad \text{(viii)}$$

where $\beta_k = \beta_0 \lambda^k, \quad 0 < \lambda < 1 \quad and \quad \sum_{k=0}^{\infty} \beta_k = \beta_0 \frac{1}{1-\lambda}$

λ = rate of decay of lags and u_t is *iid* (independently and identically distributed).

$$R_{it} = \alpha + \beta_0 R_{mt-1} + \beta_0 \lambda R_{mt-2} + \beta_0 \lambda^2 R_{mt-3} + \cdots + u_t \qquad \text{(ix)}$$

Multiply (ix) by λ.

$$\lambda R_{it} = \alpha\lambda + \beta_0 \lambda R_{mt-1} + \beta_0 \lambda^2 R_{mt-2} + \beta_0 \lambda^3 R_{mt-3} + \cdots + \lambda u_t \qquad \text{(x)}$$

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16 For more explanation on this model set-up, see section B of the chapter III of this dissertation.

17 Gujarati, D.N., *Basic Econometrics*, McGraw-Hill, 1995, 3rd ed., pp. 592~604

$E_t[R_{it+1}] = \alpha(1-\lambda) + \beta_0 R_{mt} + \lambda R_{it} - \lambda u_t$ by subtracting (x) from (viii)

As demonstrated, all the distributed lag terms are neatly captured in the AR(1) term. However, this is possible only under the restrictive condition that the rate of decay λ[18] is constant over time, which relies on the empirical strength of the assumption.

Still, Koyck modeling technique could suggest a good approximation for empirical standard CAPM, and with $\frac{U'(C_t)}{U'(C_{t-1})}$ to proxy R_{mt}, it may also serve as a good approximation for CCAPM as well. However, although the possibility for this technique was explored to a limited extent, it would have less direct relevance with the main focus of this paper, and thus, may as well be reserved for another project in the future. The schematics in the next page would help understand the flow of this dissertation.

[18] λ is a parameter that represents the speed at which the persistence or the influence of the lagged variable decays.

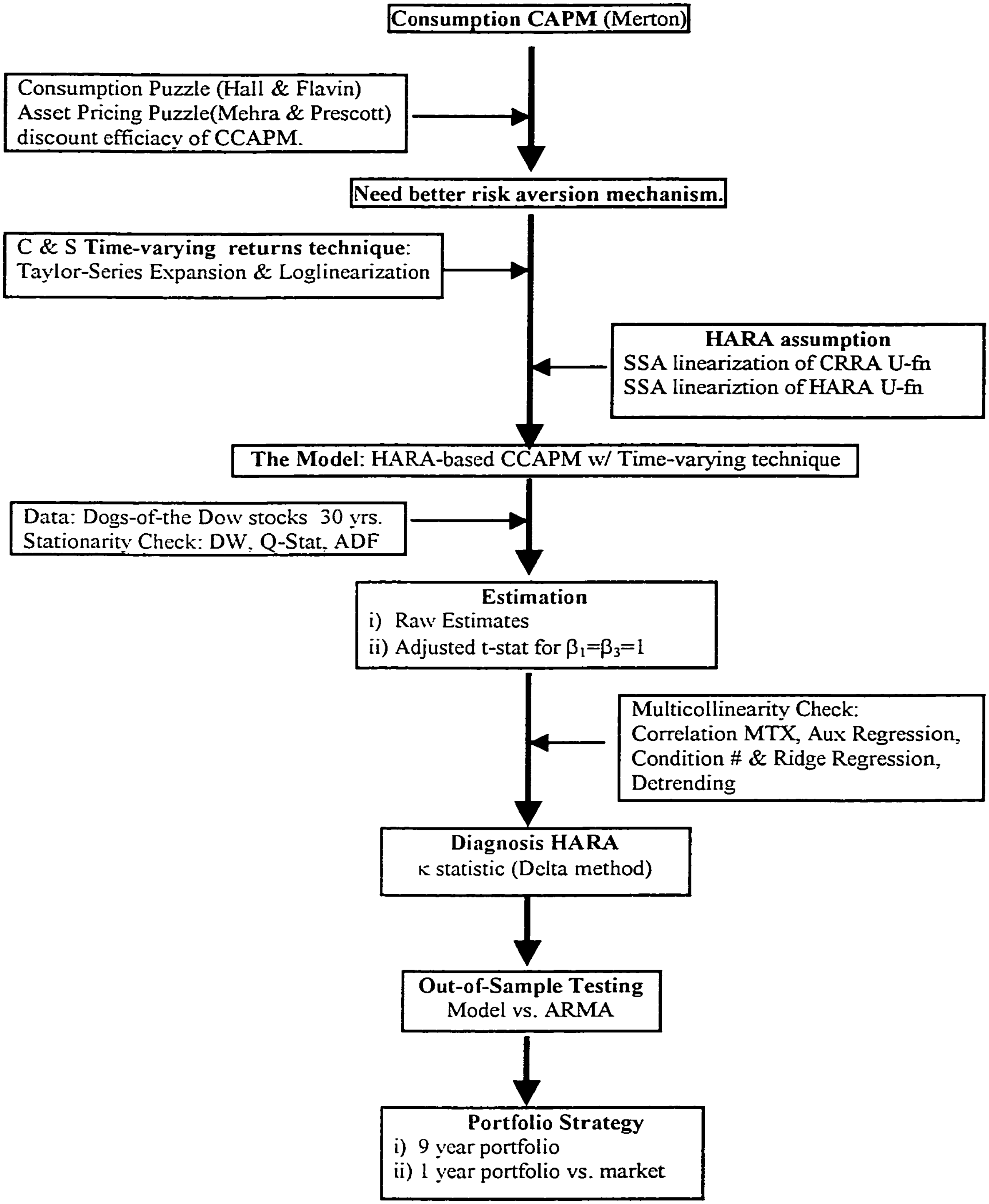

Fig 1. Schematics of this dissertation

III. Theoretical Foundations: Consumption-Based Asset Pricing Models

A. CRRA-Based Time Varying Expected Returns

Time-varying expected returns technique may be said to have been practically developed by John Campbell and Robert Shiller. They introduced and used the technique heavily in many of their studies on expected dividends, dividend-price ratio, stock prices and earnings in the late 80's. (Campbell & Shiller 1, 2, 3) The present-value relations among the variations of their model can be traced back to Gordon growth model, which is related to the utility-maximizing objective function in the following manner.

Objective function: According to Lucasian type models (1978), individuals consume to $Max E_t \sum_{\tau=0}^{\infty} \beta^{\tau} U(C_{t+\tau})$ s.t. $i)\ C_t + W_t = R_t W_{t-1}$ & $ii)\ R_t = \frac{P_t + D_t}{P_{t-1}}$ (1)

where all the variables are expressed in aggregates and in their usual notation.
Then, the first order condition will be

$$\textbf{FOC:}\quad U'(C_t) = \beta\, E_t U'(C_{t+1}) R_{t+1} \qquad (2)$$

where $U'(C_t)$ or the marginal utility of consumption is equal to the opportunity cost of the foregone current consumption, which is $E_t\beta U'(C_{t+1})R_{t+1}$ or the future market return adjusted by the marginal utility of future consumption discounted to the present.
Dividing both sides by $U'(C_t)$ we will get

$$\frac{U'(C_t)}{U'(C_t)} = \beta\, E_t \frac{U'(C_{t+1})}{U'(C_t)} R_{t+1}, \text{ where } \quad U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}, \quad U'(C_t) = \frac{(1-\gamma)C_t^{-\gamma}}{1-\gamma} = C_t^{-\gamma} \quad \text{assuming CRRA.}$$

Define the net return[19] $r_{t+1}^{\cdot} = \beta\, E_t \frac{U'(C_{t+1})}{U'(C_t)} R_{t+1} - 1 = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} - 1$ (3)

[19] For an i^{th} asset, just replace r_{t+1} with r_{it+1}. Further, according to Campbell (AER, June 1993, p492) $1 = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\sigma}}\left(\frac{1}{R_{mt+1}}\right)^{1-\theta} R_{it+1}\right]$. This may be simplified as $1 = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{it+1}\right]$ if θ=1, a simple

We may further define $h_{t+1} = \log\left[\frac{U'(C_{t+1})}{U'(C_t)}R_{t+1}\right] = \log\frac{P_{t+1}+D_{t+1}}{P_t} - \gamma\log\frac{C_{t+1}}{C_t}$ (4)

where h_{t+1} **is a utility-adjusted log return of the asset that a utility optimizing investor will rationally choose**, which is not directly observable. It can be observed only indirectly through $logR_{t+1}$ or r_{t+1} which is an observable variable.

Now, h_{t+1} is nonlinear, so we need to linearize it in terms of P and D. For this we will first redefine[20] $\log(P_{t+1}+D_{t+1}) = \log[\exp(p_{t+1})+\exp(d_{t+1})] = f(p,d)$. (5)

Since $\frac{\partial f}{\partial p} = \frac{\exp(p_{t+1})}{\exp(p_{t+1})+\exp(d_{t+1})}$ and $\frac{\partial f}{\partial d} = \frac{\exp(d_{t+1})}{\exp(p_{t+1})+\exp(d_{t+1})}$, a **first order Taylor series expansion** of $f(p,d)$ around the steady-state points p^* and d^* would give [21]

$$f(p,d) = \log[\exp(p^*)+\exp(d^*)] + (p_{t+1}-p^*)\frac{\exp(p^*)}{\exp(p^*)+\exp(d^*)} + (d_{t+1}-d^*)\frac{\exp(d^*)}{\exp(p^*)+\exp(d^*)}$$

Define $\rho = \frac{P^*}{P^*+D^*} = \frac{\exp(p^*)}{\exp(p^*)+\exp(d^*)}$, *i.e.* $\frac{\partial f}{\partial p^*} = \rho$ *and* $\frac{\partial f}{\partial d^*} = 1-\rho$. (6)

Another interpretation of ρ is that[22] $\rho = \frac{1}{1+e^{\delta}} = \frac{1}{1+e^{\ln\frac{D_t}{P_t}}} = \frac{1}{1+\frac{D_t}{P_t}} = \frac{P_t}{P_t+D_t}$.

Intuitively, ρ is the reciprocal of the return in steady state. Therefore, it could be roughly interpreted as the discount factor in steady state.

Rewriting $\log(P_{t+1}+D_{t+1})$ solely in terms of p and d,

$$\log(P_{t+1}+D_{t+1}) = f(p,d) = \log[\exp(p^*)+\exp(d^*)] + (p_{t+1}-p^*)\rho + (d_{t+1}-d^*)(1-\rho)$$

$$= \log(P^*+D^*) + \rho\, p_{t+1} + (1-\rho)d_{t+1} - \rho\log P^* - (1-\rho)\log D^*$$

$$= k + \rho\, p_{t+1} + (1-\rho)d_{t+1}, \quad (7)$$

where $k = \log(P^*+D^*) - \rho\log P^* - (1-\rho)\log D^*$

---

rescaling of the model by the ratio of i^{th} asset return to the market return. This would suggest one way of pricing i^{th}-asset relative to the entire market.

[20] $p_{t+1} = logP_{t+1}$, $d_{t+1} = logD_{t+1}$

[21] where $\frac{d\log x}{dx} = \frac{1}{x}$ for outside and $\frac{d\exp(x)}{dx} = \exp(x)$ for inside were used.

[22] Also, see Appendix 1.

Applying it to R_{t+1} in (4) for an overall view,

$$\log R_{t+1} = \log(P_{t+1} + D_{t+1}) - p_t$$
$$= f(p,d) - p_t$$
$$= (1-\rho)d_{t+1} + \rho\, p_{t+1} - p_t - k = \xi_{t+1} \quad (8)$$

Then[23], $\log R_{t+1} = \xi_{t+1} = \rho\, p_{t+1} + (1-\rho)d_{t+1} - p_t + k$

$$= \rho\, p_{t+1} + (1-\rho)d_{t+1} - p_t - p_{t+1} + p_{t+1} + k$$
$$= (1-\rho)(d_{t+1} - p_{t+1}) + (p_{t+1} - p_t) + k$$
$$= (1-\rho)\delta_{t+1} + g - \log\rho - (1-\rho)\delta_t \text{ , where } \delta_t = \log\frac{div_t}{p_t} \;\&\; g = \frac{p_{t+1}}{p_t},$$
$$= g - \log\rho = g - (g-r) = r .$$

Note that in steady state $\delta_t = \delta_{t+1} = \delta$, $k = -\log\rho - (1-\rho)\delta$ as defined above. [24] Also note that[25] $\log\rho = \log\left(\frac{P_{t+1}}{P_t}\right)\left(\frac{P_t}{P_{t+1}+D_{t+1}}\right) = \log\frac{G}{R} = g - r$ or $\rho = e^{g-r}$.

ξ_{t+1} can also be rewritten in terms of log dividend-price ratio and dividend growth as follows[26]:

$$\xi_{t+1} = \rho\, p_{t+1} + (1-\rho)d_{t+1} - p_t + k$$
$$= \rho\, p_{t+1} + d_{t+1} - \rho\, d_{t+1} - p_t + k$$
$$= -\rho(d_{t+1} - p_{t+1}) + d_{t+1} - p_t + k$$
$$= -\rho(d_{t+1} - p_{t+1}) + d_{t+1} - d_t + d_t - p_t + k$$
$$= \delta_t - \rho\delta_{t+1} + \Delta d_{t+1} + k \quad (9)$$

---

[23] $\delta_t = \log(Div_t/P_t)$. A common name for δ_t is dividend yield. The ratio variables, as we see in the financial press, are used as indicators of fundamental value relative to price. If stocks are underpriced relative to fundamental value, returns tend to be high subsequently, and the converse holds if stocks are overpriced.

[24] Also, see Appendix 2, equation (26).

[25] G and R are all in gross terms here. See also Appendix 1 and 3.

[26] See Campbell & Shiller (3), "The Div/Price Ratio & Expectations of Future Discount Factors", *Review of financial Studies*, vol 1, 1989. Also, Campbell & Shiller (2), Stock Prices , Earnings, and Expected Dividends, *Journal of Finance*, 1988, vol. 43, pp. 661~676

Intuitively ξ_{t+1} is loglinearized R_{t+1} that is exactly a linear approximation of h_{t+1}, except for $\gamma\Delta c_{t+1}$ term [27]. which is $\log\frac{U'(C_{t+1})}{U'(C_t)}$ according to the rationale of (2).

Substituting ξ_{t+1} for $\log R_{t+1}$ and rewriting h_{t+1} in terms of linearized variables

$$
\begin{aligned}
h_{t+1} &= \log R_{t+1} - \gamma \log\frac{C_{t+1}}{C_t} \\
&= f(p,d) - p_t - \gamma\,\Delta c_{t+1} \\
&= (1-\rho)d_{t+1} + \rho p_{t+1} - p_t + k - \gamma\,\Delta c_{t+1} \\
&= \delta_t - \rho\delta_{t+1} + \Delta d_{t+1} + k - \gamma\,\Delta c_{t+1} \\
&= \xi_{t+1} - \gamma\,\Delta c_{t+1} = r - \gamma\,\Delta c_{t+1}
\end{aligned}
$$

Rearranging terms will give the final structural equation between utility-adjusted log return and log dividend-price ratio, change in log dividend, and change in marginal utility of consumption:

$$
\begin{aligned}
h_{t+1} &= (1-\rho)d_{t+1} + \rho p_{t+1} + k - \gamma \log\left(\frac{C_{t+1}}{C_t}\right) \\
&= \delta_t - \rho\delta_{t+1} + \Delta d_{t+1} - \gamma\Delta c_{t+1} + k \quad \approx \quad \xi_{t+1}
\end{aligned} \tag{10}
$$

If $h = (1-\rho)d + \rho p + k - \gamma \log 1 = \log\frac{U'(C)}{U'(C)}R = \log R = \xi$ in steady state, since $-\gamma\,\Delta c_{t+1} = -\gamma \log 1 = 0$, a limiting case when $C_{t+1} = C_t$, then $h = \xi = \log R = (1-\rho)d + \rho p + k$. Therefore, $\boldsymbol{h} = \xi$ in steady state. [28]

This may provide an insight as to **why the utility function is generally overlooked in most financial asset pricing models**. Intentionally or unintentionally the financial model builders attest to an important point that utility function would drop out in the steady state under the assumption of CRRA. [29]

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[27] It is conceivable that $\gamma\,\Delta c_{t+1}$ can be any constant γg including 0 in steady state.

[28] However, if in SS $\Delta c = g$ for example, $-\gamma\Delta c = -\gamma g$. Then, $h = \xi$ only approximately.

[29] Another way to explain it is that consumption comes entirely from dividend in equilibrium as in Lucas Tree model or Cash-In-Advance model.

Campbell himself also made this point in his paper "Intertemporal Asset Pricing without Consumption Data"(AER June '93). He replaces the covariance between the return on the i^{th} asset and consumption with the weighted average of "the covariance between the return on the i^{th} asset and the market return" and "the covariance between the return on the i^{th} asset and the upward revision of expected future returns",

i.e.) $E_t r_{i,t+1} - r_{f,t+1} = -\frac{V_{ii}}{2} + \gamma\, V_{im} + (\gamma - 1)V_{ih}$. He, thereby, arrives at a real asset-pricing model without consumption variable that maps into the standard CAPM format.[30]

Another strength of (10) in terms of linearity is that it can also be viewed as an expansion around $\delta_t = \delta_{t+1} = \delta$, because the log dividend-price ratio is assumed to follow a stationary stochastic process, so that it has a fixed mean that can be used as the expansion point δ.

Then, for an i^{th} asset, $h_{it+1} \approx \xi_{it+1} = \delta_{it} - \rho\delta_{it+1} + \Delta d_{it+1} + k$, where interpretation of h_{it+1} would be **the utility-adjusted gross log return on the i^{th} asset at time t+1.**

Given $h_{t+1} \approx \xi_{t+1} = k + \delta_t - \rho\delta_{t+1} + \Delta d_{t+1}$, if we impose terminal condition $\lim_{j\to\infty} \rho^j \delta_{t+j} = 0$,

$$h_{t+1} \approx k + \delta_t + \Delta d_{t+1}$$

$$\delta_t \approx h_{t+1} - \Delta d_{t+1} - k\,.$$

Solving forward we get[31] $\delta_t = \sum_{j=0}^{\infty} \rho^j \left(h_{t+j} - \Delta d_{t+j}\right) - \frac{k}{1-\rho}$. (11)

For a multi-period return, we can define a multi-period extension of (9) and solve forward: $\xi_{j,t} = \sum_{j=0}^{n} \rho^j \xi_{t+j}$, where $\xi_{j,t}$ is a discounted j-period return at time t.[32]

Since $\xi_{t+j} = \rho\xi_{t+j+1} \approx \rho h_{t+1+j} = \rho \log\left[\frac{U'(C_{t+1+j})}{U'(C_{t+j})} R_{t+1+j}\right]$

$$\xi_{j,t} \approx h_{j,t} = \delta_t - \rho^j \delta_{t+j} + \sum_{j=0}^{n} \rho^j \Delta d_{t+j+1} + k\frac{1-\rho^n}{1-\rho} \qquad (12)$$

---

[30] Campbell, J.Y., "Intertemporal Asset Pricing without Consumption", *American Economic Review*, 1993

[31] See Campbell & Shiller (3), "The Div/Price Ratio & Expectations of Future Discount Factors", *Review of Financial Studies*, vol 1, 1989: p 200. Also, refer to Appendix 3 and 4 (33) ~ (39)

[32] Campbell & Shiller (2), Stock Prices, Earnings, and Expected Dividends, *Journal of Finance*, 1988, vol. 43, pp. 668. δ_t doesn't have to be discounted, but other terms must.

However, this multi-period forecasting model requires futures data or solves for historical return at t with historical data after t. Therefore, it is out of the scope of this study. This model is introduced just to shed light on how multi-period forecasting can be achieved.

Also dubbed "dividend-price ratio model", equation (11) is indeed a **time-varying version of Gordon growth model** or **Dynamic Gordon growth model** in that the discount rate [33] (*i.e.* return) and growth rate (*i.e.* change in log dividend) are allowed to vary over time. Campbell & Shiller proceed to test vector autoregression (VAR hereafter) generated from dividend-price ratio model, but that would not be necessary if the goal is to explain and forecast the expected utility-adjusted log return, h_{t+1} on the basis of the functional relation we have seen thus far. Campbell & Shiller's empirical rationale for the choice of VAR will be briefly hinted later in the testing part of this paper. Their theoretical rationale would probably have to do with the Granger-causality characteristic of the model.

Another rationale for my choice of utility-adjusted log returns model can also be found from Campbell & Shiller (2). In Campbell & Shiller (2) they found that the dividend-price ratio has strong forecasting power for dividend growth, and the earnings-price ratio (30 year moving average in their studies) is also highly significant. However, **their VAR tests reject more and more strongly as the return horizon increases**. In the limit, at $i = \infty$, the null hypothesis (H_0 hereafter) is that the actual δ_t equals the unrestricted VAR forecast of the present value of future real dividend growth. **This hypothesis can be rejected at better than 0.1% level.**

They also find that with their constant expected real returns model[34], **the actual dividend-price ratio, δ_t' has only a weak relation to its theoretical counterpart δ_t**, a result that strongly contradicts their model. That δ_t is less variable than δ_t' would suggest that **the dividend-price ratio is unrelated to the theoretical value implied by the constant expected real return model**. However, a short-run coherence between δ_t and δ_t' is suggested even though the overall correlation between the two is virtually zero. **From this** one can infer that **VAR may not be a good model**.

---

[33] According to Gordon present value relation definition, $P=D/(R-G)$.

[34] $E_t h_{t+i} = r$ for i=1,2,3… . The main difference in the choice of "r" is that Campbell & Shiller used commercial paper rate (or T-Bill rate), whereas I am using the market return.

The main reason for this failure of the constant expected returns model can be found in Campbell & Shiller (1). The short-term real interest rates they used for discount rate are not sufficiently variable, and do not have the appropriate correlation with stock prices to explain big movements in the log dividend-price ratio.[35] **This is another reason why I am using "*r* adjusted by the market return" which should theoretically correspond to the opportunity cost of the foregone consumption.** And **as such one doesn't necessarily have to assume *r* to be constant over time**, which is another benefit of the time varying model.

On the other hand **under the assumption that $\delta_t = \delta_t'$ holds, Campbell & Shiller also computed** $\xi_{1,t} \equiv \delta_t - \rho\delta_{t+1} + \Delta d_t$, practically a replica of my model, **to test if** $\xi_{1,t} = \xi_{i,t}'$ even if the market has superior information not available to econometricians. They found **a strong evidence that returns on stocks are far too volatile to accord with the constant expected real return present value model**, confirming the claims of the volatility literature. **However**, although return seems to be too volatile, **they found a remarkably high correlation between actual return ξ_t' and its theoretical counterpart ξ_t, equal to 0.915.** Therefore, **it is only rational to choose my time-varying returns model over VAR.**

Finally, to verify the connection between the log dividend-price ratio model and the fundamental present value relation we invoke that

$$\log R_{t+1} = \xi_{t+1} = k + \rho p_{t+1} + (1-\rho)d_{t+1} - p_t$$

and impose $\lim_{j\to\infty} \rho^j p_{t+j} = 0$,

then, $p_t = \dfrac{k}{1-\rho} + \sum_{j=0}^{\infty} \rho^j \left[(1-\rho)d_{t+1+j} - r_{t+1+j}\right]$.

Taking expectations on both sides to allow the discounted future value conditional on the current expectation in the RHS,

$$E_t[p_t] = \frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty} \rho^j \left[(1-\rho)d_{t+1+j} - r_{t+1+j}\right]\right].$$

Rewriting it in terms of log dividend-price ratio instead rather than the log stock price,

$$d_t - p_t = -\frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty} \rho^j \left[-\Delta d_{t+1+j} + r_{t+1+j}\right]\right]$$

[35] Campbell & Shiller (1) p218

we obtain the familiar log divided-price ratio model as follows:[36]

$$\delta_t = \sum_{j=0}^{\infty} \rho^j E_t(-\Delta d_{t+j} + \gamma \Delta c_{t+j}) - \frac{k}{1-\rho} . \tag{14}$$

If we simplify $E_t[p_t] = \frac{k}{1-\rho} + E_t\left[\sum_{j=0}^{\infty} \rho^j \left[(1-\rho)d_{t+1+j} - r_{t+1+j}\right]\right]$

as $$p_t = \frac{k}{1-\rho} + p_{dt} - p_{rt} ,$$

then $$p_t = \frac{k}{1-\rho} + p_{dt} - p_{rt} \approx E_t\left[\frac{P_{t+1} + D_{t+1}}{R_{t+1}}\right] \tag{15}$$

which is the basic present value relation.

It is also consistent with other findings that the relation between p_t' and its theoretical counterpart p_t in Campbell & Shiller (2), defined also in the same manner, shows that p_t is strikingly smoother than p_t' and at the same time shows that short-run movements are highly correlated. Hence, most of the short-run movements in p_t' are seen, in an attenuated form, in p_t. Since ξ_t and ξ_t' are essentially changes in p_t, their behavior is dominated by the short-run movements in the series so that they are highly correlated with each other. δ_t and δ_t' , on the other hand, are determined by the levels of p_t and p_t' and are not as much correlated as p_t and p_t'.

One final question arises as I close this chapter. What did Campbell & Shiller try to prove? It used to be thought that the expected return in the long run is constant. They tried to show that it may not, and they had some moderate success in showing that. What I am trying to do is similar except that I am not using VAR, but the utility-adjusted log return model explored in this section combined with HARA to be introduced in the following section.

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[36] $\rho^j E_t(\Delta d_{t+j} - \gamma \Delta c_{t+j})$ is the PV of h_{t+j}. *Op. cit.* pp. 200~201; Campbell & Shiller (2), Stock Prices , Earnings, and Expected Dividends, *Journal of Finance*, 1988, vol. 43: pp. 667~669. Also, refer to Campbell, Lo & McKinley, *The Econometrics of Financial Market*, Princeton, 1997: pp. 261~264 for further technical details.

B. Hyperbolic Absolute Risk Aversion-Based Asset Pricing Model

1. About HARA

According to Carroll & Kimball (1996), hyperbolic risk aversion[37] is a more realistic alternative to the power utility used in CCAPM for the following reasons:

The power utility function is defined as $U(C) = \dfrac{C^{1-\gamma} - 1}{1-\gamma}$, with $\gamma > 0$.

Then, $U' = C^{-\gamma}$, $U'' = -\gamma C^{-\gamma-1}$, where $U'' < 0$.

Since Arrow-Pratt risk aversion function is defined as $(-U'')/U' = \gamma/C$, as C increases, γ/C decreases. This means **decreasing risk aversion, which implies concavity of consumption**. This also conforms to what Keynes had argued about marginal propensity to consume (MPC hereafter) - that MPC out of transitory income or wealth declines with level of wealth.[38] It is only rational that **introduction of uncertainty requires concavity. Therefore, it is essential to allow for decreasing absolute risk aversion (DARA).** However, since $-CU''/U' = \gamma$ is a constant, power utility belongs to a CRRA family. If the only form of uncertainty is in labor income Y_L, CRRA utility implies a linear consumption function. This means that MPC stays constant, even if wealth increases. Hence the power utility function needs to be generalized.

HARA utility function is defined as

$$U(C) = H + \frac{1-\kappa}{(2-\kappa)A}[AC + B]^{(2-\kappa)/(1-\kappa)}, \quad where \quad \kappa = \frac{(1+\gamma)}{\gamma}. \qquad (16)$$

The constant of integration H can be ignored by assuming ordinal utility function.

Then, HARA $U'(C) = [AC + B]^{1/(1-\kappa)}$.

Since $1/(1-\kappa) = -\gamma$, the power utility function with $U' = C^{-\gamma}$ is a special case of HARA with B=H=0, and A=1. Further, $(U'''U')/(U'')^2 = \kappa > 0$ implies strictly concave consumption function required by the economic theory. Therefore, κ is an important parameter characterizing 3 special cases of HARA functions:

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[37] Blanchard and Fischer, Lectures on Macroeconomics. MIT Press, 1996 pp283~284.

[38] Vinod, H.D., "Concave Consumption, Euler Equation and Inference Using Estimating Functions", *Proceedings of Business and Economic Statistics section of American Statistical Association*, Alexandria, Virginia. 1997. pp118-123

i) $\kappa = 0$, then borderline case of quadratic utility function, where

$$U = C - \frac{\theta}{2}C^2 \quad .$$

ii) $\kappa = 1$, constant absolute risk aversion (CARA, hereafter) function with exponential utility $U(C) = -\gamma^{-1}e^{-\gamma C}$ *via* l'Hôpital's rule.

iii) $\kappa > 1$, HyDARA.[39]

In the financial economics literature (Huang and Litzenberger 1988 or Ingersol 1987) the HARA class has been studied in the context of discrete time intertemporal portfolio selection problem. CCAPM are common in macroeconomics. The power utility remains a common assumption for Euler Equation estimates of CCAPM. Therefore, it would be a worthy attempt to propose new estimation of CCAPM under HARA as suggested by Vinod 1999.

2. HARA-Based CCAPM [40]

Euler equation is defined[41] as $E(g_t) = E(\beta R_t c_t^{-\gamma}) = 1$ (17)

where $c_t = C_t/C_{t-1}$ assuming CRRA, a special case of HARA with B=H=0, and A=1. (Vinod 1999). However, the EuEqn is not amenable to testing as it is, because it is nonlinear in parameter. Therefore, it needs to be linearized.

a. Linearization of EuEqn

Kadane's small sigma asymptotics (SSA)[42] linearizes the Euler equation as follows.

i) Remove the E operator: $g_t = 1 + \sigma v_t$, where σ is the small σ.

ii) Taking logs, $\log(g_t) = \log(1 + \sigma v_t) = \sigma v_t - \sigma^2 v_t^2/2 + \sigma^3 v_t^3/3 - \ldots$

$$= u_t - (u_t)^2/2 + (u_t)^3/3 - \ldots .$$

SSA letting $\sigma \to 0$ justifies omission of all terms with σ^j for $j \geq 2$ (chosen number)

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[39] If $\kappa>1$, then $0<\gamma<1$, which is the normal range for risk aversion parameter values. FYR if $0<\kappa<1$, $\gamma<-1$, and if $\kappa\leq 0$, then $-1\leq\gamma<0$, which are both improbable and unrealistic values for risk aversion parameter.

[40] Carroll, C.D. and Kimball, M.S. On the Concavity of the Consumption Function, *Econometrica* 64 (4), 1996, pp. 981~992

[41] Euler Equation equals 1 for the same reason as equation (3).

[42] *Journal of American Statistical Association*, 1970 p182 and *Journal of Econometrics*, 1976 p147

iii) A linearization has $\log(g_t) = u_t$ and, therefore, $E\log(g_t) = Eu_t$.

Also, $\log E(g_t) = \log(1) = 0$ from (17).

To assure that the omission of higher powers of u_t is appropriate, we invoke Jensen's inequality on a concave (log) function that $E\log(g_t) \leq \log[E(g_t)]$.

Therefore, $E(u_t) \leq 0$, instead of $E(u_t) = 0$.

Define "Jensen's Error" ***Je*** $= E\log(g_t) - \log[E(g_t)] = E\log(g_t)$.

To evaluate ***Je*** let g=RZ, where $R = R_t$ and $Z = \beta c_t^{-\gamma}$.

Then, $\log(g) = \log R + \log Z$ which means $E\log(g) = E\log R + E\log Z$,

where $\log Z = \log\beta - \gamma\log c_t$.

Reinstating time subscripts,

$$Je = E\log(g_t) = E\log R_t + E\log Z_t \text{ where } Z_t \supset \beta \text{ and } \gamma. \quad (18)$$

If $\hat{\beta}$ *and* $\hat{\gamma}$ are such that $E\log R_t = -E\log Z_t$, then we have ***Je*** $= 0$.

This will ensure that ER_t is exactly the discounted $\log\frac{C_t}{C_{t-1}}$ *or* $\log\frac{U'(C_t)}{U'(C_{t-1})}$.

Therefore, $\log(\beta R_t c_t^{-\gamma}) = \log R_t Z_t$

Then, $\mathbf{E\log R_t = -E(\log\beta - \gamma\log c_t) \rightarrow \log R_t = -\log\beta + \gamma\log c_t + u_t}$ (19)

where a simple regression estimates $\beta = e^{-\beta 1}$.

b. Generalization of the Linear EuEqn Model in terms of HARA

When κ>1, HARA ≡ CRRA with shifted origin. Therefore, instead of $c_t = C_t/C_{t-1}$, we have $\chi_t = (AC_t+B)/(AC_{t-1}+B)$. With χ_t, SSA method

i) permits us to ignore $\sigma^j \frac{v_t^j}{j}$ *for* $j \geq 2$,[43] as $\sigma \rightarrow 0$;

ii) unlike (19) still implies a nonlinear equation in the parameters β, γ, A and B:

$$\log R_t = -\log\beta + \gamma\log\left[\frac{AC_t + B}{AC_{t-1} + B}\right] + u_t \quad (20)$$

[43] Therefore, we can consider only $E(u_t)=E\log(g_t)$ and no higher moments.

Given any reliable nonlinear (*e.g.* MLE) estimation software, it is possible to estimate these parameters from (20). Unfortunately, however, the nonlinear methods are sensitive to starting values and suffer from certain numerical inaccuracies (McCullough & Vinod 1999). **Therefore, linear approximation is in order.**

c. Linear Approximation of HARA for Linear EuEqn Model

Since $\kappa = (1+\gamma)/\gamma$, we can estimate κ from an estimate of γ. This reduces (20) to (19) if B=0 irrespective of A. Then, we can avoid estimating A and focus on estimating κ. Thus, we are trading off the difficulties of nonlinear methods in exchange for approximate formulas which can be estimated which can be estimated by simple regressions[44]. So, we proceed as follows:

i) Redefine the model in terms of ϕ=B/A:

$$\chi_t = \frac{AC_t + B}{AC_{t-1} + B} = \left[A\frac{C_t}{C_{t-1}} + \frac{B}{C_{t-1}} \right] A^{-1} \left[1 + \phi\frac{1}{C_{t-1}} \right]^{-1}$$

$$= \left[\frac{C_t}{C_{t-1}} + \frac{\phi}{C_{t-1}} \right] \times \left[1 + \phi\frac{1}{C_{t-1}} \right]^{-1}$$

ii) Substituting ϕ=B/A and taking logs,

$$\begin{aligned} \log \chi_t &= \log\left[\frac{C_t}{C_{t-1}} + \frac{\phi}{C_{t-1}} \right] - \log\left[1 + \frac{\phi}{C_{t-1}} \right] \\ &= \log\left[\frac{C_t}{C_{t-1}} + \frac{C_t}{C_t} \times \frac{\phi}{C_{t-1}} \right] - \log\left[1 + \frac{\phi}{C_{t-1}} \right] \\ &= \log\left[\frac{C_t}{C_{t-1}} \times \left(1 + \frac{\phi}{C_t} \right) \right] - \log\left[1 + \frac{\phi}{C_{t-1}} \right] \\ &= \log\frac{C}{C_{t-1}} + \log\left(1 + \frac{\phi}{C_t} \right) - \log\left(1 + \frac{\phi}{C_{t-1}} \right) \end{aligned} \qquad (21)$$

iii) Assume that C_t is such that a Taylor series expansion is valid, *i.e.*,

$$\left| \frac{\phi}{C_{t-1}} \right| < 1 \quad and \quad \left| \frac{\phi}{C_t} \right| < 1.$$

[44] For some data sets this trade-off may not be advisable and direct nonlinear estimation of (20) cannot be avoided.

This condition of convergence to a stable point is a prerequisite for a Taylor expansion.

Since $\log(1+z) = z - z^2/2 + z^3/3 + \ldots$, we can approximate the above expression to the desired level of accuracy and retain only the linear term z in expanding log(1+z) of the last two terms in (21), [45]

$$\log \chi_t = \log \frac{C_t}{C_{t-1}} + \phi\left(\frac{1}{C_t} - \frac{1}{C_{t-1}}\right) \qquad (22)$$

Therefore, Linearized Approximate SSA Regression for HARA will be

$$\log R_t = -\log \beta + \gamma \log \frac{C_t}{C_{t-1}} + \gamma\phi\left(\frac{C_t}{C_{t-1}}\right) + u_t \qquad (23)$$

If regression coefficients are β_1, β_2, β_3, the parameters of interest $\theta = (\beta, \kappa, \phi)$ can be found by using $\kappa = (1+\gamma)/\gamma$ and $\beta = e^{-\beta_1}$, $\kappa = (1+\beta_2)/\beta_2$ and $\phi = \beta_3/\beta_2$. (24)

Covariance matrix of $\hat{\theta}$, the estimates of (β, κ, ϕ), by the delta method[46] equals $Cov(\hat{\theta}) = (\tilde{D}' \tilde{V} \tilde{D})$, where $\tilde{V}$ denotes the 3*3 covariance matrix of $\hat{\beta}_i$ for i=1,2,3 in (23), and where $\tilde{D}$ denotes a 3*3 matrix of $\frac{\partial \hat{\theta}}{\partial \hat{\beta}_i}$.

A high value of $\log(C_t/C_{t-1})$ may be caused by high value of $\log R_t$. Then, the error term u_t in (23) may be correlated with $\log(C_t/C_{t-1})$. To avoid this endogeneity problem, we will need to follow Vinod (1997) and use the predicted value of $\log(C_t/C_{t-1})$ from the AR(4) as an appropriate instrument for $\log(C_t/C_{t-1})$ approved by Durbin (1960) for optimality:

$$\log\left(\frac{C_t}{C_{t-1}}\right) = b_0 + \sum_{j=1}^{4} b_j \log\left(\frac{C_{t-j}}{C_{t-1-j}}\right)$$

Note that (19) is a special case of (23) when ϕ=B/A=0. Thus, we can test whether the general HARA model involving the additional parameters A and B are needed by using the H_0: $\phi = 0$. Since (20) reduces to (19) if B=0 regardless of the value of A, $\phi = 0$ is a sufficient test for testing B=0 in (20) also. Therefore, if $t(\phi)$ is significant, meaning ϕ is significantly different from 0, HARA with A and B are legitimated.

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[45] $-z^2/2 + z^3/3 + \ldots \to 0$

[46] See Greene, 1997, p124. Delta method is used when parameters to estimate are in nonlinear functional form

C. Hybrid of Time-Varying Technique and HARA-Based CCAPM

As already shown from the loglinearized HARA-Based CCAPM, if we let g=RZ, where $R = R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$ *and* $Z = \beta c_t^{-\gamma}$, we can express log g_t as

$$\log g_t = \log R_t + \log Z_t = \log\left[\frac{P_t + D_t}{P_{t-1}}\right] + \log \beta\, c_t^{-\gamma}$$

$$= \log\left[\frac{P_t + D_t}{P_{t-1}}\right] + \log \beta - \log\frac{U'(C_t)}{U'(C_{t-1})}$$

$$= \log\left[\frac{P_t + D_t}{P_{t-1}}\right] + \log \beta - \gamma \log\frac{C_t}{C_{t-1}}$$

$$= f(p,d) - p_t + \log\beta - \gamma\,\Delta c_t$$

$$= \left[(1-\rho)d_t + \rho p_{t+1} + k' - p_t\right] - \log\beta - \gamma\Delta c_t$$

$$\approx \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\Delta c_t + k, \quad where \quad k = k' + \log\beta$$

As we have seen in the Time-Varying Expected Returns technique, we can approximate h_t as

$$h_t \approx \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\Delta c_t + k = \log 1 = 0$$

$$\approx k + \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\log\left[\frac{AC_t + B}{AC_{t-1} + B}\right] + \varepsilon_t. \tag{25}$$

***Using the linear approximation technique and SSA used in II.B.2.c. we can rewrite* (25):**

$$h_t = k + \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\left[\log\frac{C_t}{C_{t-1}} + \phi\left(\frac{1}{C_t} - \frac{1}{C_{t-1}}\right)\right] + \varepsilon_t$$

$$= k + \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\,\Delta c_t - \gamma\phi\left(\frac{1}{C_t} - \frac{1}{C_{t-1}}\right) + \varepsilon_t$$

$$= k + \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\log R_{mt} - \gamma\phi\left(\frac{1}{C_t} - \frac{1}{C_{t-1}}\right) + \varepsilon_t$$

$$= \beta_0 + \beta_1\delta_{t-1} + \beta_2\delta_t + \beta_3\Delta d_t + \beta_4\log R_{mt} + \beta_5\left(\frac{1}{C_t} - \frac{1}{C_{t-1}}\right) + \varepsilon_t. \tag{26}$$

Now, **even a simple OLS can estimate** $\beta_1=\beta_3=1$, $\beta_2=-\rho$, $\beta_4=-\gamma$, $\beta_5=\beta_4\phi$, and $\kappa=(1+\gamma)/\gamma$.

Then, HARA assumption will be affirmed if $\kappa > 1$. For testing the significance of this κ parameter, we need a κ statistic defined as $t_\kappa = \frac{\hat{\kappa} - 1}{\sigma(\hat{\kappa})}$, where **Delta Method** is used to compute the variance of κ, where $\kappa = f(\gamma) = \frac{1+\gamma}{\gamma}$. Then, $\frac{df}{d\gamma} = \frac{\gamma-(1+\gamma)}{\gamma^2}$ and $\sigma^2(\kappa) = \left[-\frac{1}{\gamma^2}\right]^2 \sigma(\gamma)$. Also, as discussed in the previous chapters, Δc_t can be proxied by $logR_{mt}$. This framework would certainly imply what are the relevant variables in constructing any utility-maximizing CCAPM-based asset-pricing model.

This model has strengths in several points. First, HARA assumption is quite reasonable and conceivable in the light that the existing CRRA-based CCAPM models have not been able to replicate the volatility of the market to satisfaction as exhibited by asset pricing puzzles. **As HARA assumes concave consumption with increase in income, it would naturally embrace the progressively smoothing consumption**, not necessarily a sensitive one such as under CRRA with permanent income hypothesis (PIH hereafter). **In this sense HARA might reasonably replicate the volatility of the market in response up to a certain point in transitory income.**

Second, **this is a very solid model** in the sense that none of the variables are arbitrary, but **all derived solidly from the fundamental present value relations**. Therefore, it would be the strength of the model that **it is theoretically complete**. Another strength in a related issue would be **the parsimony in the choice of variables**. Adding extra variables cannot make stock returns unpredictable if they were already found to be predictable using fewer variables. Therefore, attempts to bring any other variables into the model would be totally unnecessary unless they are theoretically and mathematically derived.

Third, another useful aspect of this model is that it casts light on the role of the dividend. Unlike the prevailing tendency in finance to treat the dividend as nothing more than a signal (synonymous to decoy) and disregard utility function, **this model certainly attests to the realistic possibility that dividends do have a significantly material role in determining the return**. Indeed, the proposed model is a sound loglinearized version of the Gordon return equation $R = \frac{D}{P} + G$ having the growth rate of dividend and the growth rate of market return for G term.

Fourth, **the market return** which proxies the consumption growth rate instead of constant expected return of Campbell & Shiller model **has the advantage of self-**

adjustment with time. Intuitively it is only reasonable to assume that the stock market return converges to the macroeconomic output growth rate in the long run. Therefore, consumption is quite a relevant variable in the asset valuation model if we are using a long time series. This stipulates a fundamental component to be a legitimate part of any long-term asset valuation model. Campbell & Shiller must have based their assumption on the postulate that the return on reasonably long time series such as 10+ years must converge to the long mean by stationarity. However, the assumption of this long mean is not infallible empirically in that until the economy reaches a steady state, the macro-variables would hardly manifest long means. Moreover, *ad hoc* exogenous shocks to the economic fundamentals make it hard to exactly determine when the market would attain the steady state. This sort of imperfection in the long-run constant return might be complemented by allowing it to change over time.

Fifth, the model reflects more realistic and comprehensive risk aversion mechanism that decreases with concavity (DARA) as the transitory income/wealth increases. Combined with HyDARA **the model is also easily amenable to testing as even a simple OLS can do the job**.

The model also makes intuitive sense in terms of algebraic signs because if $D_t = C_t$ in equilibrium according to Lucas Tree Model, then, if D_t goes down, C_t goes down as well. This will lead to an increase in R_{t+1}. because D_t or C_t will be reinvested. Therefore, a decrease in δ_t will cause h_{t+1} to increase and Δd_{t+1} as well. Δd_{t+1} will also increase because if D_t falls, D_{t+1} will be relatively higher regardless of the level of pay-out. However, in an incomplete substitution between D_t and C_t, D_t and C_t are in a complementary relation, so a drop in C_t would mean an increase in D_t and an increase in R_{t+1}. Then, the following should be the correct signs: $h_{t+1} = \delta_t - \rho\delta_{t+1} + \Delta d_{t+1} - \Delta c_{t+1}$. δ_{t+1} has negative sign because if D_{t+1} is low, h_{t+2} will be high, but for h_{t+2} to be high h_{t+1} will have to be relatively low.

IV. Testing the Model

A. Sample and Data Sources

1) **Number and Types of Data**: The sample will consist of 10~30 years time series of quarterly stock prices[47], dividends and consumption. The selected stocks include 1997, 1998, 1999 and 2000 makers of so-called "Dogs of the Dow" stocks. As previously explained in the Introduction, the Dogs of the Dow[48] strategy is nothing more than a dividend yield strategy for the Dow Jones Industrial Average. The strategy is simple: once each year, adjust your portfolio so you own only the 10 highest yielding stocks in the Dow Jones Industrial Average. These stocks, therefore, rely heavily on the strategy that maximizes dividend-yield ratios. Historically these stocks reportedly have done better than the market during the down market and at least as well as the market during the up market.

What I am trying to do is mainly threefold. First, I will **estimate the parameters of the equation (26) for each of these stocks and diagnose how well the HARA assumption applies by checking if the parameter** $\kappa \geq 1$ as proposed by the model. Second, I will **conduct an out-of-sample testing of the proposed model to measure its *ex post* forecasting power** and **compare its results with the forecast of a completely atheoretical DGP** such as ARIMA. Third, for a more practical application I will **construct a portfolio consisting of these stocks** and **compare the estimated log returns on the portfolio with the log return on S&P 500 and DJIA**. This will test the model's performance for the short-run forecasting horizon such as 1 year.

2) **Choice of Time Horizon**: The decision on time horizon is fairly constrained by the availability of data, and the data available[49] to me mostly date back from 1970 and some from 1989 in quarterly form. Therefore, while Campbell & Shiller studies were done over a fairly long-term market data such as more than 100 years, I am basically interested in looking into relatively short to medium-term data within 10~30 year range. Besides, i) long-horizon

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[47] Quarterly stock price is taken from the monthly closing price of the quarter.

[48] The term "dogs" should not be interpreted in negative connotation, but rather as in "top-dog vs. under-dog" context.

[49] If the data are not readily available for every period, some techniques like "Winsorizing" that truncates values above and below the upper and lower bounds into the bound vlaues may be used.

forecasts will inevitably involve high degree of averaging. (*i.e.* law of large numbers); ii) there are those securities that have been in existence for relatively short period of time.

3) **Data Sources**: Stock price and dividend time series are available on-line from such on-line sources as CRSP, Compustat[50], Yahoo Finance, Marketguide.com, Bigcharts.com, Bloomberg, Reuters, ...etc. From these sources, returns series are obtained by the theoretically straightforward definition[51], $R_t = \frac{P_t + D_t}{P_{t-1}}$. Consumption Data are also available from on-line GDP data series provided by such websites as BEA-NPIA, NBER, FRED, BLS, and U.S. Census Bureau, Citibase...etc.

B. Stationarity Issues

The standard theory of inference in regression requires that all variables be stationary. Therefore, it is only natural to be concerned about the stationarity of the data when we are dealing with time series, because if the data series is non-stationary, then the estimation is not reliable. Normally, we should try to check for the stationarity of all the data series to be used in this study, which alone would take up a good deal of time and efforts.

Besides, it is not the main objective of this study especially when we can legitimately assume that all the variables used in the testing of the model are all in the first difference of the log as they pertain to the rate of change in these variables. Particularly the log return is defined as the log of the sum of the price and dividend minus log of the previous period price, which is roughly the first difference of the price. Therefore, we can reasonably assume that most of these variables already have the stationarity taken care of.

In addition, what supports the stationarity is that the tests already conducted by Campbell & Shiller confirm this assumption (Campbell & Shiller (1)). They used Phillips-Perron (1988) test, a modification of the Dickey-Fuller (1981) ***F***-statistic. According to the Campbell & Shiller's test, the **H_0 of unit root is generally not rejected for level values** of

[50] Compustat data were made available to me thanks to the generous and unsparing support from my mentor Dr. H. Vinod.

[51] I prefer to use theoretical returns over the reported returns data, because corporate earnings report practices are quite often dubious and fictitious and may contain fabricated data through "creative accounting procedures" all in an effort to present their performance favorably to the public.

price and dividends. **However**, the exception is that the **H_0 of unit root can be rejected for the real dividend on Cowles/S&P500 stocks at 5% level. Therefore**, the **H_0 of unit root is strongly rejected at least for the growth rates of the stock market variables and for the log dividend-price ratio**.

Empirically dividend series are often found to be relatively stationary as discussed earlier in the theory part. Also empirically, return is generally considered a mean-reverting stationary process. So, we may somehow proceed under the assumption that **the log dividend-price ratio and growth rates of real dividends and prices are stationary, so that log dividends and prices are cointegrated processes.**[52]

To verify this point I have also taken some measures to check stationarity. The following is the result of Durbin-Watson (DW hereafter) and Portmanteau *Q statistic* for the residuals of the 14 stock portfolio. This residual check for white noise and autocorrelation among the residuals supports stationarity as suggested by Campbell & Shiller.

ESS =		0.095076063
ρ =		0.000633528
DW		1.998732944
dL & dU @ 5% & 1%	1.59 & 1.76	1.46 & 1.63
Q stat		5.61901E-06
χ^2 w/ 4 df @1%		13.2767

Table 1. Stationarity check by DW and Q-statistic

DW is close to 2 and *Q statistic* is under critical χ^2 value indicating that this residual series is not autocorrelated and quite likely white noise.

Finally, the Dickey-Fuller (DF hereafter) test on residuals of the 14 stock portfolio over 10 year data range (from 1989 through 1999) was not necessary as the variables are in the 1st difference form. However, as there can be no absolute guarantee for staionarity, it would not hurt to double check. The DF on residuals also produced the following results reaffirming that the residuals are stationary. The test was based on the model:

$$\Delta u_t = \gamma_1 + \gamma_2 t + \delta\, u_{t-1} + \zeta_i \sum_{i=1}^{\infty} \Delta u_{t-1} + \upsilon_t \, .$$

[52] Note that this is a conservative assumption in the sense that it leads to greater variability in the rational forecast of expected futures dividends. and less evidence of excess volatility in stock prices, than does the assumption that dividends and prices are stationary around a deterministic trend.

H_0: $\delta = \rho - 1 = 0$ indicates random walk (nonstationarity).

Dickey-Fuller for Residuals					
Multiple R	0.715671838				
R Square	0.51218618				
Adjusted R Square	0.51058679				
Standard Error	0.029119016				
Observations	613				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	0.543071478	0.271535739	**320.23854**	8.26991E-96
Residual	610	0.517229443	0.000847917		
Total	612	1.06030092			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	
Intercept	-0.000260925	0.001176153	-0.221846555	0.824507585	
u_{t-1}(H_0: $\delta=\rho-1=0$)	-1.008799622	0.057264966	**-17.616349**	1.84396E-56	
Δu_{t-1}	-0.01504834	0.039632764	-0.379694423	0.704304411	

Table 2. Dickey-Fuller test for stationarity of the Portfolio residuals

Coefficient for u_{t-1} has a significant t-value rejecting H_0. This indicates that the residuals (u_t) are stationary, where DF critical τ at 1% with 2 df = −4.07. [53]

Therefore, judging from the above evidences we can conclude that the variables used in the testing of my model are cointegrated stationary processes.

C. Regression Results

1. Estimation of Parameters

The following individual regression results of 14 stocks used S&P 500 index to calculate the market return. They all present significantly high ***t*** and ***F*** ratios, R^2s and very small standard errors (*se* hereafter). The table presents t values of estimates based on the H_0: β_i=0. β_4 is $-\gamma$, the coefficient of r_{mt}, and β_5 is $-\gamma\phi$, the coefficient of $\frac{1}{C_t} - \frac{1}{C_{t-1}}$. If β_5 =0, then, the model reduces to CRRA-based one. However, it is interesting to note that only in one in 14 cases, β_4 was significant, seriously weakening the CRRA assumption. On the

[53] However, estimated u is based on the estimated cointegrating parameter β. Therefore, DF and ADF τ_c and F_c are not quite appropriate. One needs to find critical values in the following references:

1. Engel & Granger, Econometrica vol. 55 1987, pp. 251-276
2. Engel & Yoo, B.S., Journal of Econometrics vol. 35, pp. 143-159
3. Long-run Economic Relationship: Readings in cointegration, Oxford Univ. Press, 1991, Chapter 12.

contrary, in 8 cases out of 14, β_5, the addition to the model by assuming HARA, was found to be significant, strongly supporting the HARA.

Table 3. Regression Results [54]
Bold italic indicates t significant @ 1% level. Boldface indicates t significant @ 5% level.

		AT&T			Caterpillar	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.0403504	0.000870401	**46.35840368**	0.1153128	0.00911073	**12.6568**
δ_{it-1}	**0.99985908**	0.000382716	**2612.537871**	**0.99292**	0.00767727	**129.333**
δ_{it}	**-0.99316579**	0.000378098	**-2626.738674**	**-0.97011**	0.00731731	**-132.578**
Δd_{it}	**0.999206739**	0.003371166	**296.3979471**	**0.99048**	0.00895636	**110.589**
r_{mt}	0.000660775	0.000752846	0.877702987	0.009528	0.01513176	0.6296714
$(1/C_t - 1/C_{t-1})$	-12.28583862	40.48811735	-0.30344307	-19.65477	435.085036	-0.0451745
F	**1879766.061**			**4214.72**		
R Square	**0.999995957**			**0.99688**		
Adjusted R	**0.999995425**			**0.99664**		
df (k, n-k)	5, 38			5, 66		

		Chevron			Du Pont	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.041226759	0.005996151	**6.8755366**	0.306364488	0.051296728	**5.9723983**
δ_{it-1}	**0.9246365**	0.035258702	**26.224349**	**0.9472273**	0.130611466	**7.2522527**
δ_{it}	**-0.9169311**	0.035297468	**-25.977249**	**-0.8840685**	0.130836721	**-6.7570364**
Δd_{it}	**0.9268854**	0.035338184	**26.229006**	**1.0518353**	0.136812447	**7.6881548**
r_{mt}	0.12346752	0.045002076	**2.7435961**	-0.18682533	0.186913601	-0.999527744
$(1/C_t - 1/C_{t-1})$	-5111.457304	290.331276	**-17.605603**	-5599.729307	1834.330507	-3.052737381
F	**336.99304**			**47.74824181**		
R Square	**0.9371512**			0.676816893		
Adjusted R	**0.9343703**			0.662642195		
df (k, n-k)	5, 113			5, 114		

		Exxon			GM	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.049073766	0.000580863	**84.484299**	0.183732312	0.015169703	**12.111794**
δ_{it-1}	**0.9982327**	0.000637797	**1565.1256**	**1.0040958**	0.026587084	**37.7662989**
δ_{it}	**-0.989768**	0.000632436	**-1565.0082**	**-0.964449**	0.026605491	**-36.250014**
Δd_{it}	**1.0009805**	0.00154995	**645.81486**	**1.0265366**	0.026726874	**38.4084066**
r_{mt}	-1.65844E-05	0.000627697	-0.026420962	-0.06546763	0.043482762	**-1.5055996**
$(1/C_t - 1/C_{t-1})$	-79.8182461	34.40803055	**-2.3197563**	-1463.58924	394.5504635	**-3.7095109**
F	**640706.03**			**473.3979**		
R Square	**0.9999881**			**0.9544353**		
Adjusted R	**0.9999866**			**0.9524192**		
df (k, n-k)	5, 38			5, 113		

[54] In the earlier version of the paper, I ran the regression with 3 different softwares, *i.e.*, Excel, SAS, & Gauss, and each of them all came up with practically the same estimates.

		Goodyear			Int'l Paper	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.127870131	0.006337964	**20.175268**	0.154546024	0.006565884	**23.537733**
δ_{it-1}	**1.0025474**	0.008233144	**121.76969**	**0.9714867**	0.01181025	**82.25793**
δ_{it}	**-0.9779884**	0.008266476	**-118.30778**	**-0.9393902**	0.011936918	**-78.696213**
Δd_{it}	**1.0028502**	0.010696469	**93.755254**	**0.9790171**	0.015414652	**63.512109**
r_{mt}	-0.020614447	0.015410053	**-1.3377272**	0.026371506	0.017768714	1.48415389
$(1/C_t - 1/C_{t-1})$	-273.3185545	141.9836115	-1.92500072	-644.1647136	155.5426339	**-4.1414029**
F	**3880.1303**			**2467.4407**		
R Square	**0.9941582**			**0.9908443**		
Adjusted R	**0.993902**			**0.9904427**		
df (k, n-k)	5, 114			5, 113		

		JP Morgan			Kodak	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.065094262	0.001377322	**47.261481**	0.158195352	0.007702217	**20.538938**
δ_{it-1}	**0.9998868**	0.000748677	**1335.5378**	**0.9920884**	0.01330993	**74.537461**
δ_{it}	**-0.9879338**	0.000775753	**-1273.5153**	**-0.9602367**	0.013391195	**-71.706572**
Δd_{it}	**0.9989132**	0.001785735	**559.38503**	**1.009159**	0.013732946	**73.484524**
r_{mt}	-0.000701589	0.001291558	-0.543210959	-0.004624746	0.021160858	-0.21855193
$(1/C_t - 1/C_{t-1})$	-42.71987096	45.69379179	-0.934916305	407.7764428	144.9531625	2.813160029
F	**727009.75**			**1624.1684**		
R Square	**0.9999873**			**0.9908443**		
Adjusted R	**0.999986**			**0.9904427**		
df (k, n-k)	5, 46			5, 113		

		3M			Philip Morris	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.157801589	0.005176658	**30.483295**	0.432595599	0.026440296	**16.361224**
δ_{it-1}	**0.994659**	0.012415152	**80.116537**	**0.914615**	0.06563431	**13.935014**
δ_{it}	**-0.9619646**	0.012457401	**-77.220334**	-0.790331256	0.066741719	**-11.841638**
Δd_{it}	**1.0011354**	0.015526984	**64.477131**	**0.9386661**	0.076546639	**12.262669**
r_{mt}	0.013817702	0.016312316	0.847071727	0.243161418	0.115281191	**2.1092896**
$(1/C_t - 1/C_{t-1})$	134.2197	107.7232327	1.245967993	-6732.27522	1111.628563	**-6.0562273**
F	**2570.8715**			**266.79974**		
R Square	**0.9913623**			**0.9219073**		
Adjusted R	**0.9909766**			**0.9184519**		
df (k, n-k)	5, 112			5, 113		

		SBC			Texaco	
Variables	Estimates	se	t	Estimates	se	t
Intercept	0.157801589	0.005176658	**30.483295**	0.076039167	0.008474924	**8.9722535**
δ_{it-1}	**0.994659**	0.012415152	**80.116537**	**0.9733498**	0.042060391	**23.141721**
δ_{it}	**-0.9619646**	0.012457401	**-77.220334**	**-0.9647292**	0.041899541	**-23.024816**
Δd_{it}	**1.0011354**	0.015526984	**64.477131**	**0.9726338**	0.042093901	**23.106288**
r_{mt}	0.013817702	0.016312316	0.847071727	-0.00295811	0.047307923	-0.062528848
$(1/C_t - 1/C_{t-1})$	134.2197	107.7232327	1.245967993	-3511.542915	354.7763682	**-9.8979054**
F	**2570.8715**			**146.716**		
R Square	**0.9913623**			**0.8665218**		
Adjusted R	**0.9909766**			**0.8606157**		
df (k, n-k)	5, 53			5, 113		

With regard to the prescribed parameter values of β_1 and β_3, the H_0 for β_1 and β_3 are not 0, but 1 as specified by the model. Therefore, we need additional test statistic for β_1 and β_3, because H_0: $\beta_1=\beta_3=1$. The new adjusted t-statistic is "$t=(\beta_i-1)/se_i$. Accordingly the new t-statistic for β_1 and β_3 are as follows.

Table 4. Adjusted Regression Statistic for β_1 and β_3.

	AT&T			Caterpillar		
Variables	Estimates	se	t	Estimates	se	t
δ_{it-1}	**0.99985908**	0.000382716	-0.368210277	**0.992920628**	0.007677268	-0.922121273
Δd_{it}	**0.999206739**	0.003371166	-0.235307496	**0.990476779**	0.008956358	-1.063291606

	Chevron			Du Pont		
Variables	Estimates	se	t	Estimates	se	t
δ_{it-1}	**0.9246365**	0.035258702	-2.137443971	**0.9472273**	0.130611466	-0.404043394
Δd_{it}	**0.9268854**	0.035338184	-2.068997094	**1.0518353**	0.136812447	0.378878539

	Exxon			GM		
Variables	Estimates	se	t	Estimates	se	t
δ_{it-1}	**0.9982327**	0.000637797	**-2.77094436**	**1.0040958**	0.026587084	0.154052246
Δd_{it}	**1.0009805**	0.00154995	0.632601052	**1.0265366**	0.026726874	0.992880799

	Goodyear			Int'l Paper		
Variables	Estimates	se	t	Estimates	se	t
δ_{it-1}	**1.0025474**	0.008233144	0.309407925	**0.9714867**	0.01181025	-2.414284202
Δd_{it}	**1.0028502**	0.010696469	0.266461764	**0.9790171**	0.015414652	-1.36123086

	JP Morgan			Kodak		
Variables	Estimates	se	t	Estimates	se	t
δ_{it-1}	**0.9998868**	0.000748677	-0.15120005	**0.9920884**	0.01330993	-0.594413344
Δd_{it}	**0.9989132**	0.001785735	-0.60860094	**1.009159**	0.013732946	0.666936286

	3M			Philip Morris		
Variables	Estimates	se	t	Estimates	se	t
δ_{it-1}	**0.994659**	0.012415152	-0.43020013	**0.914615**	0.06563431	-1.300920205
Δd_{it}	**1.0011354**	0.015526984	0.073124311	**0.9386661**	0.076546639	-0.801261829

	SBC			Texaco		
Variables	Estimates	se	t	Estimates	se	t
δ_{it-1}	**0.994659**	0.012415152	-0.43020013	**0.9733498**	0.042060391	-0.633617505
Δd_{it}	**1.0011354**	0.015526984	0.073124311	**0.9726338**	0.042093901	-0.650122686

This time the low t values for β_1 and β_3 confirm our adjusted H_0: $\beta_1=\beta_3=0$ (that adjusted β_1 and β_3 are not significantly different from zero.) which confirms our original H_0 except for Exxon and International Paper. [55]

[55] *t critical @ 1% =2.617 for 120 df & 2.576 for infinite df*

2. Diagnosis HARA

Next, we proceed to the main objective of this paper – to diagnose if HARA is the correct assumption for risk aversion mechanism. The following table summarizes parameters to diagnose HARA. As long as $\kappa>0$, we have strictly concave consumption required by the economic theory and will exhibit 3 cases of HARA: i) quadratic utility function if $\kappa=0$, ii) CARA if $\kappa=1$, iii) HyDARA if $\kappa>1$. However, the κ estimates are in their raw values and must not be taken as they are, for their statistical significance can only be determined by the appropriate test statistic, which follows the table.

Table 5. Raw Estimates of κ

AT&T			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.99985908	0.999206739	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$ [56]	-0.993165791		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	0.000660775	$\kappa=(1+\gamma)/\gamma$	-1512.373825
$\beta_5=\beta_4*\phi$	-12.28583862	$\phi=\beta_5/\beta_4$	-18593.06659

Caterpillar			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.992920628	0.990476779	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.970113559		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	0.009528039	$\kappa=(1+\gamma)/\gamma$	-0.009614784
$\beta_5=\beta_4*\phi$	-19.65477045	$\phi=\beta_5/\beta_4$	0.009619649

Chevron			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.9246365	0.9268854	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.9169311		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	0.12346752	$\kappa=(1+\gamma)/\gamma$	-7.099296074
$\beta_5=\beta_4*\phi$	-5111.457304	$\phi=\beta_5/\beta_4$	-41399.20607

t critical @ 5% =1.980 for 120 df & 1.960 for infinite df
F critical @1%=3.17 for n-k=120, k=5 & 3.02 for n-k = infinite, k=5
t critical @ 1% =2.704 for 40 df & 2.576 for infinite df
t critical @ 5% =2.021 for 40 df & 1.960 for infinite df
F critical @1%=3.51 for n-k=40, k=5 & 3.02 for n-k = infinite, k=5

[56] The estimates of parameter ρ from Campbell & Shiller (1) range from 0.937 for the Cowles/S&P annual data to 0.933 for the NYSE annual data, which are closely in line with the range of ρ for the quarterly 14 stocks which mostly run from 0.910 to 0.993.

Du Pont			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.9472273	1.0518353	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.8840685		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	-0.18682533	$\kappa=(1+\gamma)/\gamma$	6.352593248
$\beta_5=\beta_4*\phi$	-5599.729307	$\phi=\beta_5/\beta_4$	29973.07328

Exxon			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.9982327	1.0009805	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.989768		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	-1.66E-05	$\kappa=(1+\gamma)/\gamma$	60298.6291
$\beta_5=\beta_4*\phi$	-79.8182461	$\phi=\beta_5/\beta_4$	4812850.999

GM			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	1.0040958	1.0265366	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.964449		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	-0.06546763	$\kappa=(1+\gamma)/\gamma$	0.025850613
$\beta_5=\beta_4*\phi$	-1463.58924	$\phi=\beta_5/\beta_4$	-0.063775252

Goodyear			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	1.0025474	1.0028502	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.9779884		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	-0.020614447	$\kappa=(1+\gamma)/\gamma$	0.002842099
$\beta_5=\beta_4*\phi$	-273.3185545	$\phi=\beta_5/\beta_4$	-0.020555859

Int'l Paper			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.9714867	0.9790171	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.9393902		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	0.026371506	$\kappa=(1+\gamma)/\gamma$	-0.021432618
$\beta_5=\beta_4*\phi$	-644.1647136	$\phi=\beta_5/\beta_4$	0.026936716

JP Morgan			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.9998868	0.9989132	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.9879338		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	-0.000701589	$\kappa=(1+\gamma)/\gamma$	-0.001087982
$\beta_5=\beta_4*\phi$	-42.71987096	$\phi=\beta_5/\beta_4$	-0.000702352

Kodak			$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.9920884	1.009159	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.9602367		$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	-0.004624746	$\kappa=(1+\gamma)/\gamma$	0.009075874
$\beta_5=\beta_4*\phi$	407.7764428	$\phi=\beta_5/\beta_4$	-0.004582772

3M				$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.994659		1.0011354	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.9619646			$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	0.013817702	$\kappa=(1+\gamma)/\gamma$		0.001134112
$\beta_5=\beta_4*\phi$	134.2197	$\phi=\beta_5/\beta_4$		0.013802031

Philip Morris				$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.914615		0.9386661	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.790331256			$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	0.243161418	$\kappa=(1+\gamma)/\gamma$		-0.065341552
$\beta_5=\beta_4*\phi$	-6732.27522	$\phi=\beta_5/\beta_4$		0.259049962

SBC				$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.994659		1.0011354	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.9619646			$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	0.013817702	$\kappa=(1+\gamma)/\gamma$		23.53951069
$\beta_5=\beta_4*\phi$	134.2197	$\phi=\beta_5/\beta_4$		299992.7325

Texaco				$\kappa>1 \rightarrow$ HyDARA
$\beta_1=\beta_3$	0.9733498		0.9726338	$\kappa=1 \rightarrow$ CARA
$\beta_2=-\rho$	-0.9647292			$\kappa=0 \rightarrow$ quadratic U fn
$\beta_4=-\gamma$	-0.00295811	$\kappa=(1+\gamma)/\gamma$		-0.02813618
$\beta_5=\beta_4*\phi$	-3511.542915	$\phi=\beta_5/\beta_4$		-0.00304134

We need a test statistic to check how significant these κ estimates are, and as κ is a nonlinear parameter, delta method is the best choice. The ***t*** statistic for kappa was calculated according to the following steps at the top of the table and the results are shown below.

Table 6. κ Statistic

Company	$\sigma\gamma$	$\sigma^2\kappa = (-1/\gamma^2)^2\sigma\gamma$	$\sigma\kappa$	$t\kappa = (\kappa-1)/\sigma\kappa$
AT&T	0.000660775	3466080579	58873.42847	**-0.0257055**
Caterpillar	0.015131763	1836011.838	1354.995143	**-0.077456657**
Chevron	0.045002076	193.6518495	13.91588479	**-0.582018**
Du Pont	0.186913601	153.4256148	12.38650939	**0.4321309**
Exxon	0.000627697	8.29757E+15	91091006.03	**0.0006619**
GM	0.043482762	2367.067515	48.65251808	**0.313955473**
Goodyear	0.015410053	85333.0712	292.1182487	**0.166061755**
Int'l Paper	0.017768714	36738.0643	191.6717619	**-0.197836734**
JP Morgan	0.001291558	5330682653	73011.52411	**0.019522068**
Kodak	0.021160858	46257454.59	6801.283304	**0.031792249**
3M	0.016312316	447478.6684	668.9384639	**-0.10818773**
Philip Morris	0.115281191	32.97461893	5.742353083	**-0.716168871**
SBC Comm	0.016312316	447478.6684	668.9384639	**-0.10818773**
Texaco	0.047307923	617840826.6	24856.40414	**0.013600265**

Since γ is insignificant in 13 out of 14 cases, the relevance of r_{mt} is seriously in doubt. However, the $\gamma\phi$, significant in 8 out of 14 cases, shows that HARA assumption is needed. The delta method gives rather a wide confidence interval for κ. It suggests that in future work we should use a bootstrap for this case. Hence, the test of H_0: κ=1 or κ=0 (adjusted) would also have low power. However, we have enough evidence based on statistically insignificant γ and statistically significant κ that $\hat{\gamma} < 1$ and $\kappa = \frac{1+\gamma}{\gamma} > 1$. Hence, **HyDARA assumption is the most appropriate and flexible approach**. The traditional assumption implied in CRRA that A=1 and B= is clearly rejected as HyDARA holds. Also, CARA based on the assumption that κ=1 is rejected due to high standard error. Therefore, the flexibility introduced here seems worth the effort.

D. Precautionary Check for Multicollinearity

The regression results all show very high R^2s, which would normally make one suspicious of multicollinearity. So, I have checked correlation between the regressors, **run auxiliary regressions, checked condition numbers,** and run a demeaned regression. The results show that the correlation between δ_{t-1} and δ_t for each of the individual stocks is high as can be reasonably expected.

Table 7. Correlation Coefficients between δ_{t-1} & δ_t

Company	AT&T	CAT	Chev	DP	Exxon	GM	GT	IP	JPM	Kodak	3M	Ph Mo	SBC	Texaco
$\rho(\delta_{t-1}, \delta_t)$	**0.872**	**0.926**	**0.953**	**0.98**	**0.981**	**0.858**	**0.977**	**0.991**	**0.87**	0.7815	**0.984**	**0.985**	**0.987**	0.631

The bold face indicates that ρ>0.8.

The following correlation coefficient matrix of the 14 companies pooled also shows that the only possible source of multicollinearity would be the overall close correlation between δ_{it-1} and δ_{it} which is slightly above the general criterion of 0.8. However, this close correlation is unavoidable because of the way the model is set up, and also as they are the lead and lag series of the same variable. Therefore, this close correlation wouldn't render any adverse effects on the estimation as the estimates are still best linear unbiased estimator (BLUE hereafter).

Table 8. Correlation Matrix of Portfolio

Correlation Matrix	h_{it}	δ_{it-1}	δ_{it}	Δd_{it}	r_{mt}	$(1/C_t - 1/C_{t-1})$
h_{it}	1					
δ_{it-1}	0.2296	1				
δ_{it}	0.1154	**0.8871**	1			
Δd_{it}	0.065	-0.2518	0.1978	1		
r_{mt}	0.1764	0.0103	-0.0028	0.042272	1	
$(1/C_t - 1/C_{t-1})$	0.0166	-0.2333	-0.237	0.0158942	-0.2453106	1

Also, auxiliary regressions for individual stocks show only 6 instances with R_i^2 greater than the overall R^2, which, according to **Klein's rule of thumb,**[57] **affirms that multicollinearity is not a general phenomenon across the entire sample. Therefore, it is evident that multicollinearity is not significant for the most part of the sample.**

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[57] Auxiliary Regression

Regress X_i on the rest of X's and if $R_i = \dfrac{R^2_{x1\cdot x2x3\cdots xk}/(k-2)}{(1-R^2_{x1\cdot x2x3\cdots xk})/(n-k+1)} \sim F \rangle F_{critical}$,

then multicollinearity. Or if $R^2_{Aux} > R^2_{overall}$, then multicollinearity. (Klein's rule of thumb)

Table 9. Auxiliary Regression

Dependent Variable	δ_{it-1}	δ_{it}	Δd_{it}	r_{mt}	$(1/C_t - 1/C_{t-1})$	Overall R^2
R_i^2 AT&T	0.9907	0.9935634	0.9858	0.0682	0.0255	**0.999998596**
R_i^2 Caterpillar	0.96713	0.9615884	0.7762	0.1254	0.383	**0.996907343**
R_i^2 Chevron	**0.9997**	**0.99971**	**0.997**	0.0759	NA	**0.936**
R_i^2 Du Pont	**0.9951**	**0.99513**	**0.873**	0.1146	NA	**0.741080629**
R_i^2 Exxon	0.96516	0.9655074	0.3201	0.2419	NA	**0.999988145**
R_i^2 GM	**0.9896**	**0.98962**	**0.99**	**0.99**	NA	**0.953581806**
R_i^2 Goodyear	0.98387	0.9839325	0.6347	0.1025	NA	**0.994077455**
R_i^2 Int'l Paper	0.99023	0.9905932	0.9902	0.9902	NA	**0.990831283**
R_i^2 JP Morgan	0.83939	0.8410203	0.1561	0.2577	NA	**0.999987296**
R_i^2 Kodak	0.98286	0.9831643	0.961	0.1334	NA	**0.986280335**
R_i^2 3M	0.99013	0.9904491	0.6524	0.2611	NA	**0.991348158**
R_i^2 Philip Morris	**0.993**	**0.99323**	0.7564	0.1407	NA	**0.920452678**
R_i^2 SBC	**0.9959**	**0.99594**	0.8432	0.2498	NA	**0.898504837**
R_i^2 Texaco	**0.9991**	**0.99905**	**0.999**	0.0284	NA	**0.867217193**

Bold face indicates that R_i^2 > overall R^2. 6 out of 14 stocks produce R_i^2 > overall R^2. This would indicate that these 6 stocks show some degree of multicollinearity. R_i^2 for $(1/C_t - 1/C_{t-1})$ can largely be ignored due to the insignificance of their low values.

This conclusion is also supported by the following condition number check. Defined $k = \sqrt{\frac{\lambda \max}{\lambda \min}}$, where λ = eigenvalue, 10<k<30 indicates moderate to strong multicollinearity and 30<k indicates severe multicollinearity.

Table 10. Condition Number

Company	AT&T	Caterpillar	Chevron	Du Pont	Exxon	GM	Goodyear	Int'l Paper
eigenva1	0.109112	0.016919	8.52E-05	0.00242	0.017438	0.004555	0.007993	0.004794
eigenva2	0.585961	0.474232	0.908056	0.260104	0.522661	0.326481	0.389371	0.346003
eigenva3	0.893555	0.788739	0.980586	0.997372	0.705647	0.999953	1.004198	0.958288
eigenva4	1.395973	1.240404	1.157944	1.023536	1.520979	1.138159	1.023292	1.030051
eigenva5	2.015399	2.479707	1.953329	2.716568	2.233274	2.530853	2.575145	2.660864
$k=(\lambda_{Max}/\lambda_{min})^{1/2}$	4.297773	12.10634	**151.4506**	**33.50591**	11.31664	23.57257	17.94904	23.55855

Company	JP Morgan	Kodak	3M	Phil Morris	SBC	Texaco		Portfolio
eigenva1	0.080784	0.007042	0.005346	0.003416	0.002008	0.000345		0.01003
eigenva2	0.648579	0.703968	0.553255	0.339024	0.353148	0.76256		0.703573
eigenva3	0.938968	0.994185	1.001066	0.995516	1.021575	1.002612		1.091521
eigenva4	1.084368	1.223972	1.021912	1.010779	1.042008	1.38041		1.188357
eigenva5	2.247302	2.070834	2.418421	2.651264	2.581261	1.854073		2.006518
$k=(\lambda_{Max}/\lambda_{min})^{1/2}$	5.274354	17.14805	21.27009	27.85729	**35.84974**	**73.28557**		14.14362

From the above result **the condition number indicates that the multicollinearity is severe (k>30) only in 4 cases. Therefore, the multicollinearity is not a general**

phenomenon across the entire sample, and certainly not in the portfolio case. Then, a separate remedy - ridge regression - is in order for these 4 stocks. The following are the result of Hoerl, Kennard & Baldwin[58] estimates of ridge regression by Gauss.[59]

Table 11. Ridge Regression Results

Chevron bias parameter	HKB-noniterative Ridge Estimator 7.7E-07				
	δ_{it-1}	δ_{it}	Δd_{it}	r_{mt}	$(1/C_t - 1/C_{t-1})$
Est	0.95135901	-0.94366194	0.95355504	0.062304414	-4943.8819
Std. Err.	0.031344282	0.031384813	0.031428402	0.027602577	285.88833
bias parameter	HKB-iterative 2.2E-10		Ridge Estimator		
Est	0.95654307	-0.94885269	0.95874563	0.061071492	-4941.8925
Std. Err.	0.031515827	0.03155658	0.031599921	0.027613647	285.89135

Du Pont bias parameter	HKB-noniterative Ridge Estimator 0.000110455				
	δ_{it-1}	δ_{it}	Δd_{it}	r_{mt}	$(1/C_t - 1/C_{t-1})$
Est	0.994991	-0.92748265	1.095589	-0.50980277	-5251.4528
Std. Err.	**0.09224507**	**0.092437987**	**0.098615592**	0.096001951	1626.963
bias parameter	HKB-iterative Ridge Estimator 0.000121728				
Est	0.99072101	-0.92320395	1.0913308	-0.5083978	-5247.1482
Std. Err.	**0.091837469**	**0.092029498**	**0.098236631**	0.095958754	1626.8719

SBC bias parameter	HKB-noniterative Ridge Estimator 0.000630533				
	δ_{it-1}	δ_{it}	Δd_{it}	r_{mt}	$(1/C_t - 1/C_{t-1})$
Est	0.84400845	-0.86377713	0.74431066	-0.23874042	-10402.341
Std. Err.	0.14588019	0.14796447	0.17851535	0.17007192	13268.183
bias parameter	HKB-iterative Ridge Estimator 0.000938238				
Est	0.79394156	-0.81298021	0.69539839	-0.21393848	-10513.855
Std. Err.	0.13728515	0.13924262	0.17185623	0.16825098	13261.826

Texaco bias parameter	HKB-noniterative Ridge Estimator 3.0E-06				
	δ_{it-1}	δ_{it}	Δd_{it}	r_{mt}	$(1/C_t - 1/C_{t-1})$
Est	0.95912871	-0.95059942	0.9584117	0.024856601	-3516.2241
Std. Err.	0.03973242	0.039612156	0.039781682	0.030846887	347.55476
bias parameter	HKB-iterative Ridge Estimator 3.0E-06				
Est	0.95898713	-0.95045827	0.95826997	0.024874491	-3516.1597
Std. Err.	0.039726542	0.039606296	0.039775799	0.030846764	347.5546

[58] Amemiya, *Advanced Econometrics*. Harvard. 1985. pp 60~69.

[59] Gauss programs are provided in the Appendix.

Table 12. Comparison btwn OLS Estimator & Ridge Estimator

Chevron	*Coefficients(OLS)*	*Coefficients(ridge)*	***se(OLS)***	***se(ridge)***
δ_{it-1}	0.9246365	0.95135901	0.035258702	0.031344282
δ_{it}	-0.9169311	-0.94366194	0.035297468	0.031384813
Δd_{it}	0.9268854	0.95355504	0.035338184	0.031428402
r_{mt}	0.12346752	0.062304414	0.045002076	0.027602577
$(1/C_t - 1/C_{t-1})$	-5111.457304	-4943.8819	290.331276	285.88833

Ridge se are not significantly < OLS se.

Du Pont	*Coefficients(OLS)*	*Coefficients(ridge)*	***se(OLS)***	***se(ridge)***
δ_{it-1}	0.9472273	0.994991	0.130611466	**0.09224507**
δ_{it}	-0.8840685	-0.92748265	0.130836721	**0.092437987**
Δd_{it}	1.0518353	1.095589	0.136812447	**0.098615592**
r_{mt}	-0.18682533	-0.50980277	0.186913601	**0.096001951**
$(1/C_t - 1/C_{t-1})$	-5599.729307	-5251.4528	1834.330507	1626.963

Boldface indicate ridge se < OLS se.

SBC	*Coefficients(OLS)*	*Coefficients(ridge)*	***se(OLS)***	***se(ridge)***
δ_{it-1}	0.96918062	0.84400845	0.16737636	**0.14588019**
δ_{it}	-0.99077298	-0.86377713	0.16977691	**0.14796447**
Δd_{it}	0.86660971	0.74431066	0.19574922	**0.17851535**
r_{mt}	-0.30077081	-0.23874042	0.17497802	**0.17007192**
$(1/C_t - 1/C_{t-1})$	-10121.552	-10402.341	13281.579	**13268.183**

Ridge se are not significantly < OLS se.

Texaco	*Coefficients(OLS)*	*Coefficients(ridge)*	***se(OLS)***	***se(ridge)***
δ_{it-1}	0.9733498	0.95912871	0.042060391	0.039726542
δ_{it}	-0.9647292	-0.95059942	0.041899541	0.039612156
Δd_{it}	0.9726338	0.9584117	0.042093901	0.039781682
r_{mt}	-0.00295811	0.024856601	0.047307923	0.030846887
$(1/C_t - 1/C_{t-1})$	-3511.542915	-3516.2241	354.7763682	347.55476

Ridge se are not significantly < OLS se.

Only one of the 4 stocks came up with significantly smaller *se* than the OLS estimates. The rest of the sample produced *se* not significantly smaller than OLS *se*. This conforms to the Monte Carlo simulation report by R. Mittelhammer, G. Judge & D. Miller that ridge regression generally produces smaller mean squared error (MSE hereafter) than OLS when condition number $k(x'x) \geq 10$.[60] However, most of the ridge *se*'s are not significantly smaller than the model *se*'s, which **attests to the efficiency of the model.** One should also note that ridge is not a completely satisfactory remedy as even the ridge generally gives slightly biased estimates only with a smaller variance and MSE.

[60] Mittelhammer, R., Judge, G. & Miller, D., *Econometric Foundation*, Cambridge University Press, 2000: pp 550~521

Another possible action to take would be detrending. If correlation within the data series itself or multicollinearity is the most probable cause, then detrending might correct the problem. The rationale is that **if the correlation between δ_t and δ_{t-1} is gotten rid of, then the main source of multicollinearity is out of the way. With the main source of the problem out of the way the multicollinearity is taken care of.** Of course, this is only under the assumption that return is a mean-reverting stationary process in the long run and therefore, demeaning is effectively detrending. The demeaned regression of the 14 stocks pooled together gives the following result.

Table 13. Demeaned Regression

Demeaned regression					
Multiple R	0.97				
R Square	**0.941**				
Adjusted R Square	0.9405				
Standard Error	0.0298				
Observations	615				
ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	5	8.6067	1.7213394	**1940.67**	0
Residual	609	0.5402	0.000887		
Total	614	9.1469			
	Coefficients	*Standard Error*	*t Stat*	*P-value*	
Intercept	2E-15	**0.001**	1.366E-12	1	
$\delta_{it-1}-\mu$	**1.007**	**0.011**	**95.638**	0	
$\delta_{it}-\mu$	**-0.98**	**0.011**	**-92.4335**	0	
$\Delta d_{it}-\mu$	**1.024**	**0.011**	**91.505**	0	
$r_{mt}-\mu$	-0.0334	**0.013**	**-2.58874**	0.009863	
$(1/C_t - 1/C_{t-1})-\mu$	399.47	1128.7	0.3539289	0.7235147	

It is not surprising to see that even the demeaned regression still gives quite consistent estimates for $\beta_1=\beta_3$ and also for β_2 as in the normal regressions. Besides, its standard errors and ***t*** and ***F*** statistics, and R^2 are also still quite significant. Theoretically the demeaned regression statistic should not be different from normal regression statistic as demeaning doesn't affect the properties of the sample distribution. This will be verified in the portfolio section to follow. Therefore, it may be considered, overall, a good evidence for non-significant multicollinearity. The plot of this regression is provided in the Appendix.

On the other hand, what makes multicollinearity less of our concern is that all of the regression results show **significant *t* ratios**, **very small standard errors,** and **very high *F* ratios.** Therefore, the estimators are still **BLUE**. Although there may be slight multicollinearity, it should not adversely affect the model. Furthermore, β_1 and β_3 are always one as prescribed by the model ($\beta_1=\beta_3=1$ from $h_t = \delta_{t-1} - \rho\delta_t + \Delta d_t - \gamma\,\Delta c_t + k$).

This may be owing to the fact that this model is a straightforward extension of Gordon growth model that includes return, price and dividend all within itself. So, it may seem tautological to rewrite the present value relation $P = \dfrac{D}{R-G}$ as $R = \dfrac{D}{P} + G$ of the proposed model. However, the proposed model is a sound loglinearization of the return equation above having the growth rate of dividend and market return for G term. Therefore, it is clear that **there is no structural source of multicollinearity except for δ_{t-1} and δ_t for which the close correlation is not surprising due to lag and lead nature of these variables**. This also supports the largely held consensus that multicollinearity is mostly data problem, not the model specificaiton problem.

Even if the multicollinearity were severe, completely satisfactory cure is not available, as the remedy will inevitably involve modifying the model by dropping one or more variables. This certainly is not a desirable solution when the model is theoretically complete. Any **arbitrary** variable modification will result in specification error. Even the ridge regression generally gives slightly biased estimates only with a smaller variance and MSE and therefore, it is not a completely satisfactory solution, either. Besides, all of the Excel, SAS and Gauss regression results already indicated that **R^2 are extremely high, *F* statistics are also high and *t* ratios are generally high as well.** This doesn't fit the strong criterion for multicollinearity.

A. Out-of-Sample Testing

Our next concern is how good the model is in terms of explaining or forecasting *ex post* our variable of interest. For this one needs some other forecasting method for comparison. Then, one can apply root mean squared error (RMSE hereafter) to compare which model produces smaller errors, where $RMSE = \sqrt{\frac{1}{T}\sum_{i=1}^{T} e_i^2}$. The other forecasting method of choice had better be a pure atheoretical data generating process such as ARMA(p,q) to highlight the difference between the structural models and pure DGP.

1. Out-of-Sample Forecast

The following are 5-quarter out-of-sample forecasts by the Model followed by ARMA. The results are self-explanatory and the more straightforward comparison of these two sets of forecasts follows in the next section.

Table 14. The Model Tested by SAS

Quarter	AT&T Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	0.10936467	0.000962039	9.25519E-07	0.000805217
Dec-99	0.16348851	0.000563746	3.1781E-07	
Sep-99	-0.23937327	0.000277456	7.69817E-08	
Jun-99	0.05090522	0.000862836	7.44487E-07	
Mar-99	0.05569408	0.000727117	5.28699E-07	

Quarter	CAT Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	-0.27803491	-0.000873598	7.63173E-07	0.008411008
Dec-99	-0.14302672	0.00349676	1.22273E-05	
Sep-99	-0.08774853	0.008740785	7.64013E-05	
Jun-99	0.26447959	0.012574975	0.00015813	
Mar-99	0.00583705	0.005954693	3.54584E-05	

Quarter	Chevron Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	0.0910305	-0.010656958	0.000113571	0.018878357
Dec-99	0.01997516	-0.029709422	0.00088265	
Sep-99	-0.04681874	-0.008520388	7.2597E-05	
Jun-99	0.09788597	-0.016254774	0.000264218	
Mar-99	0.09210223	-0.00961947	9.25342E-05	

Quarter	Du Pont Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	-0.21762379	0.102897	0.010587793	0.128729533
Dec-99	0.03734811	-0.057657654	0.003324405	
Sep-99	-0.12009421	0.176741096	0.031237415	
Jun-99	0.12467203	0.14438853	0.020848048	
Mar-99	0.06701182	-0.01695613	0.00028751	

Quarter	Exxon Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	-0.02136585	0.000256723	6.59069E-08	0.000425439
Dec-99	0.06936549	0.000349524	1.22167E-07	
Sep-99	-0.00461796	0.000428906	1.8396E-07	
Jun-99	0.0937462	0.000572548	3.27811E-07	
Mar-99	-0.02993606	0.000155392	2.41468E-08	

Quarter	GM Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	0.11635737	0.026381605	0.000695989	0.069199542
Dec-99	0.1302923	0.027919487	0.000779498	
Sep-99	-0.15645274	0.124992829	0.015623207	
Jun-99	0.15839957	-0.038598167	0.001489819	
Mar-99	0.18403603	0.023786421	0.000565794	

Quarter	GT Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	-0.5160664	-0.004235512	1.79396E-05	0.005079097
Dec-99	-0.18931773	0.000550891	3.03481E-07	
Sep-99	0.16954863	0.006528967	4.26274E-05	
Jun-99	-0.00323235	0.003184462	1.01408E-05	
Mar-99	-0.01476612	0.005672535	3.21777E-05	

Quarter	IP Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	-0.27522215	0.009044054	8.17949E-05	0.02325495
Dec-99	0.14228424	0.027332803	0.000747082	
Sep-99	-0.05625828	0.02160043	0.000466579	
Jun-99	0.1590263	0.025659951	0.000658433	
Mar-99	-0.06282653	0.014466584	0.000209282	

Quarter	JPM Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	0.05465801	0.001129461	1.27568E-06	0.001103416
Dec-99	0.11755114	0.000918638	8.43895E-07	
Sep-99	-0.19028279	0.000368081	1.35483E-07	
Jun-99	0.14316175	0.001426238	2.03416E-06	
Mar-99	0.17634497	0.000762162	5.8089E-07	

Quarter	Kodak Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	-0.18781385	0.004833486	2.33626E-05	0.011757591
Dec-99	-0.12925087	0.010413954	0.00010845	
Sep-99	0.10726909	0.014448323	0.000208754	
Jun-99	0.05997977	0.011811067	0.000139501	
Mar-99	-0.11482363	0.008537882	7.28954E-05	

Quarter	3M Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	0.01195853	0.017992334	0.000323724	0.017417936
Dec-99	0.0933969	0.017991708	0.000323702	
Sep-99	0.199495	0.019415235	0.000376951	
Jun-99	0.00057119	0.009519369	9.06184E-05	
Mar-99	-0.0304927	0.009926864	9.85426E-05	

Quarter	Phil Morris Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	-0.09253272	0.03659502	0.001339196	0.088943441
Dec-99	-0.35777985	0.001429837	2.04443E-06	
Sep-99	-0.23770633	0.103272877	0.010665287	
Jun-99	0.03641413	0.118428164	0.01402523	
Mar-99	-0.46965707	0.074913188	0.005611986	

Quarter	SBC Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	-0.12727803	0.085897215	0.007378332	0.098330159
Dec-99	-0.20706838	0.093828298	0.00880375	
Sep-99	0.12812864	0.087191552	0.007602367	
Jun-99	-0.21013176	0.091647899	0.008399337	
Mar-99	0.11809331	0.080569819	0.006491496	

Quarter	Texaco Out-of-Sample	Forecast Error	ESS	RMSE
Mar-00	0.04774359	-0.041030387	0.001683493	0.044173777
Dec-99	-0.09051522	-0.043920153	0.00192898	
Sep-99	0.06584253	-0.039186796	0.001535605	
Jun-99	0.1463111	-0.037413414	0.001399764	
Mar-99	0.12180428	-0.03546053	0.001257449	

The last row of the ARMA below explains the type of process and the number of lags used.

Table 15. ARMA Out-of-Sample Tested by SAS

Quarter	AT&T	Forecast	Std Error	ESS	RMSE
Mar-00	0.08037	0.0466	0.1327	0.001140652	
Dec-99	-0.0097	0.0486	0.1321	0.003402887	
Sep-99	-0.0553	0.0156	0.1278	0.00503236	
Jun-99	0.08163	-0.0327	0.1272	0.013071622	
Mar-99	0.08248	-0.0186	0.111	0.010217725	
e p=(1,2);	indicates	AR w/ spikes at	1,2 lags.	0.032865245	0.081074343

Quarter	CAT	Forecast	Std Error	ESS	RMSE
Mar-00	-0.2789	0.047	0.1414	0.106216354	
Dec-99	-0.1395	0.0427	0.1405	0.033207758	
Sep-99	-0.079	0.0203	0.1405	0.009862027	
Jun-99	0.27705	0.0279	0.1404	0.062077996	
Mar-99	0.01179	0.0358	0.1404	0.000576396	
e q=(1,4,8);	indicates	MA w/ spikes at	1,4,8 lags.	0.211940531	0.20588372

Quarter	Chevron	Forecast	Std Error	ESS	RMSE
Mar-00	0.08037	0.0992	0.1289	0.000354435	
Dec-99	-0.0097	0.0898	0.1289	0.00990707	
Sep-99	-0.0553	0.1005	0.1289	0.024285833	
Jun-99	0.08163	0.1327	0.1271	0.002608023	
Mar-99	0.08248	0.2285	0.1199	0.021321034	
e p=(1,2,19);	indicates	AR w/ spikes at	1,2,19 lags.	0.058476395	0.108144714

Quarter	Du Pont	Forecast	Std Error	ESS	RMSE
Mar-00	-0.1147	-0.0456	0.1552	0.004778513	
Dec-99	-0.0203	-0.0307	0.1552	0.000107962	
Sep-99	0.05665	0.0191	0.1686	0.001409768	
Jun-99	0.26906	-0.0521	0.1686	0.103144106	
Mar-99	0.05006	0.0213	0.1686	0.00082689	
e q=(2,5);	indicates	MA w/ spikes at	2, 5 lags.	0.110267238	0.148504033

Quarter	**GM**	Forecast	Std Error	ESS	RMSE
Mar-00	0.14274	0.1037	0.1413	0.001524041	
Dec-99	0.15821	0.1388	0.139	0.000376818	
Sep-99	-0.0315	0.1604	0.1373	0.036810226	
Jun-99	0.1198	0.1332	0.1373	0.000179522	
Mar-99	0.20782	0.1086	0.1373	0.009845094	
e q=(3,4);	indicates	MA w/ spikes at	3, 4 lags.	0.048735701	0.098727606

Quarter	**GT**	Forecast	Std Error	ESS	RMSE
Mar-00	-0.1774	0.0011	0.2021	0.031867435	
Dec-99	-0.5203	0.0011	0.2021	0.271859956	
Sep-99	-0.1888	0.0011	0.2021	0.036049418	
Jun-99	0.17608	0.0011	0.2021	0.030617159	
Mar-99	-5E-05	0.1436	0.1429	0.020634716	
e q=(1)	indicates	MA w/ spikes at	1st lag.	0.391028684	0.279652886

Quarter	**JPM**	Forecast	Std Error	ESS	RMSE
Mar-00	0.05579	-0.0028	0.1864	0.003432492	
Dec-99	0.11847	-0.0028	0.1864	0.014706358	
Sep-99	-0.1899	-0.0028	0.1864	0.035011915	
Jun-99	0.14459	-0.0028	0.1864	0.021723219	
Mar-99	0.17711	-0.1226	0.1424	0.089824366	
e q=(1)	indicates	MA w/ spikes at	1st lag.	0.164698349	0.181492892

Quarter	**Kodak**	Forecast	Std Error	ESS	RMSE
Mar-00	-0.183	0.0208	0.1622	0.041526438	
Dec-99	-0.1188	0.0039	0.1618	0.015064349	
Sep-99	0.12172	-0.0647	0.1526	0.034751451	
Jun-99	0.07179	0.0795	0.149	5.94312E-05	
Mar-99	-0.1063	0.0038	0.1318	0.012118872	
e p=(1,2,12)	indicates	MA w/ spikes at	1,2,12 lags.	0.103520542	0.143889223

Quarter	**3M**	Forecast	Std Error	ESS	RMSE
Dec-99	0.02995	0.0007	0.1522	0.000855613	
Sep-99	0.11139	0.0007	0.1522	0.012251968	
Jun-99	0.21891	0.0007	0.1522	0.047615707	
Mar-99	0.01009	-0.147	0.1076	0.024677443	
e q=(1)	indicates	MA w/ spikes at	1st lag.	0.085400731	0.130691033

Quarter	Philip Morris	Forecast	Std Error	ESS	RMSE
Mar-00	-0.0559	0.0083	0.175	0.004126482	
Dec-99	-0.3564	0.0083	0.175	0.132969629	
Sep-99	-0.1344	0.0083	0.175	0.020372837	
Jun-99	0.15484	0.0083	0.175	0.021474645	
Mar-99	-0.3947	-0.0508	0.1448	0.118297394	
e q=(1)	indicates	MA w/ spikes at	1st lag.	0.297240988	0.243820011

Quarter	SBC	Forecast	Std Error	ESS	RMSE
Mar-00	-0.2439	0.2373	0.1121	0.231555104	
Dec-99	-0.0414	0.2756	0.1089	0.100476834	
Sep-99	-0.1132	0.3209	0.1036	0.188477607	
Jun-99	0.21532	0.2984	0.102	0.006902255	
Mar-99	-0.1185	0.2867	0.0931	0.164173962	
e p=(1,3,26)	indicates	MA w/ spikes at	1,3,26 lags.	0.691585763	0.37191014

Quarter	Texaco	Forecast	Std Error	ESS	RMSE
Mar-00	0.00671	0.0062	0.1284	2.63377E-07	
Dec-99	-0.1344	-0.0134	0.1284	0.014649562	
Sep-99	0.02666	-0.0034	0.1268	0.000903347	
Jun-99	0.1089	0.0002	0.1253	0.011815186	
Mar-99	0.08634	-0.1007	0.0975	0.034985364	
e p=(1,2,22) q=(1)	indicates AR	at 1,2,22 lags &	MA at 1 lag	0.062353723	0.111672488

2. Model vs. ARMA

The following is a comparison of the Model RMSE and ARMA RMSE. The last column shows the difference obtained by subtracting ARMA RMSE from the Model RMSE. Any negative value would indicate that ARMA RMSE>Model RMSE meaning that the Model has a better out-of-sample forecasting power.

Table 16. Model vs. ARMA

Company	**Model RMSE**	**ARMA RMSE**	**Model RMSE - ARMA RMSE**
AT&T	0.000805217	0.081074343	**-0.080269126**
Caterpillar	0.008411008	0.20588372	**-0.197472713**
Chevron	0.018878357	0.108144714	**-0.089266357**
Du Pont	0.128729533	0.148504033	**-0.019774499**
Exxon	0.000425439	N/A	N/A
GM	0.069199542	0.098727606	**-0.029528064**
Goodyear	0.005079097	0.279652886	**-0.274573789**
Int'l Paper	0.02325495	N/A	N/A
JP Morgan	0.001103416	0.181492892	**-0.180389476**
Kodak	0.011757591	0.143889223	**-0.132131631**
3M	0.017417936	0.130691033	**-0.113273097**
Philip Morris	0.088943441	0.243820011	**-0.15487657**
SBC	0.098330159	0.37191014	**-0.273579981**
Texaco	0.044173777	0.111672488	**-0.067498711**

In all of the cases where ARMA process could be identified, the model produced smaller RMSE than ARMA indicating that this model has better out-of-sample forecasting power than ARMA.

V. Empirical Applications

I have constructed a hypothetical portfolio consisting of 1 share each of these 14 Dogs of the Dow stocks and compared its performance with the return on the market index so as to evaluate the Dogs of the Dow strategy using real asset-pricing model involving consumption.

Table 17. HARA Estimates of Portfolio

portfolio parameter estimates			
			$\kappa>1 \rightarrow$ **HyDARA**
$\beta_1=\beta_3$	**1.007331729**	**1.023517657**	$\kappa=1 \rightarrow$ **CARA**
$\beta_2=-\rho$	**-0.981918725**		$\kappa=0 \rightarrow$ **quadratic U fn**
$\beta_4=-\gamma$	-0.033374462	$\kappa=(1+\gamma)/\gamma$	30.96302991
$\beta_5=\beta_4*\phi$	399.4724302	$\phi=\beta_5/\beta_4$	-11969.40438

First, to diagnose whether HARA is a relevant assumption I have checked the following kappa statistic, which turned out to support HARA.

Table 18. Portfolio Kappa Statistic

	σ_γ	$\sigma^2_\kappa=(-1/\gamma^2)^2\sigma_\gamma$	σ_κ	$t_\kappa=(\kappa-1)/\sigma_\kappa$
portfolio	**0.0128922**	10391.29175	101.9376856	**0.2939348**

Even for the multicollinearity, the following tests checked out favorably indicating only moderate degree of multicollinearity.

Table 19. Correlation Check for Portfolio

Correlation Matrix	h_{it}	δ_{it-1}	δ_{it}	Δd_{it}	r_{mt}	$(1/C_t - 1/C_{t-1})$
h_{it}	1					
δ_{it-1}	0.2296	1				
δ_{it}	0.1154	**0.887**	1			
Δd_{it}	0.065	-0.2518	0.1978	1		
r_{mt}	0.1764	0.0103	-0.0028	0.042272	1	
$(1/C_t - 1/C_{t-1})$	0.0166	-0.2333	-0.237	0.0158942	-0.2453106	1

Table 20. Auxiliary Regression of the Portfolio

Dependent	δ_{it-1}	δ_{it}	Δd_{it}	r_{mt}	$(1/C_t - 1/C_{t-1})$	Overall R^2
R_i^2 portfolio	**0.978179597**	**0.977802944**	0.902097451	0.115410416	0.129685914	**0.9409447**

Table 21. Condition Number of the Portfolio

Eigen values	Condition No.
eigenva1	0.01003
eigenva2	0.703573
eigenva3	1.091521
eigenva4	1.188357
eigenva5	2.006518
$k=(\lambda_{Max}/\lambda_{min})^{1/2}$	14.14362

Overall, the portfolio is a well-balanced and well-behaved one meeting all the theoretical and statistical criteria, which means that we may safely proceed with the Dividend-Yield strategy set out in the beginning of this paper.

A. Portfolio by Dividend Yield Strategy

The following are the forecast of the portfolio for 5 quarters out of sample and their RMSE.

Table 22. Portfolio Returns over 5 Quarter Out-of-Sample Forecasting Horizon

14 stock portfolio	μh_{it}	$\mu\delta_{it-1}$	$\mu\delta_{it}$	$\mu\Delta d_{it}$	r_{mt}	$(1/C_t - 1/C_{t-1})$
Mar-00	-0.06752	-4.599883882	-4.517981364	0	-0.10774704	-2.98769E-06
Dec-99	-0.04506	-5.023362515	-4.968706	0.010298665	0.180387388	-2.67431E-06
Sep-99	-0.05384	-4.867151798	-5.023362515	-0.228853752	-0.108604648	-2.89253E-06
Jun-99	0.154911	-4.981885146	-4.867151798	0.242718717	0.008624896	-3.19655E-06
Mar-99	-0.00111	-4.987607797	-4.981885146	0.001287036	-0.132738588	-2.44952E-06

All the variables are in natural log.

Table 23. Accuracy of the Out-of-Sample Forecasts of the Portfolio

Out-of-Sample Forecast	Forecast Error	ESS	RMSE
-0.0645	-0.0030559	9.339E-06	**0.01240291**
-0.0469	0.0018668	3.485E-06	
-0.0719	0.0180684	0.0003265	
0.1388	0.0161406	0.0002605	
0.0028	-0.0039393	1.552E-05	

This 5 period out-of-sample testing is based on the regression result over 9 year data range (1989~1998) of the portfolio and produced RMSE of 0.01240291, which is curiously slightly higher than the RMSE of 0.011304839 from the regression over just 1 year data range reported in the following section. This is probably for the same reason as Campbell & Shiller's finding about the short-term coherence between the theoretical and actual variables although the coherence decreases greatly over the long run. (See pp. 12~14 of this dissertation.)

B. Out-of-Sample Performance of Portfolio '98 vis-à-vis Market

The following is the corresponding test result to the above except for the fact that the data range used is limited to only 1 year of 1998.

Table 24. '98 Dogs Portfolio Returns over 5 Quarter Out-of-Sample Forecasting Horizon

14 stock portfolio	μh_{it}	$\mu \delta_{it-1}$	$\mu \delta_{it}$	$\mu \Delta d_{it}$	r_{mt}	$(1/C_t - 1/C_{t-1})$
Mar-00	-0.067515495	-4.599883882	-4.517981364	0	-0.10774704	-2.98769E-06
Dec-99	-0.045061575	-5.023362515	-4.968706	0.010298665	0.180387388	-2.67431E-06
Sep-99	-0.053835424	-4.867151798	-5.023362515	-0.228853752	-0.108604648	-2.89253E-06
Jun-99	0.154910543	-4.981885146	-4.867151798	0.242718717	0.008624896	-3.19655E-06
Mar-99	-0.001113337	-4.987607797	-4.981885146	0.001287036	-0.132738588	-2.44952E-06

This type of forecasting should be effective for the case where one has only limited amount of data available for such stocks that are relatively new in the market especially as the test produces low RMSE.

Table 25. Accuracy of the Out-of-Sample Forecasts of the '98 Dogs Portfolio

Out-of-Sample Forecast	Forecast Error	ESS	RMSE
-0.070887475	0.00337198	1.13702E-05	**0.011304839**
-0.049280047	0.004218472	1.77955E-05	
-0.067209277	0.013373853	0.00017886	
0.145010952	0.009899591	9.80019E-05	
0.013210416	-0.014323753	0.00020517	

The following table shows the performance of this portfolio vis-à-vis the market.

Table 26. '98 Dogs Portfolio performance vis-à-vis the Market

Date	μh_{it}	DJI	SP500	DJI–portfolio	SP500–portfolio
Mar-00	-0.067515495		0.022674962	0.067515495	0.090190457
Dec-99	-0.045061575	0.106365138	0.138710235	0.151426713	0.183771809
Sep-99	-0.053835424	-0.059507506	-0.064478896	-0.005672082	-0.010643472
Jun-99	0.154910543	0.114263968	0.068109889	-0.040646576	-0.086800655
Mar-99	-0.001113337	0.063793532	0.048622402	0.064906868	0.049735739

For the overall observed period, the Dogs of the Dow portfolio seems to have slightly under-performed the market. However, during the 3rd quarter of '99, when the markets and the portfolio all show negative log returns, the portfolio clearly fared better reaffirming the popular consensus of the financial market that the Dogs do better than the market in the down-market and underperform the market in the up-market. However, during the 2nd quarter the portfolio overperformed the market even in the up-market.

It is not surprising that the portfolio did not do much better than the market if the random walk holds in the short run for the quarterly return. However, **doing better than the market during the down-market is the strength of this portfolio. This might suggest a practical strategy to beat the downside risk of the short run random walk.**

On the other hand, if random walk absolutely holds, then strategies are pointless. As long as the efficient market prevents strategies from beating the market in the short run, it would rather be a wise choice to **ride along the market during the up-market** and **minimize the losses during the down-market than trying to beat the market at all times**. This portfolio seems to provide exactly this kind of protection.

VI. Conclusion

Despite limited multicollinearity in some data series, the overall test results proved to be quite robust in the light of the different other variants of tests performed such as auxiliary regressions, data pooling, and detrending. All of the regressions estimated β_1 and β_3 to be not significantly different from 1 exactly as prescribed by the model and β_2 to be within the reasonably consistent range with Campbell & Shiller's estimates. The parameter of our interest β_5, which adds the contribution of HARA to the model actually proved to be quite significant in 8 out of 14 cases. On the other hand, β_4, the parameter that represents the traditional explanatory variable based on CRRA, actually performed poorly producing only 1 significant estimate out of 14 cases.

However, the auxiliary regressions of the portfolio over the entire data range that produced higher R^2s for δ_{t-1} and δ_t as dependent variables might most likely indicate that especially in the portfolio case all the given variables explain the log dividend-price ratio better than the other ways around. This may also be the reason why Campbell & Shiller chose the VAR model. As the lag and lead relation is most explicit in the dividend price ratio, there may be endogeneity between each of the regressors as discussed in the chapters on HARA of this dissertation. When these two aspects are both present in the model, it would be necessary to encompass all the possible Granger causalities. Then, VAR would come up as a logical solution to these issues.

I also considered other possible reasons why Campbell & Shiller chose VAR out of all the possible modeling options. However, they did not explicitly provide any compelling rationale as to why VAR would make their best choice among all the competing models. It is quite conceivable that in studying the portfolio performance the log dividend-price ratio might be an ideal candidate for the dependent variable if it happens to be the variable of our interest as well. Perhaps that is why Campbell & Shiller deliberately chose to model it with respect to log dividend-price ratio. Therefore, the dividend-price ratio model based on HARA assumption could possibly be the next step in further research.

However, as discussed previously in chapter III of this dissertation, Campbell & Shiller's VAR did not quite produce desirable results as their VAR tests rejected more and more as the returns horizon increased. In the limit, at $i=$□ the H_0: "δ_t equals the unrestricted VAR forecast of the present value of future real dividend growth." was rejected at better than 0.1% level. And the weak relation between the theoretical δ_t and the actual δ_t strongly contradicted their model. In this light, my **HARA-based CCAPM with time-varying technique was concluded to be a better model than VAR at least for this study**.

Also, earnings ratio or its variants has been empirically an important variable in a number of popular asset-pricing models. However, it seems that so far they have been rather added in the model arbitrarily on empirical ground without sufficient theoretical corroboration to legitimately derive them into the model. Therefore, further studies to theoretically incorporate earnings variable into the HARA-based CCAPM type model would be in order.

I have also diagnosed HARA possibility and validated theoretically and empirically the popular Dogs-of-the Dow strategy. With HARA I have reinstated the relevance of risk aversion in the asset-pricing model. Transient income does not affect consumption. Therefore, change in transient income is not matched by a commensurate change in consumption, and consumption tends to be smooth. **The built-in progressively decreasing curvature of the HARA-based consumption ensures this smooth response of consumption to income**. Therefore, **it is only natural that the variance of consumption becomes smaller than the stock market volatility past a certain point on the curve**. This means that HARA is a more realistic explanation for why consumption tends to be smooth vis-à-vis the market. Even if HARA-based consumption may not completely track income in reality, **the model still closely tracked and forecasted the returns path**. This may be another strength of HARA-based model

CRRA cannot explain progressively smoothing consumption as income increases, because CRRA assumes proportional changes in consumption in response to changes in relative income. This would imply that there is a point or a separating equilibrium where MPC_{HARA} is tangent to MPC_{CRRA} somewhere along the curvature of the consumption. Past this point MPC_{HARA} will become less than MPC_{CRRA}.

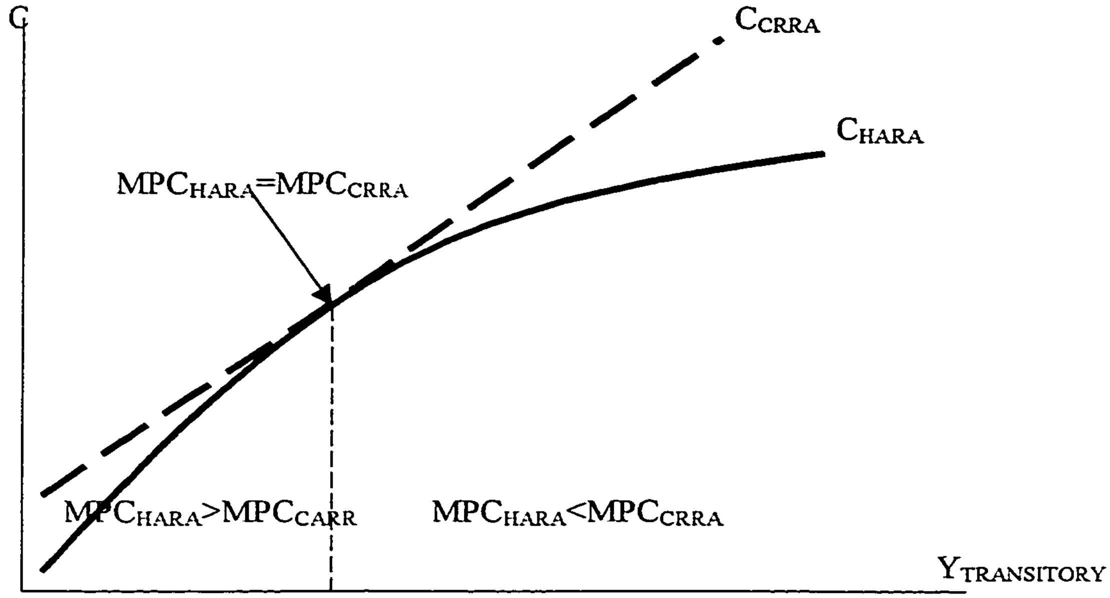

Figure 2. Separating Equilibrium between HARA and CRRA.

Certainly it is not a puzzle in itself that consumption is too smooth, but that the risk premium is too high to be explained by the smooth consumption is. This dissertation might only partially answer the second puzzle, but it may answer at least the first puzzle. Under the permanent income hypothesis, CRRA may be a relevant assumption, but with changes in transitory income, HARA may be the relevant one. Intuitively, it indicates that no matter whether the windfall gain in income is great or small, as long as it is transient, people's general attitude toward risk should be absolutely averse, not relatively averse.

This study also examined how and why consumption variable may drop out in steady state or under the assumption of the Lucas Tree model. In steady state consumption growth rate is a constant such as g including 0. If it is 0 under CRRA assumption, utility function may be irrelevant. In non-equilibrium, however, due to incomplete substitution between consumption and dividend, which is more realistic, utility function may still be a relevant variable as suggested by the HARA assumption and kappa statistic.

The model also outperformed a simple DGP such as ARMA in an out-of-sample testing, which is rare for a structural model. Empirically, the hypothetical portfolio using dividend yield strategy also fared well vis-à-vis the market suggesting the practical value of this model. Therefore, the main contribution of this dissertation may be summed up in 3 points: i) The risk-aversion mechanism in the utility function was diagnosed and HARA was found to be quite a relevant factor. This also conforms to Vinod's (1999) test result that HARA models are statistically significantly different from the traditional CRRA.; ii) It was verified that the utility function is an integral part of the long run asset-pricing. Even aside from the theoretical exercise, it is also logically quite probable that the more the true consumption path is revealed in the long run, and thus the permanent income path as well, the better the asset return would track the consumption.; iii) The popular Dividend yield strategy is theoretically and empirically reaffirmed to be a very effective strategy in portfolio management. It is also suggested to be a strategy to beat the downside risk of the short-run random walk.

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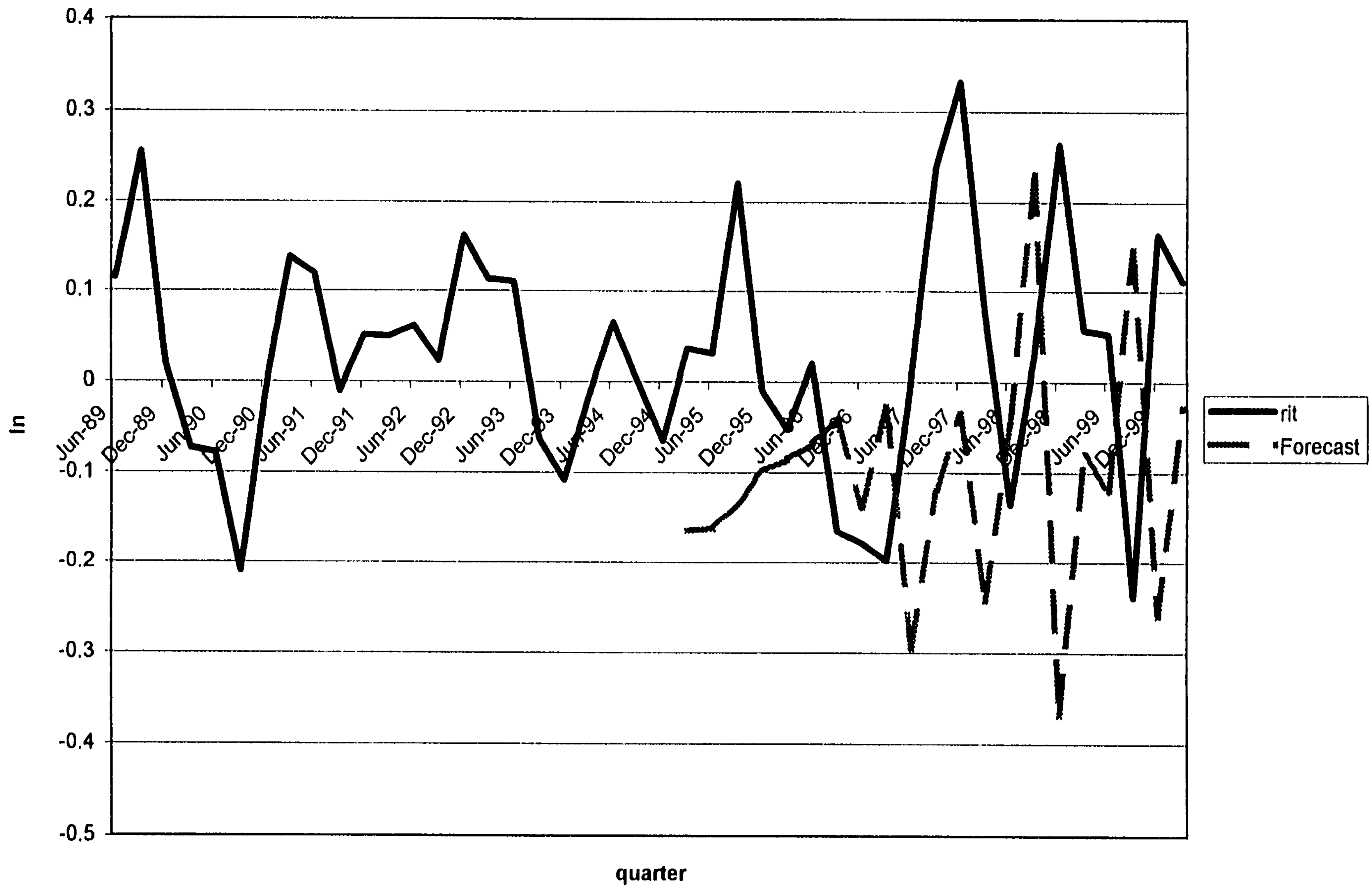
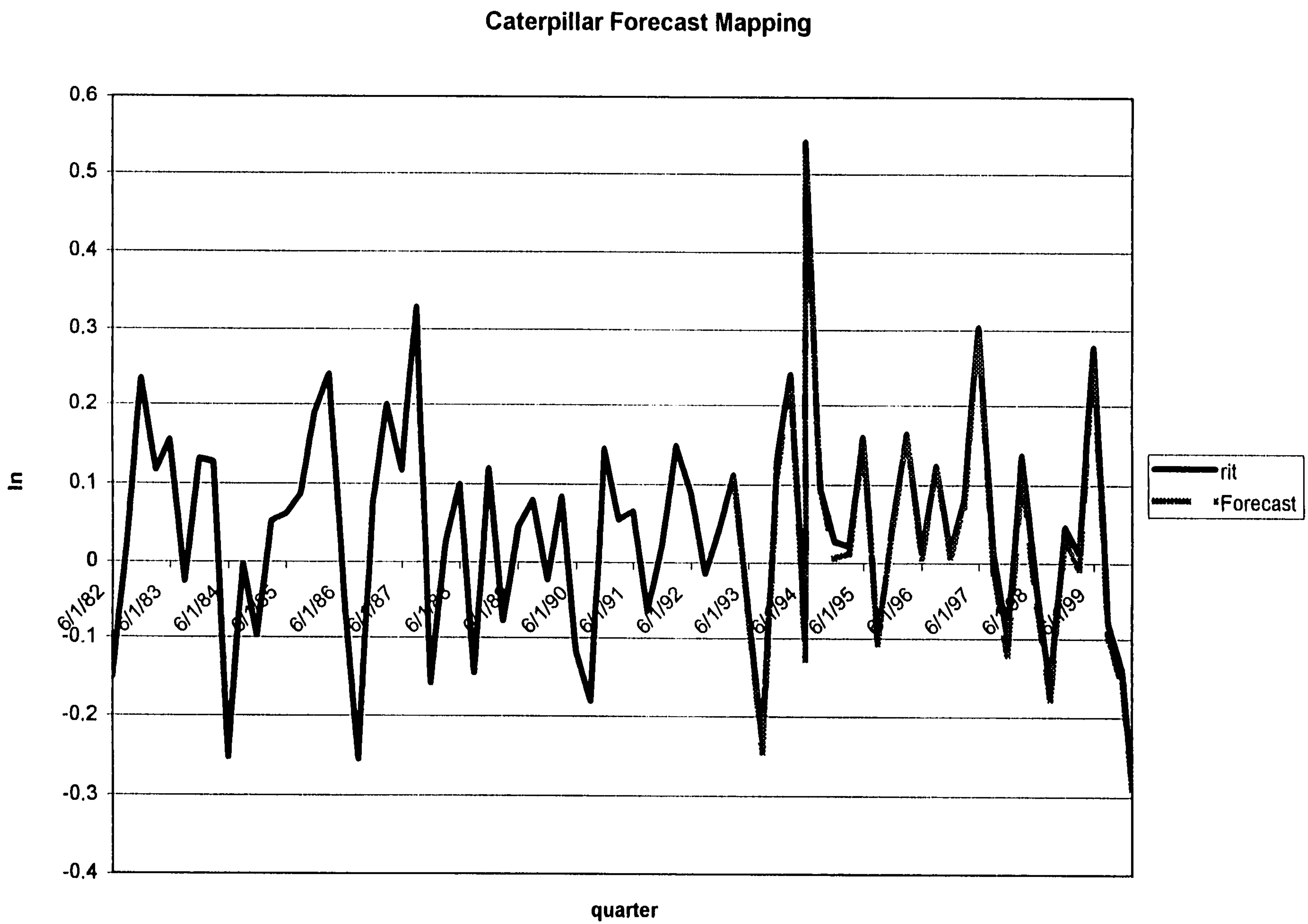
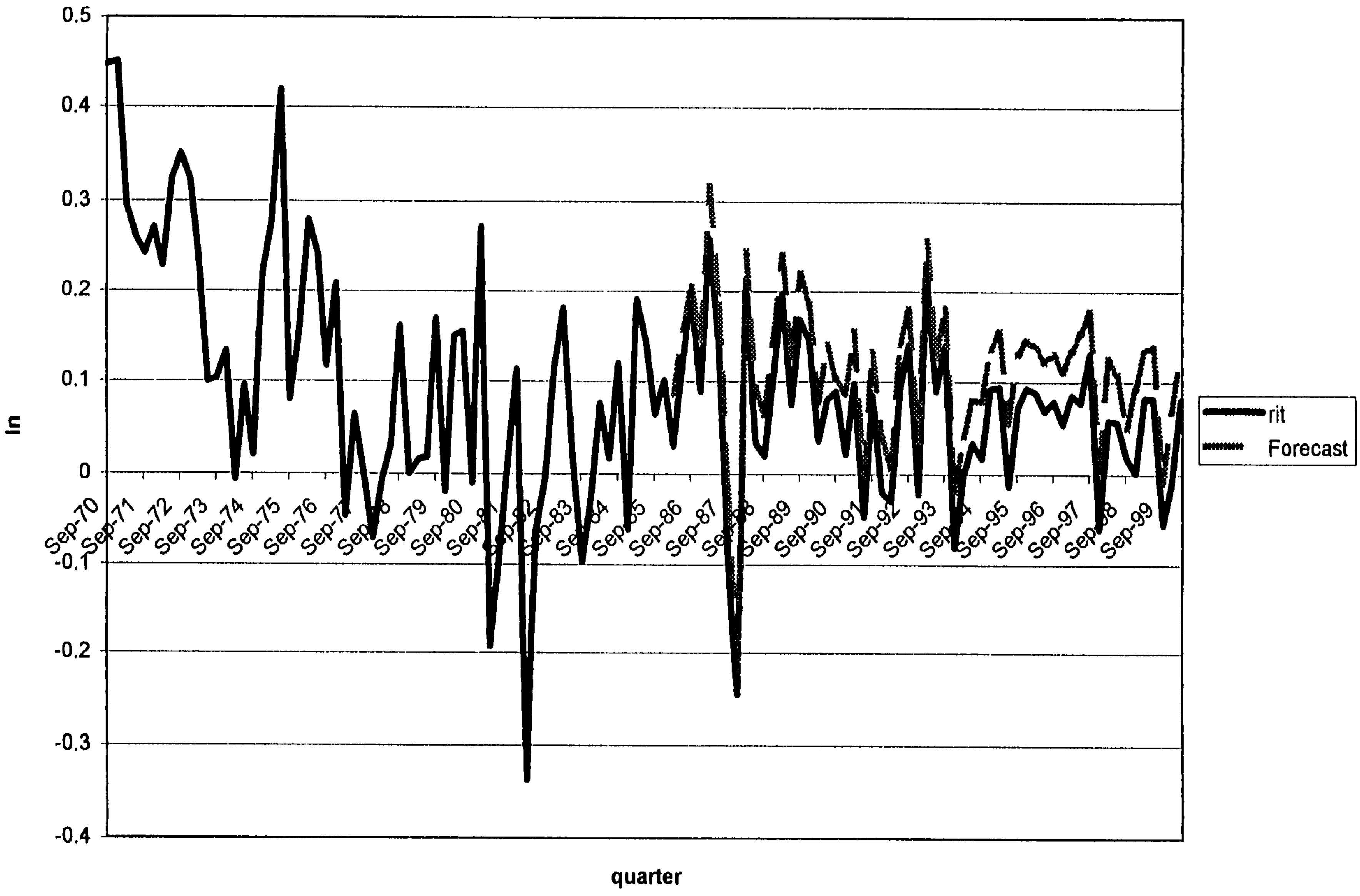
AT&T Forecast Mapping

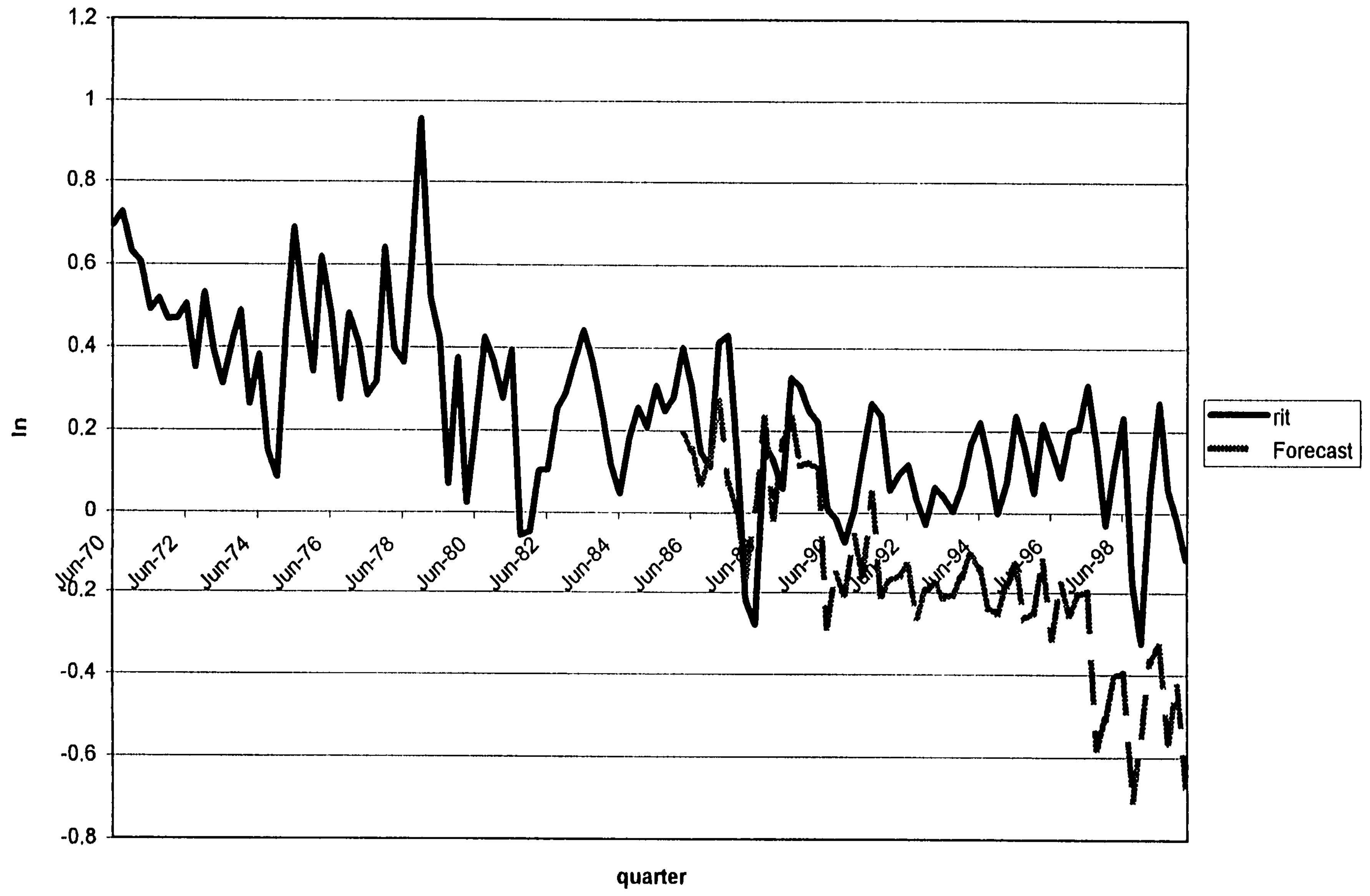



Caterpillar Forecast Mapping
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
ln
6/1/82
6/1/83
6/1/84
6/1/86
6/1/90
6/1/91
6/1/92
6/1/93
6/1/95
6/1/97
quarter
rit
Forecast

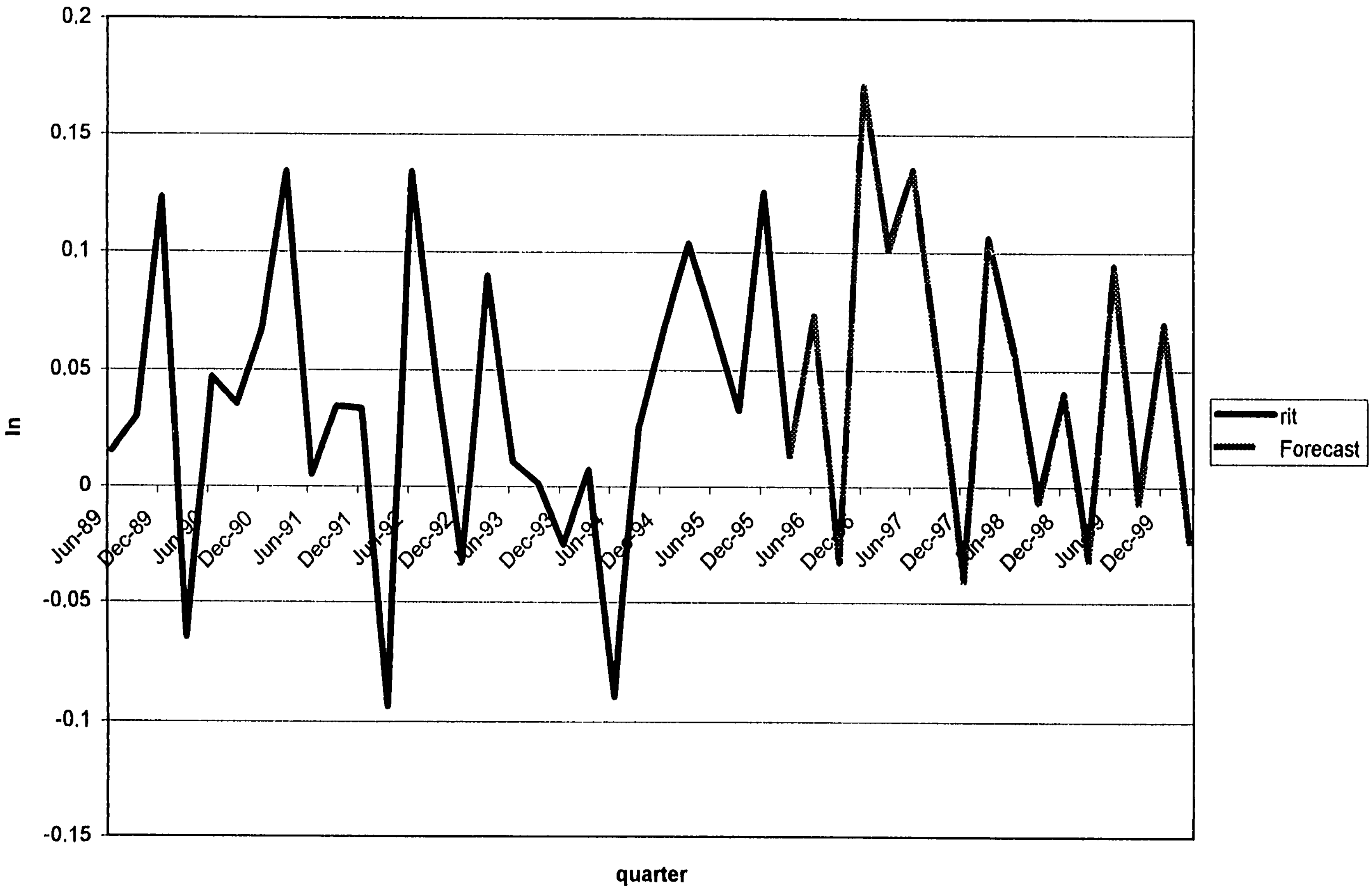
Chevron Forecast Mapping



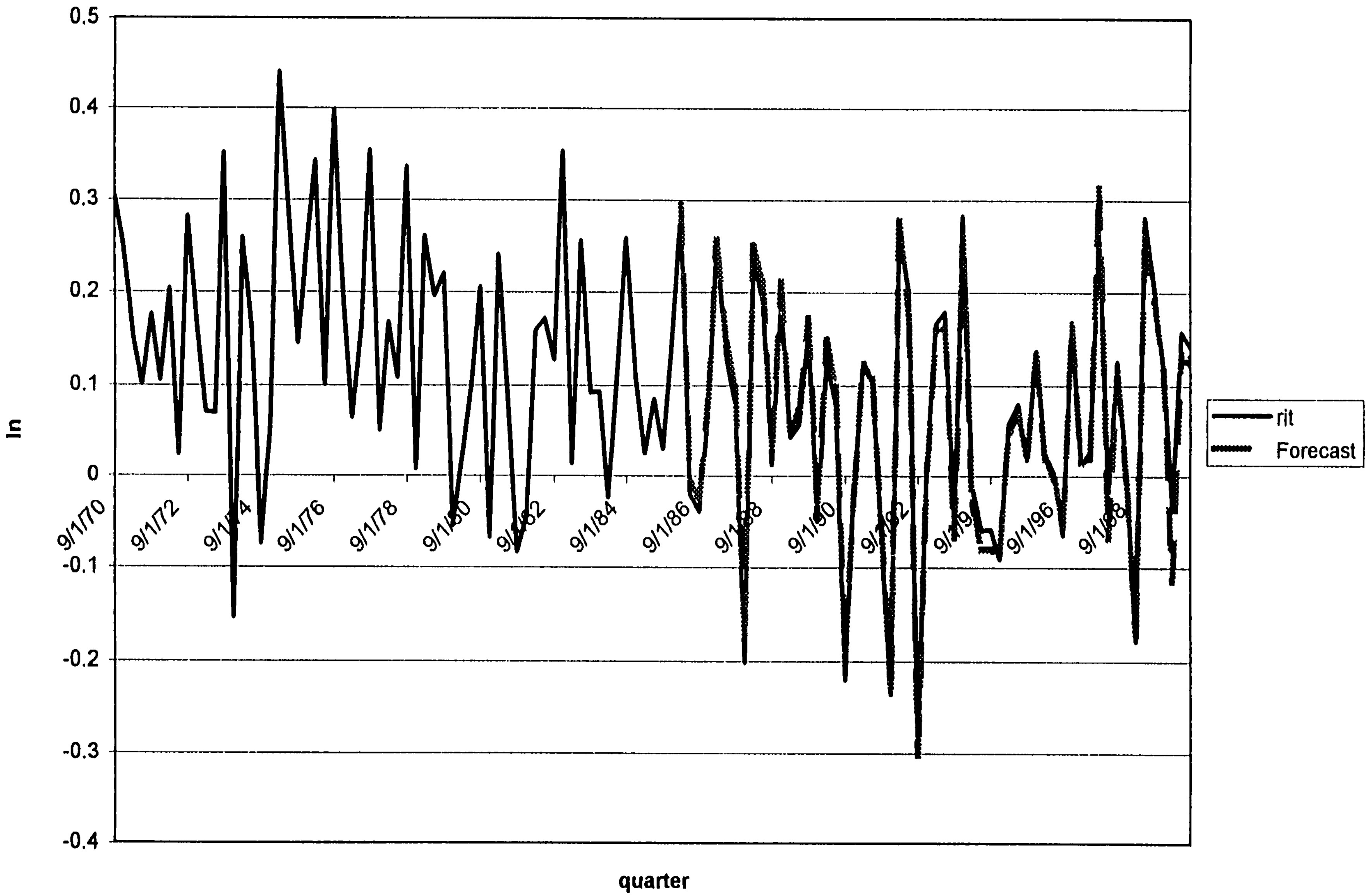
Du Pont Forecast Mapping

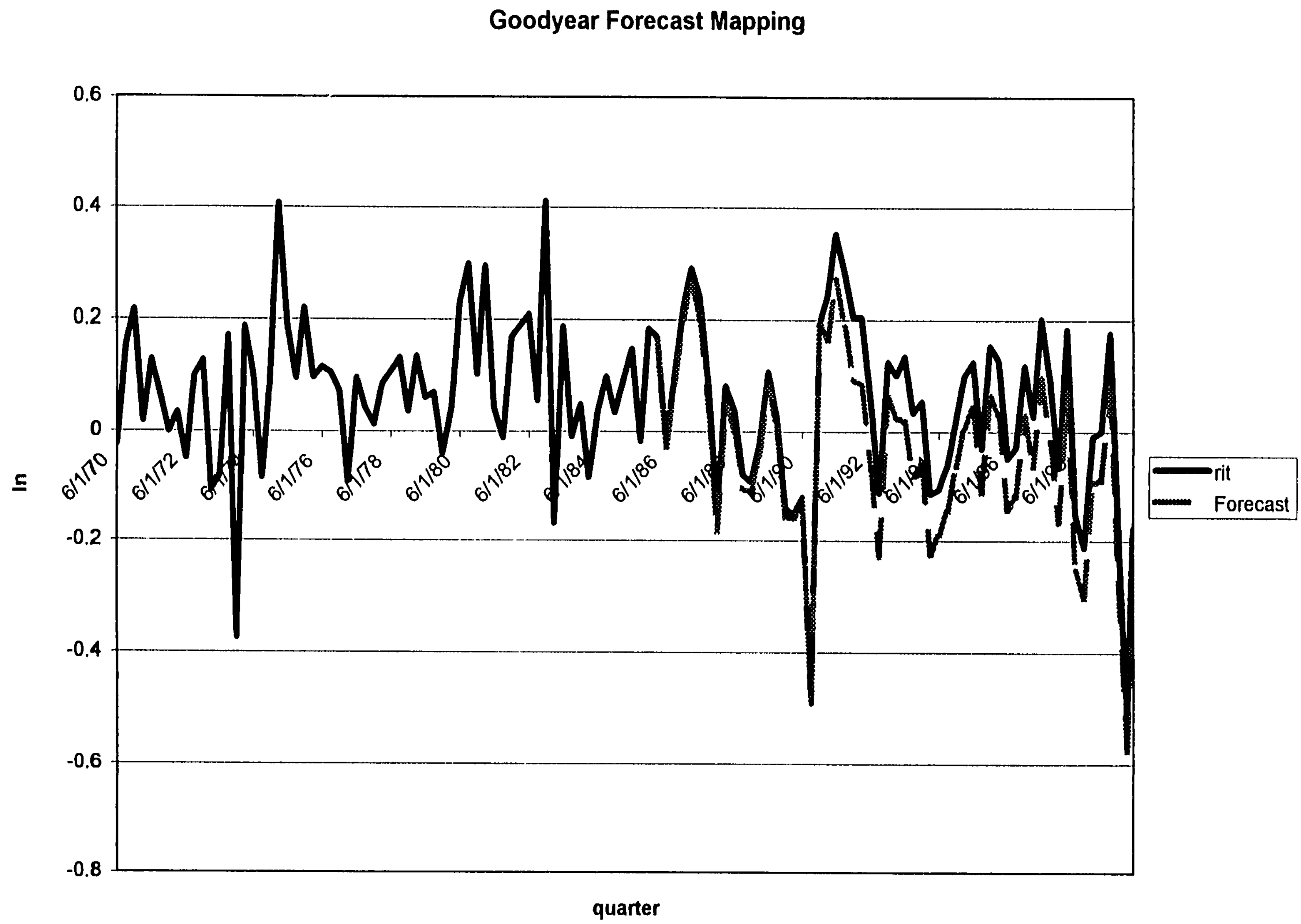


Exxon Forecast Mapping



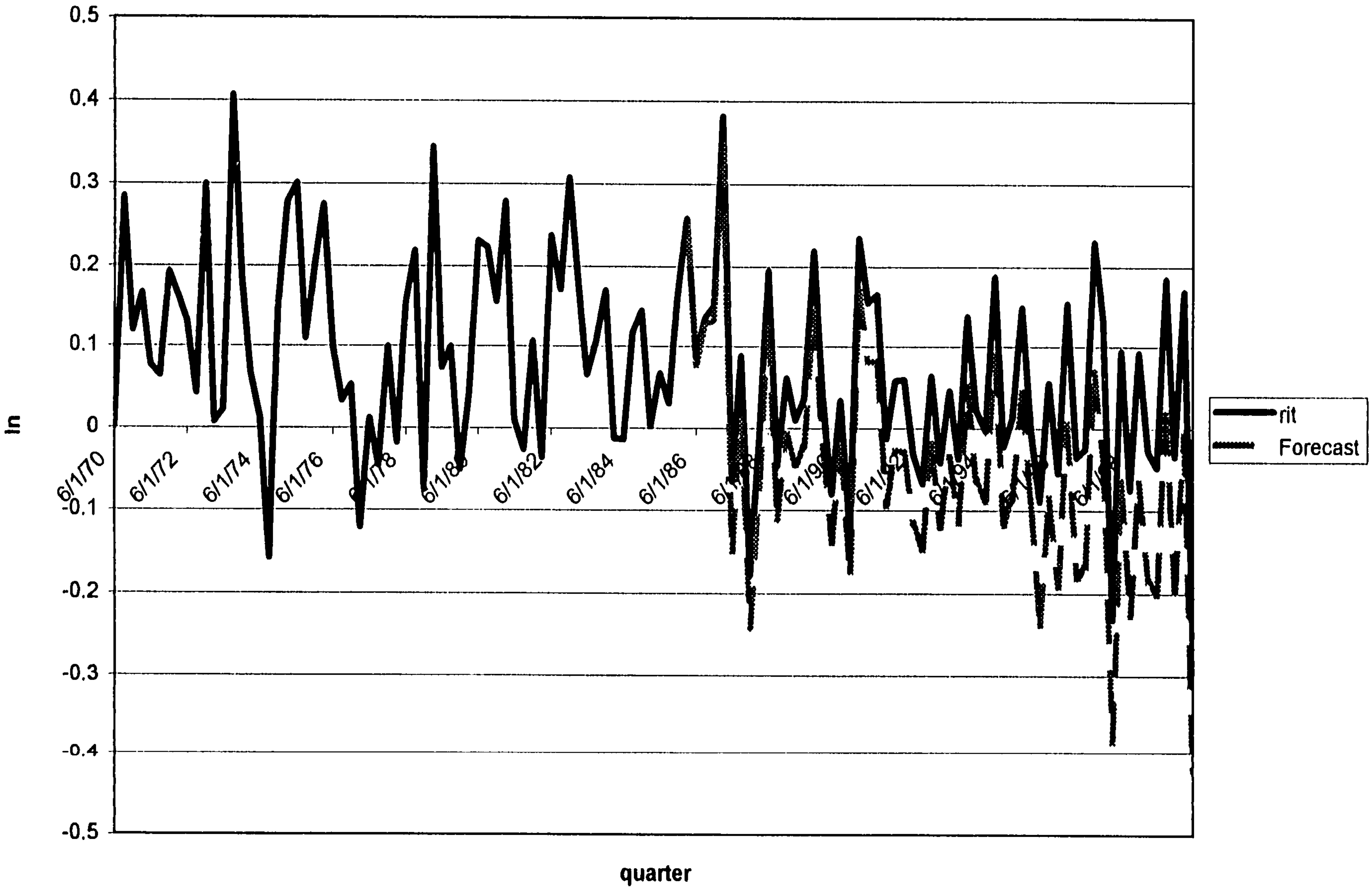
GM Forecast Mapping



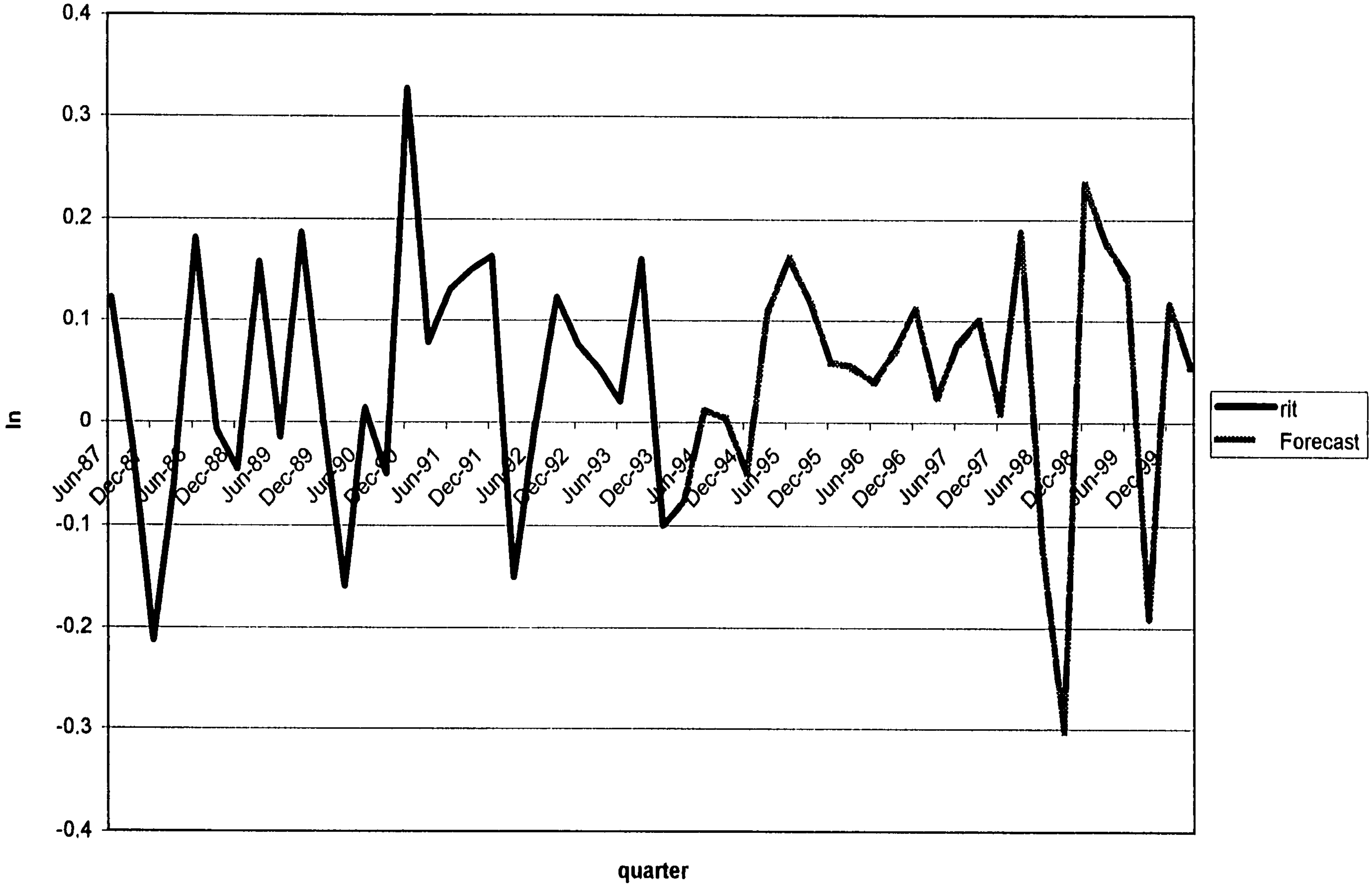


Goodyear Forecast Mapping
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
ln
6/1/70
6/1/72
6/1/76
6/1/78
6/1/80
6/1/82
6/1/86
6/1/92
rit
Forecast
quarter

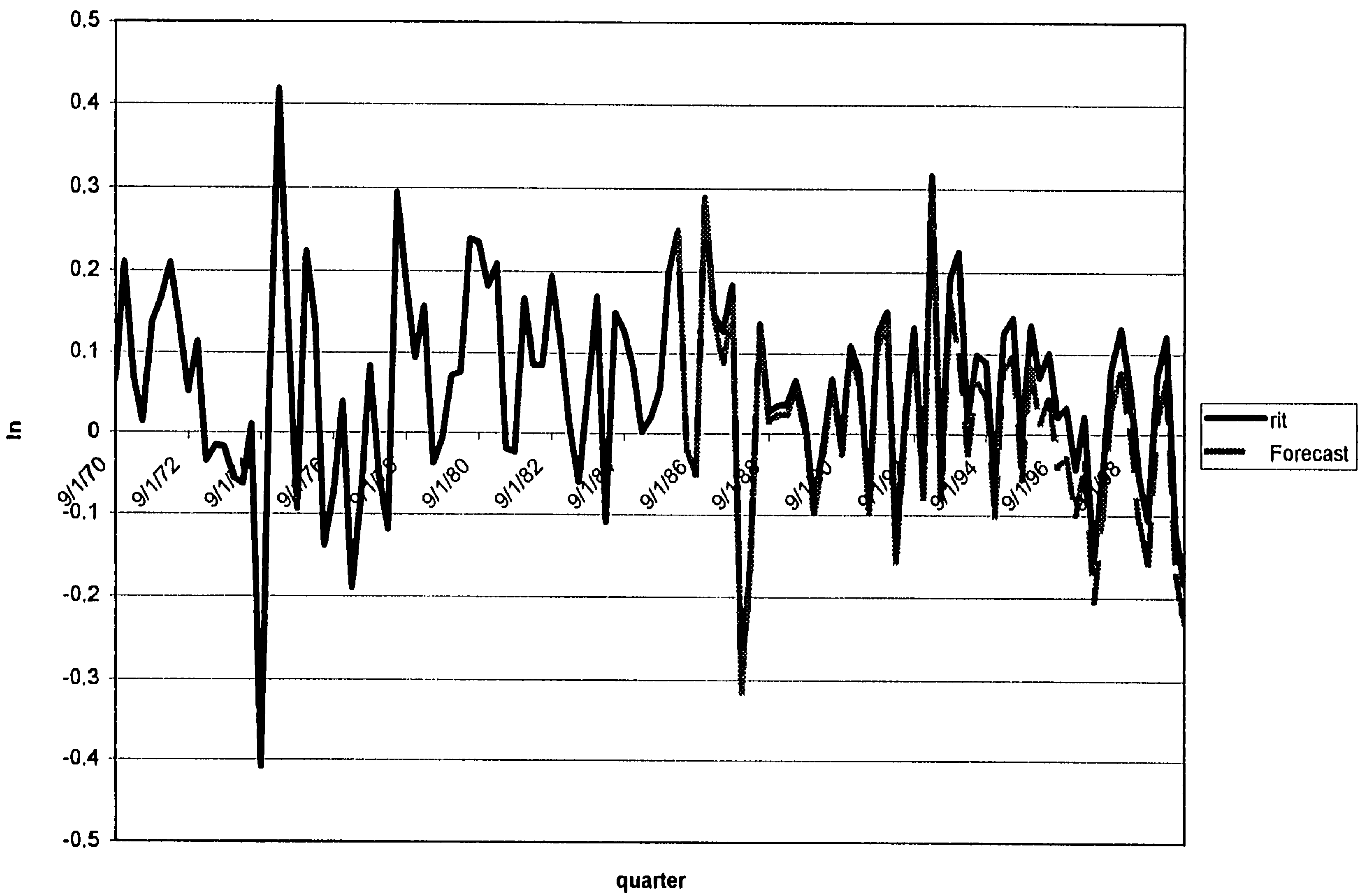
Int'l Paper Forecast Mapping



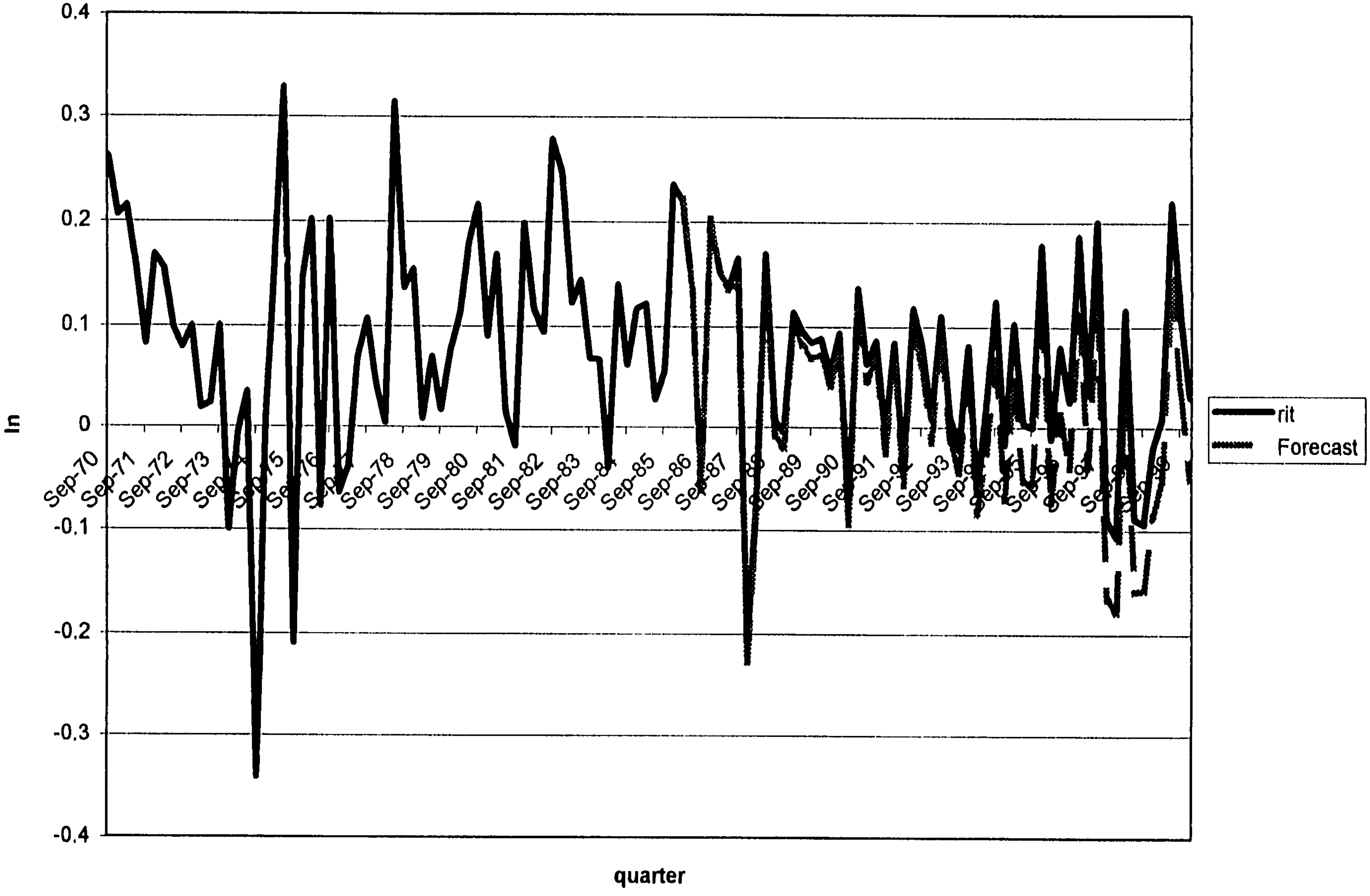
JP Morgan Forecast Mapping



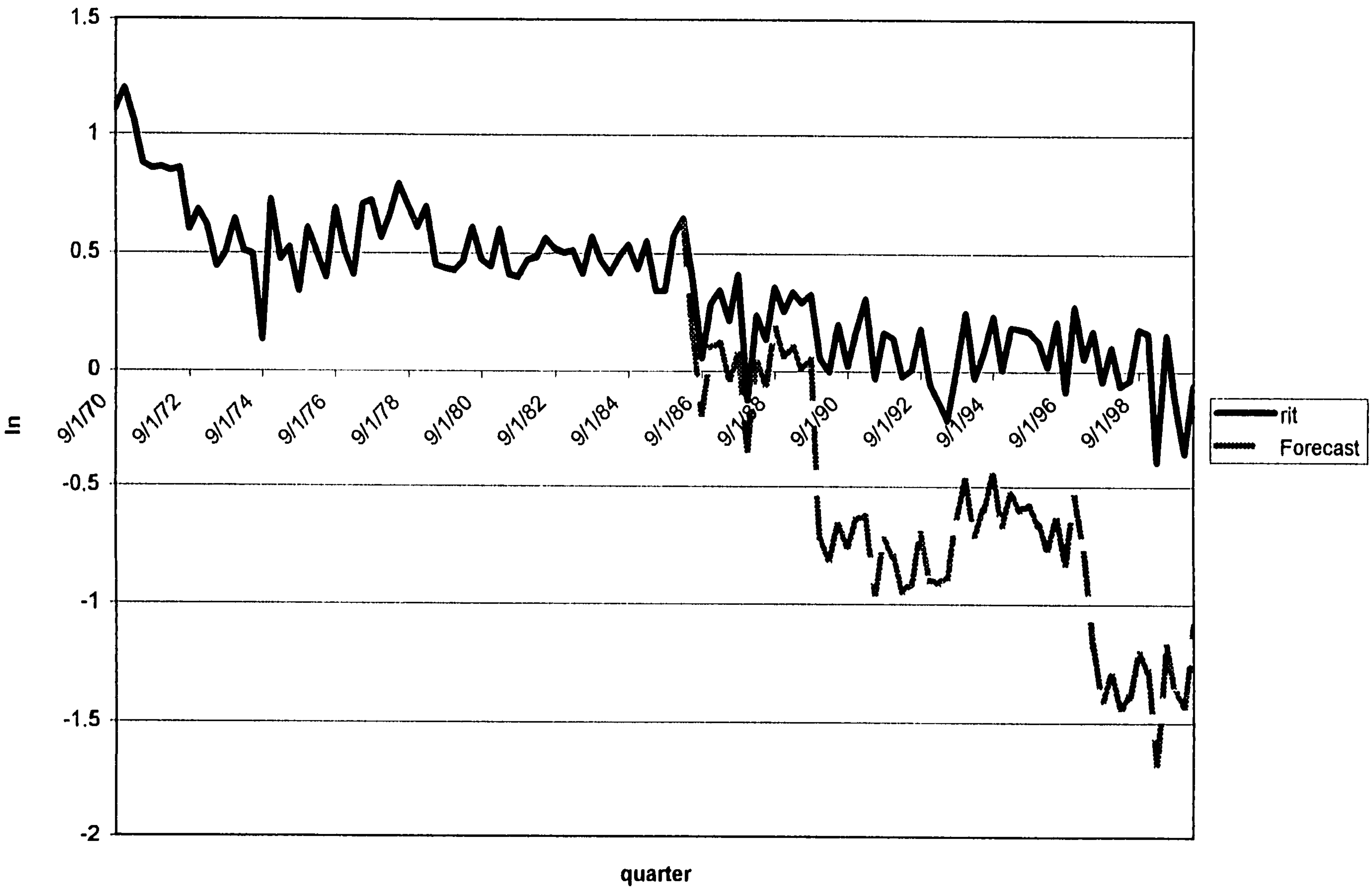
Kodak Forecast Mapping

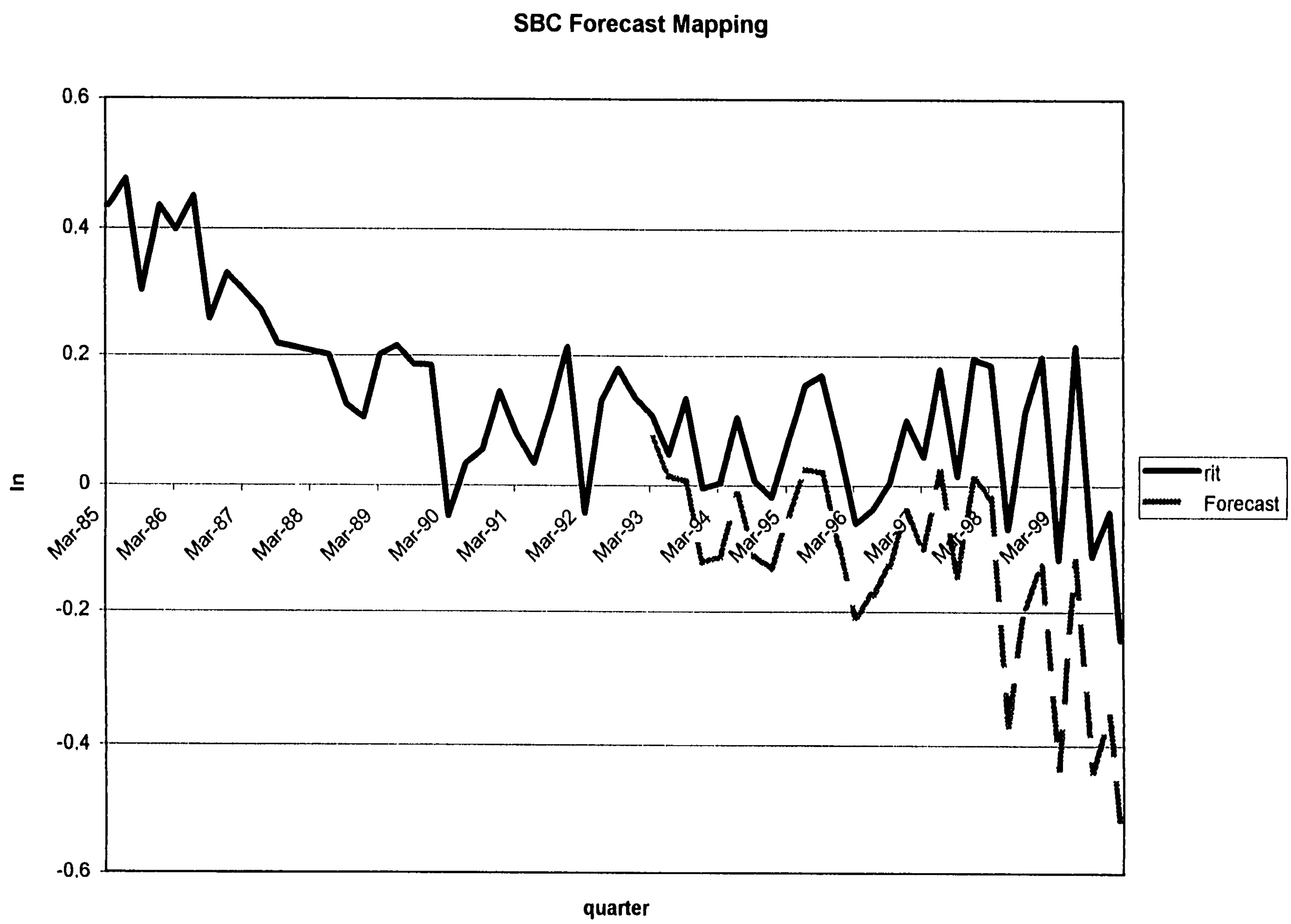


3M Forecast Mapping

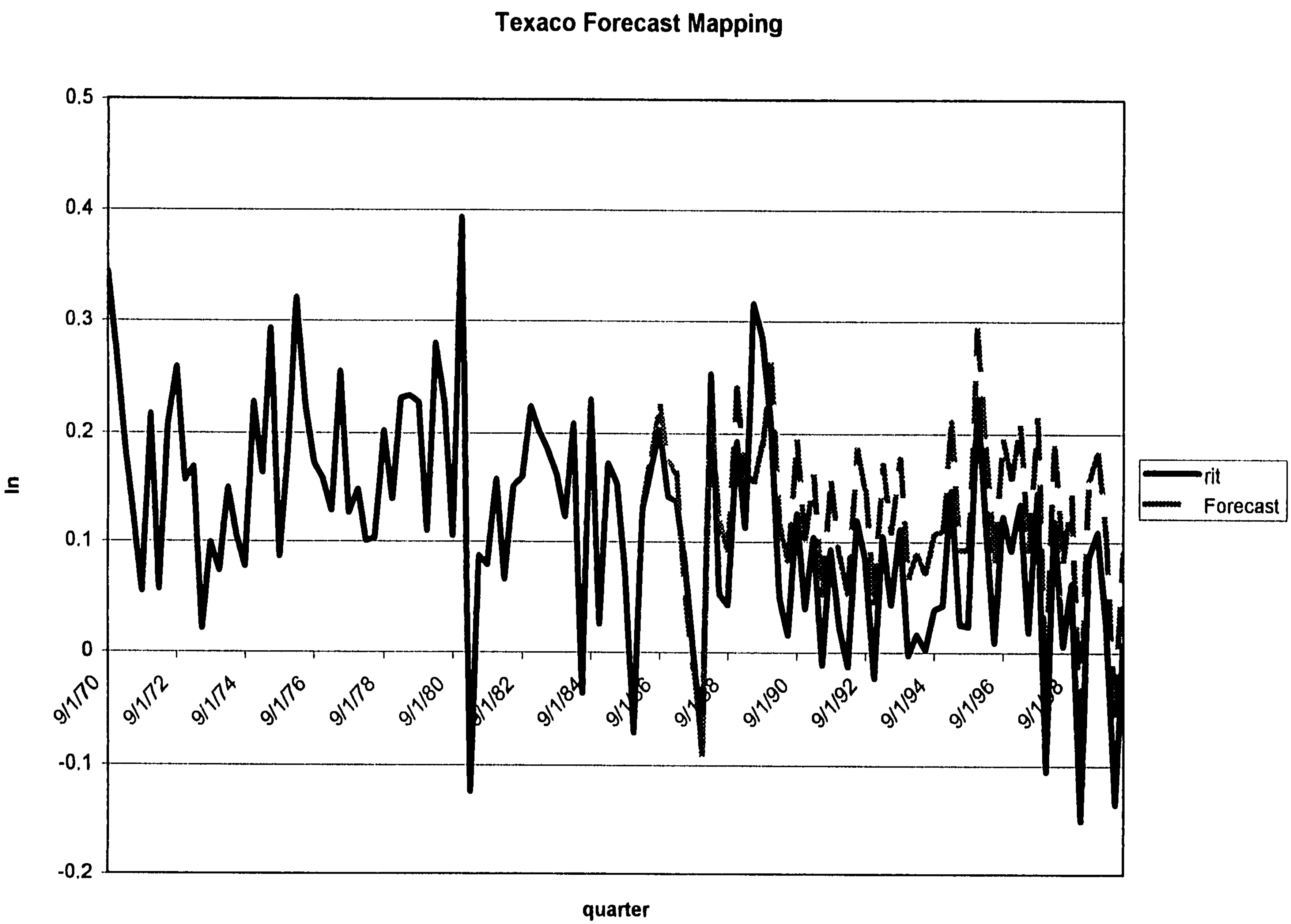


Philip Morris Forecast Mapping



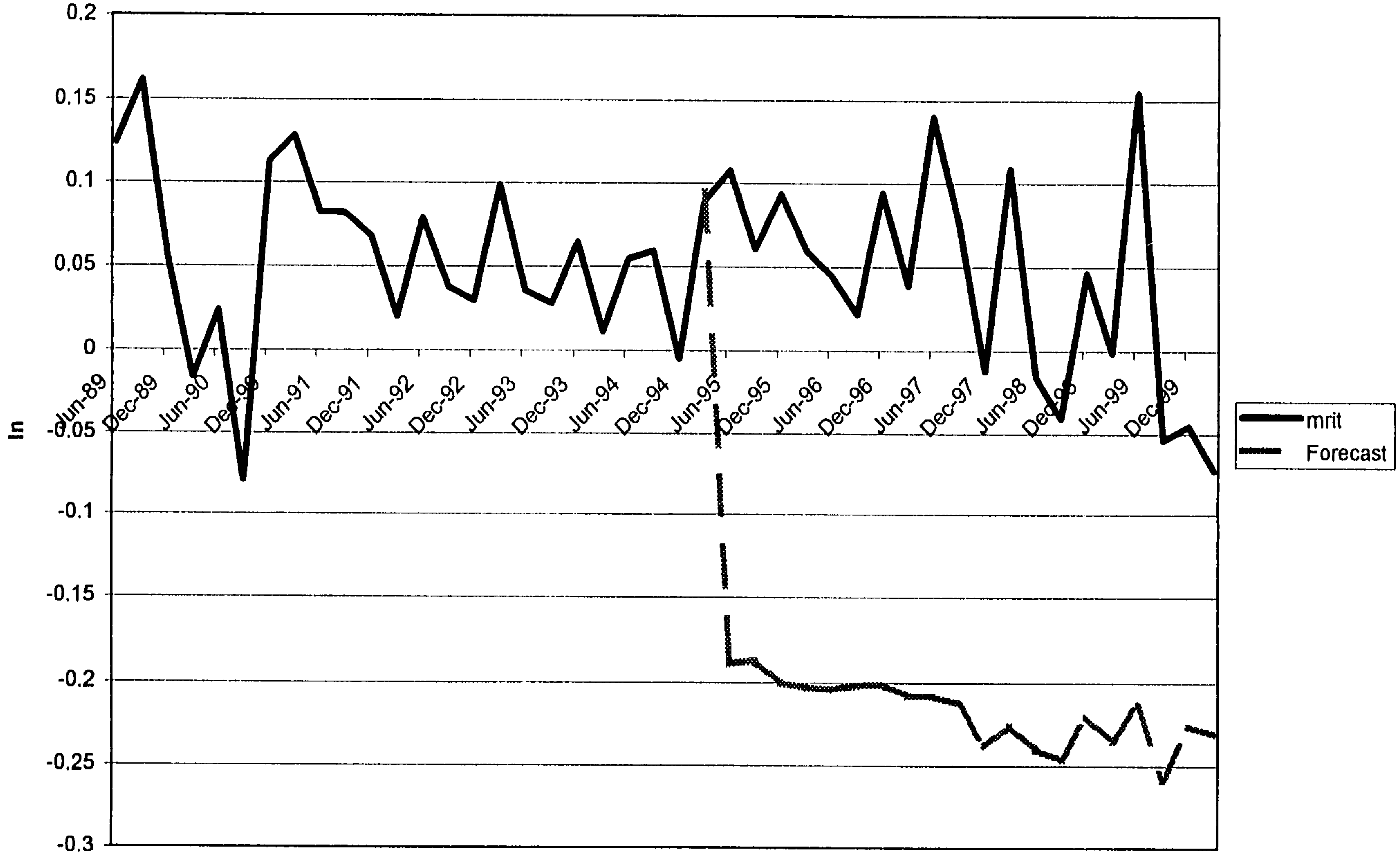


SBC Forecast Mapping
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
ln
Mar-85
Mar-86
Mar-87
Mar-88
Mar-89
Mar-90
Mar-91
Mar-92
Mar-93
Mar-94
Mar-95
Mar-96
Mar-97
Mar-98
Mar-99
quarter
rit
Forecast



Texaco Forecast Mapping
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
ln
9/1/70
9/1/72
9/1/74
9/1/76
9/1/78
9/1/80
9/1/84
9/1/90
9/1/92
9/1/94
9/1/96
quarter
rit
Forecast

Portfolio Mean Forecast Mapping



ABSTRACT

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M.A. French Linguistics, Hankuk University of Foreign Studies

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Hyperbolic Utility Consumption Capital Asset Pricing Model with Time Variation

Dissertation directed by Hrishikesh Vinod, Ph.D.

In the past the studies done about consumption and the financial market generally produced results that would only work to discount the credibility of consumption-based asset pricing models. This study was motivated by the idea that a model based on hyperbolic utility risk aversion mechanism might be an alternative solution to close this gap between consumption and actual financial market. Accordingly, this study showed that Hyperbolic Utility CCAPM with time-varying technique was a better model than Vector Autoregression model of Campbell & Shiller at least for the scope of this study.

The model also reinstated relevance of risk aversion & utility function in constructing an asset-pricing model. It also showed that Hyperbolic Absolute Risk Aversion offers a more realistic explanation why consumption tends to be smooth vis-à-vis the market. Even if

HARA consumption may not completely track income, the model still closely tracked and forecasted the returns path.

On the other hand, the traditional risk aversion mechanism of Constant Relative Risk Aversion was shown to be unable to explain progressively smoothing consumption, because CRRA assumes proportional changes in consumption in response to changes in relative income. From this, we may infer that there may exist a point or a separating equilibrium where $MPC_{HARA}=MPC_{CRRA}$ past which $MPC_{HARA}<MPC_{CRRA}$ somewhere on the consumption curve. Therefore, it answers at least the puzzle why consumption is too smooth, and partially the puzzle why risk premium is too high to be explained by the smooth consumption.

This dissertation also shows how and why consumption may drop out in steady state and/or under Lucas Tree model. The model also outperformed a simple Data Generating Process (ARMA) in an out-of-sample testing. The hypothetical portfolio consisting of assets selected by the criteria that the model suggests also fared well vis-à-vis the market, supporting the practical value of Model.

VITA

Seung-Mo Jeff Hong, son of Soon-Hwa Hong and Yon-Gu Shim, was born on July 19, 1961 in Seoul, South Korea. After receiving BA in French from Hankuk University of Foreign Studies, Seoul in 1984, he worked for a South Korean conglomerate for 2 years mainly on international infrastructure and power plant projects. He also received MA in French Linguisitcs from his *alma mater* in 1988 with a thesis in Chomskian analysis. During the Seoul Olympics in 1988, he worked as a member of editorial staff for The Seoul Olympian, the official newsletter of the Seoul Olympics. From 1989 through 1992, he worked for Electronics & Telecommunications Research Institute, a government research arm of high-tech R&D, on international joint projects management and telecommunications policy research.

Jeff Hong enrolled in the graduate program in International Studies at University of Denver in January 1993 and received MA with a field in International Economics and a concentration in Technology Assessment in November 1994. He briefly worked in foreign exchange trading in 1995 in Seoul before he returned to the U.S. In 1996 he entered the graduate program in Economics at Fordham University and was awarded Presidential Scholarship, Graduate Assistantship and Teaching Fellowship. He has been teaching at Fordham as an Adjunct Faculty since 1999.